Averaged Equations for Sheared Liquid-Solid Suspensions Influenced by Both Inertia and Viscosity

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The rheology of particle suspensions in very viscous fluids has been studied extensively both experimentally and computationally and it is known that the particles enhance the viscosity of the suspension and give rise to normal stresses whose magnitude can be predicted as a function of volume fraction. At the opposite extreme, Bagnold's experiment suggests that large particles at high volume fractions can be described by granular flow theories even when they are suspended in a liquid. Although the latter result is still somewhat controversial, I believe the greatest uncertainty in modeling the behavior of sheared liquid-solid suspension comes at intermediate particle Reynolds numbers (based on particle size and shear rate) and at lower particle volume fractions. In such situations, one can neglect neither the particle and fluid inertia nor the viscous stresses and there is a complex interplay between the disturbance fields produced by the particles and the imposed flow. The microstructure of the particles can vary within this regime from an approximate hard-sphere distribution at high particle inertia to a structure with clusters of particles separated by small lubrication gaps and this structure couples with the rheology in a non-trivial way. Furthermore, at finite Reynolds numbers, it is important to understand the coupling between the shearing motion and the mean relative velocity of the two phases that arises from the nonlinearity of the Navier-Stokes equations. I am interested in investigating these issues using lattice-Boltzmann simulations of particle suspensions. We would extract from the simulations not only the overall average properties of the suspension but also measures of the microstructure and the individual (viscous, Reynolds, particle kinetic, and particle collisional) components of the stress. This detailed information can form the basis of physically based models of the averaged properties of liquid-solid suspensions.

Can averaged equations of motion accurately predict the fluctuations in an unstable suspension?

It is clearly not feasible to perform direct numerical simulations of the many hydrodynamically interacting particles or bubbles constituting a typical engineering scale multiphase flow. It is desirable instead to solve averaged equations of motion describing the multiphase flow. Since simple homogeneous flows of multiphase systems are typically subject to instabilities however, even a detailed solution of the averaged equations of motion on the scale of an industrial application may be prohibitively expensive. As a result, Sundaresan has proposed that we should develop subgrid models for an approximate engineering scale simulation by extracting information from smaller scale flows from simulations of the multiphase flow equations. To assure that such an approach is well founded, it is important to establish that the solutions of the averaged equations of motion actually produce an accurate representation of the particle concentration and particle and fluid velocity fluctuations in an unstable suspension and the effect of these fluctuations on the mean properties of the system. To establish this, I would be interested in performing many particle (or bubble) simulations using the lattice Boltzmann method for unstable systems such as liquid-solid and gas-solid fluidized beds and suspensions of rising bubbles and comparing the results with solutions of the averaged equations which will be developed by Ashok Sangani.