Improvements to the Eulerian/Lagrangian Multiphase Coupling Algorithm for Polyhedral/Polygonal Control Volume Mesh Topologies

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Outline

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Algorithm Review
Limitations
SIMPLE Algorithm
Interpolation
Force Mapping
Industrial Application
Summary

- Eulerian/Lagrangian multiphase algorithm
  - Fundamental perspective
  - Numerical stability
  - Emerging mesh topology technology
- General motivation
- Review general Eulerian/Lagrangian algorithm
- Discuss some limitations to the current algorithm
- Propose three improvements to the algorithm
- Discuss each improvement in isolation
- Review an industrial application with improvements

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- Worked with the Eulerian/Lagrangian method in the mid 1990's
  - Cartesian mesh topologies
  - Staggered velocity storage/solution techniques
  - Bilinear interpolation methods
- Early 2000 began working with commercial CFD software
  - Unstructured mesh topologies
  - Colocated velocity storage/solution techniques
  - Gradient projection based interpolation
- Experienced solution instability
- Began investigating source of instability
- Pursued possible remedies

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Eulerian/Lagrangian algorithm

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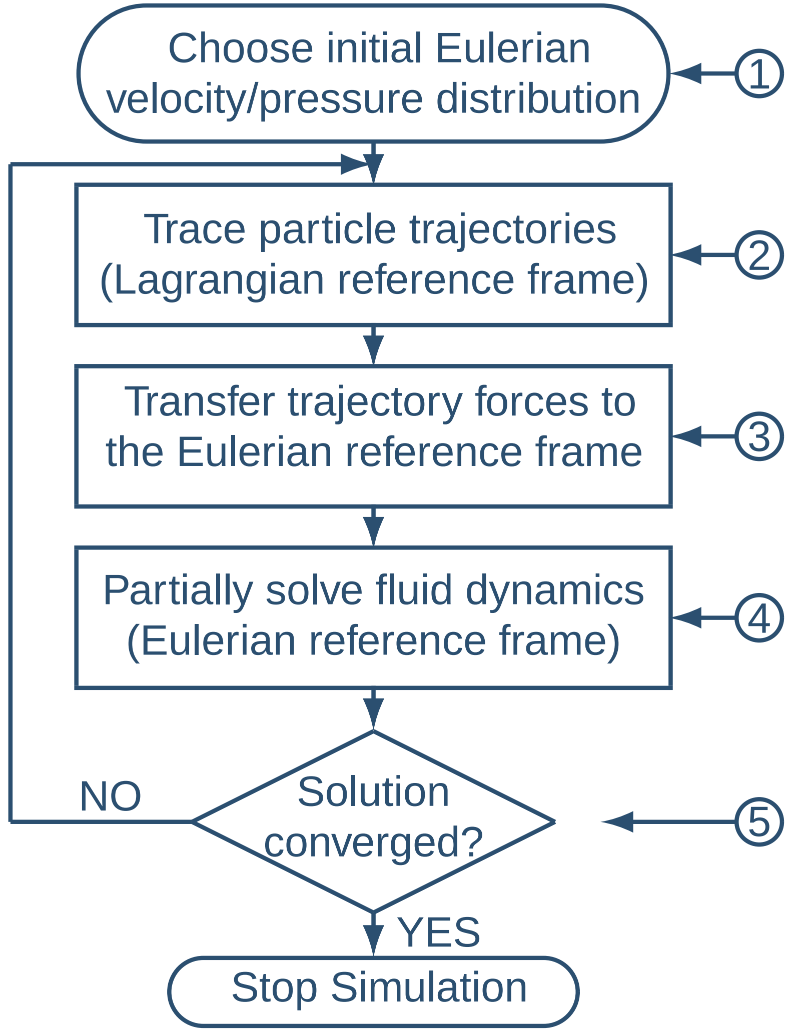

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Eulerian Reference Frame

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- Need coupled momentum/continuity solution
- Lagrangian coupling forces are non-isotropic (Forces vary for each coordinate direction)
- Want linearized forces to improve stability
  - Traditional Cartesian SIMPLE algorithm
    - Staggered velocity storage
    - Allow non-isotropic linearized sources
    - Requires multiple mesh topologies
  - Colocated SIMPLE algorithm
    - Colocated velocity storage
    - Limited to isotropic linearized source terms
- Lagrangian source term distribution
  - Observe unnecessary discontinuities

Lagrangian Reference Frame

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- Compute particle trajectory paths and forces
- Use ODE solvers to compute trajectories
- Interpolated data required from Eulerian reference
- Poor quality interpolation data
  - Confuse ODE solvers
  - Forces not implied by the fluid mechanics may result

Target Limitations

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- Colocated SIMPLE algorithm does not allow linearized momentum source terms
  - Propose new SIMPLE algorithm which is multiphase aware
- Interpolation from Eulerian to Lagrangian reference frames hampers ODE solvers
  - Propose improved interpolation method applicable to all mesh topologies
- Lagrangian source terms mapped to the Eulerian reference frame can be unnecessarily discontinuous
  - Propose ”Influence/Dependence” for mapping source terms

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SIMPLE Algorithm (Eulerian Reference)

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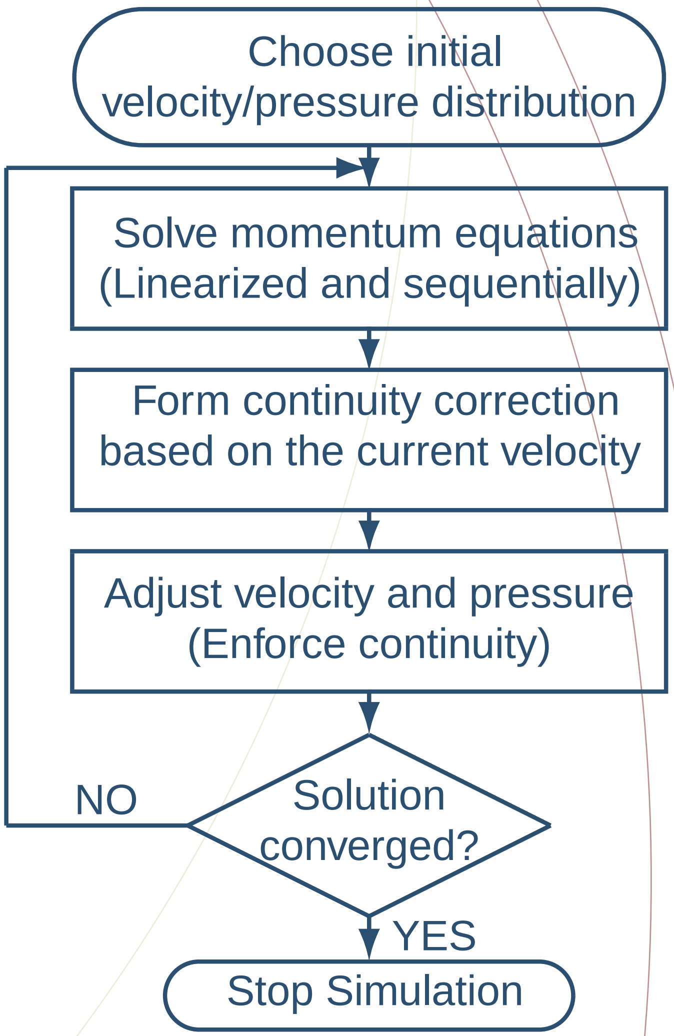
- SIMPLE algorithm review
- Previous derivations developed for fluid only applications
- Linearized non-isotropic forces not considered in algorithm
- New SIMPLE method freshly derived
- Resulting algorithm is multiphase aware
- Improved stability for multi-physic applications

SIMPLE Algorithm



Stability Test Case

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- Propose a particle driven cavity to evaluate stability
- Compare the SIMPLE algorithms
- Standard SIMPLE Algorithm
  - Requires isotropic implicit terms
  - Assume average of non-isotropic implicit terms
- Proposed SIMPLE algorithm
  - Retain the non-isotropic implicit terms
  - Base the continuity correction on the non-isotropic aware correction algorithm

Particle Driven Cavity

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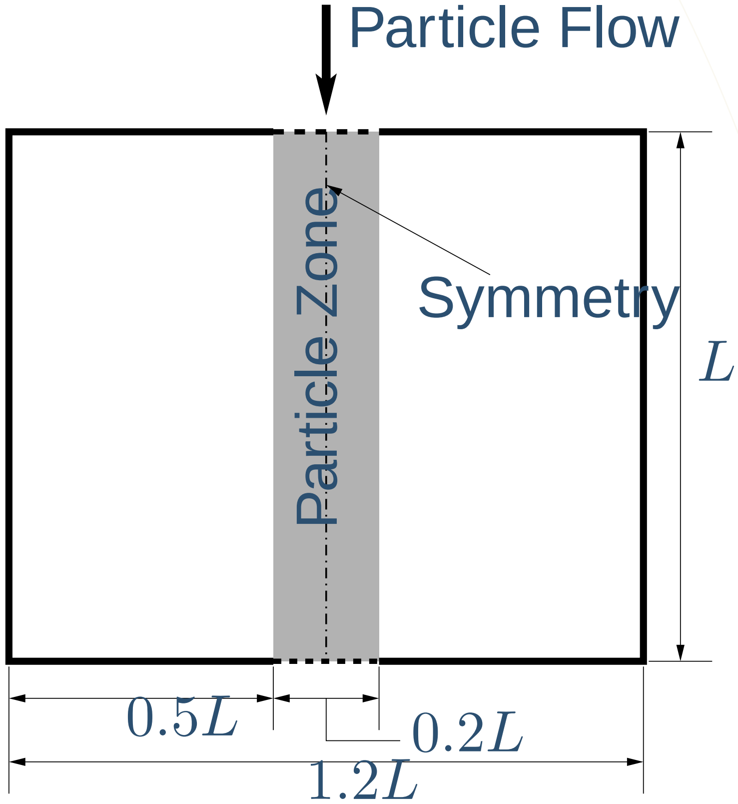
- Test case setup
- Vary particle loading
- Observe residuals
- Four cases considered

Particle zone source terms

Case #	$F^y_{explicit}$	$F^y_{implicit}$
1	50	-50
2	100	-100
3	150	-150
4	200	-200



Test Case Geometry
($\rho = 1$ and $\mu = 0.001$)

Case 1 Residuals

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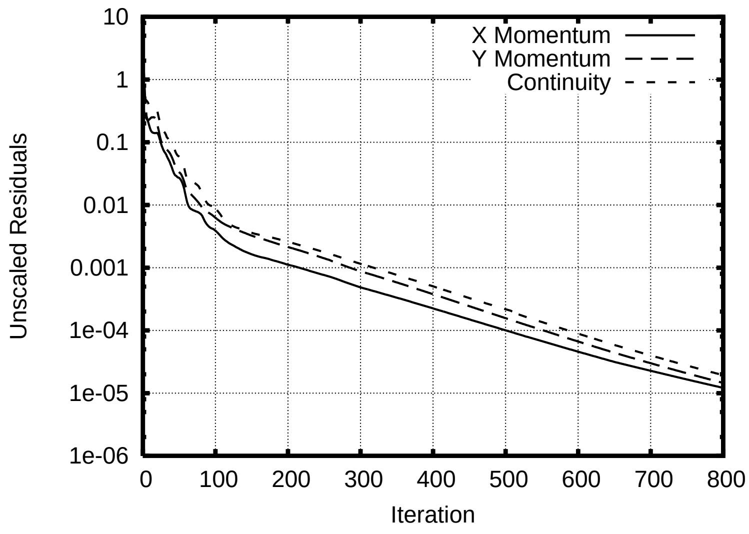
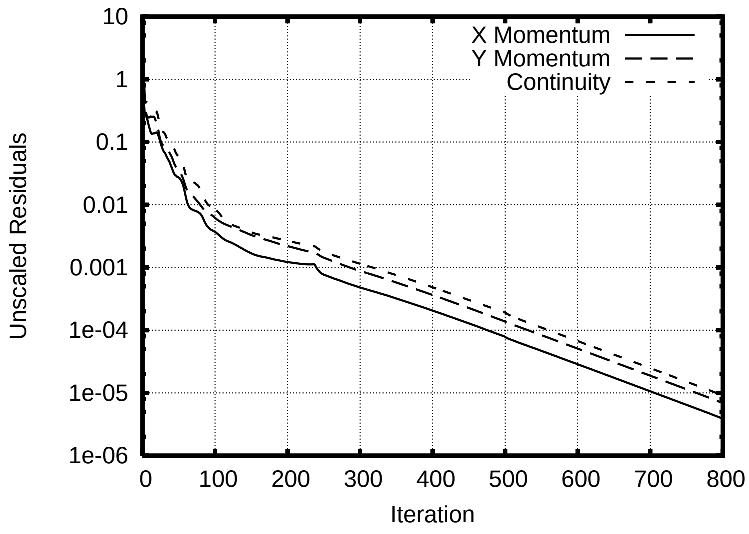


(a) implicit terms (averaging)



(b) implicit terms (proposed)

Case 2 Residuals

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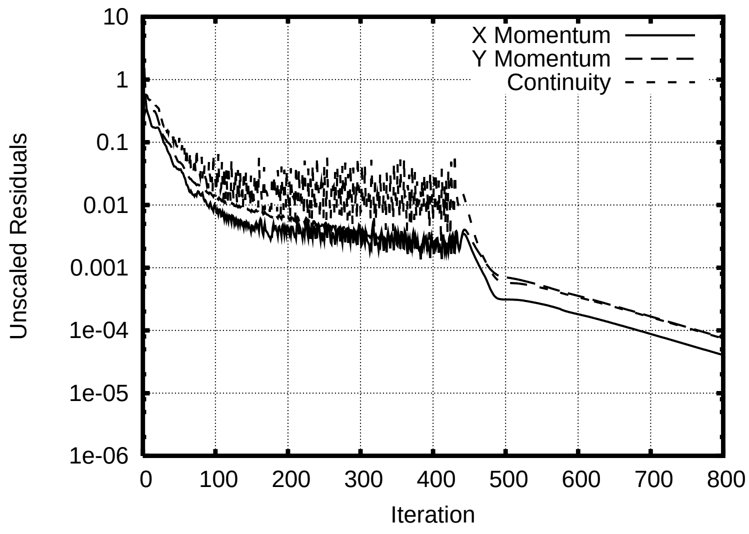
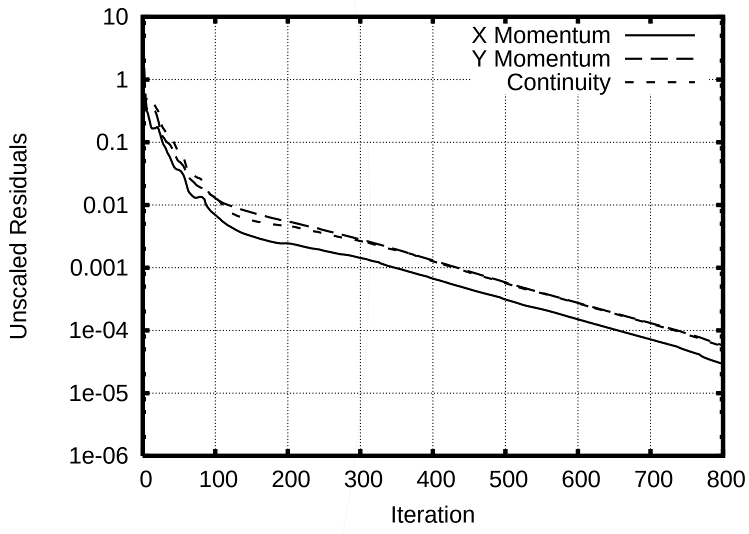


(c) implicit terms (averaging)



(d) implicit terms (proposed)

Case 3 Residuals

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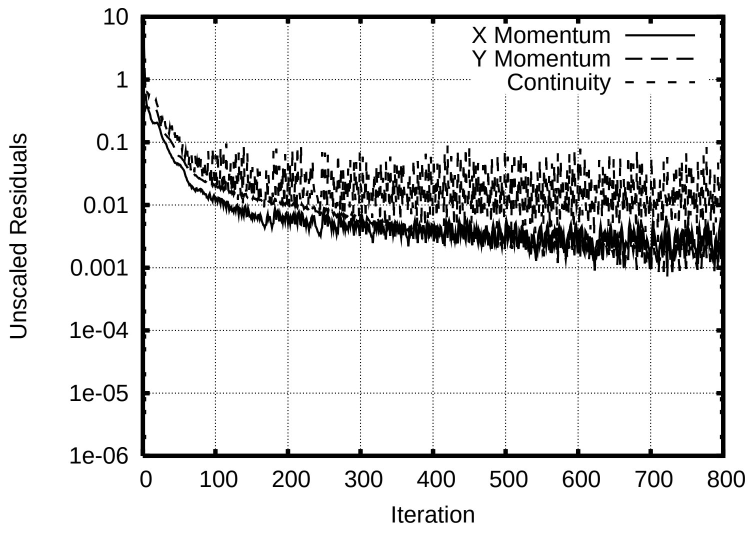
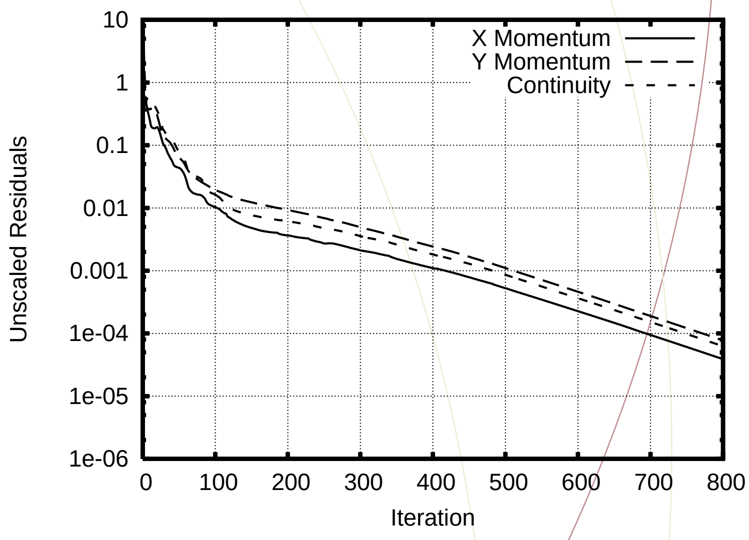


(e) implicit terms (averaging)



(f) implicit terms (proposed)

Case 4 Residuals

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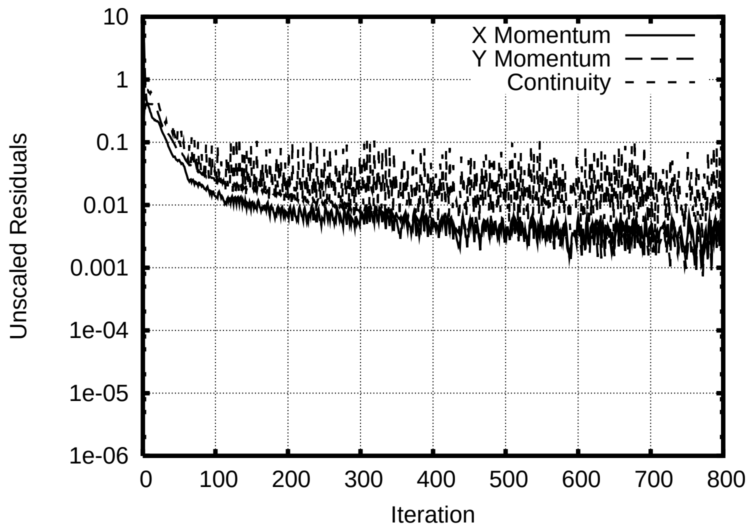
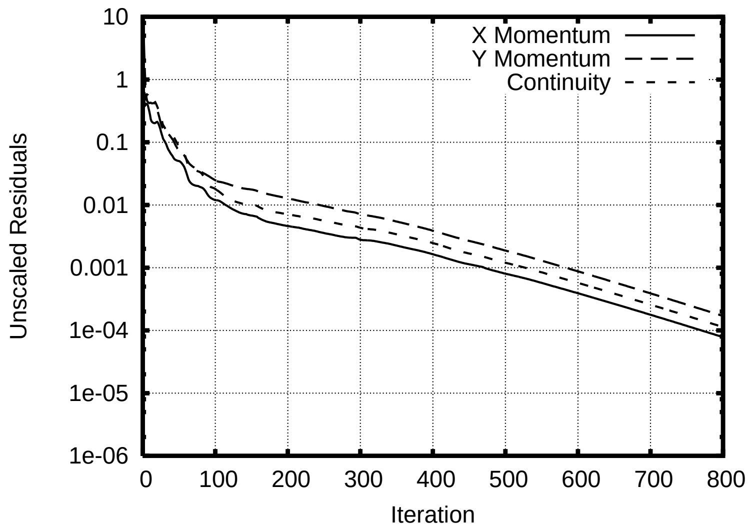


(g) implicit terms (averaging)



(h) implicit terms (proposed)

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Provide high quality data to Lagrangian integration

Important factors to consider

- Continuous variation of interpolated data
- Linear approximation between data points
- Applicable to polyhedral control volume data
- Mimic behavior of control volume discretization method
- Low operation count
- Interpolation method should be dependent on the location of the data
- Reproduce the discrete data

Proposed interpolation method

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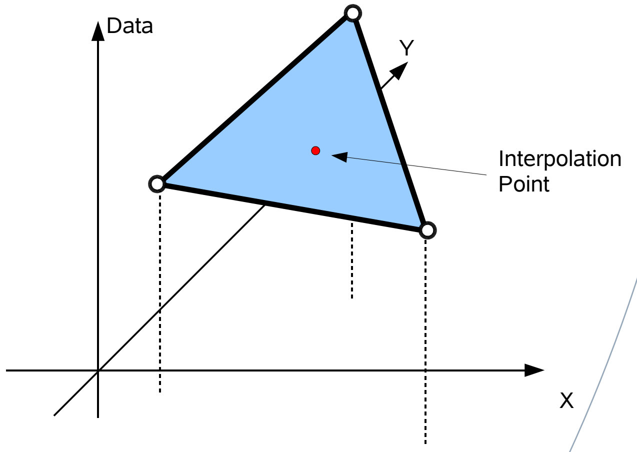
- Preprocessing stage
  - Perform a face based decomposition of the control volume mesh dividing volume into triangles/tetrahedra
  - Generate linear approximate weighting values for control volume vertex data
- Interpolation stage
  - Perform interpolation using the triangular/tetrahedron primatives using vertex data



Interpolation test case

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- Evaluate proposed interpolation method
- Compare to the frequently used gradient projection technique
  - Interpolation is based on a projection of the Eulerian gradient
- Consider a velocity field with an anomaly
- Calculate a particle trajectory through the field using both methods
- Observe the differences between the two trajectory paths

Sample trajectory calculation

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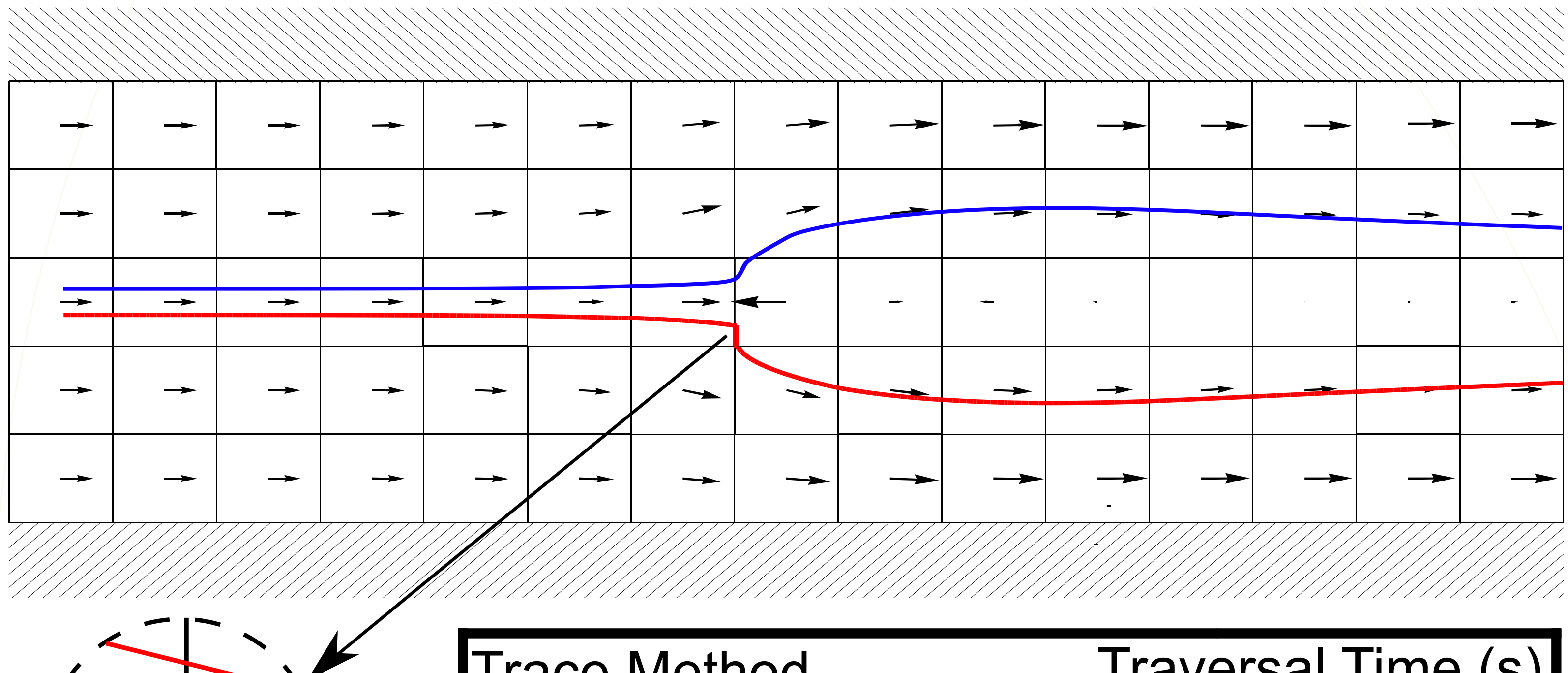

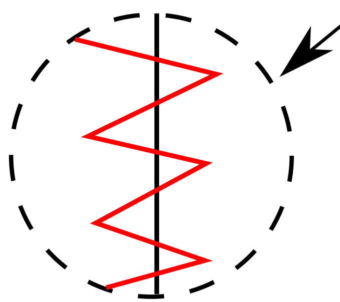

Trace Method	Traversal Time (s)
Nominal Fluid Particle	0.56
Lagrangian Calculations (Proposed Interpolation)	0.77
Lagrangian Calculations (Gradient Projection)	4.64

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The Lagrangian particle calculations provide forces to the Eulerian reference frame

Primary mapping objectives

- Smooth distribution of forces
  (Help the Eulerian Solution Process)
- Consistent with interpolation method
- Linearized and retaining its non-isotropic nature

Proposed Mapping Method

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Propose Influence/Dependence perspective for mapping

- Particle path depends on the Eulerian data (Interpolation)
- Particle forces should influence the same data (Force mapping)
- Use a mapping method consistent with interpolation technique
- Use interpolation weights to distribute forces
- This method provides an intelligent smoothing process

Mapping Test Case

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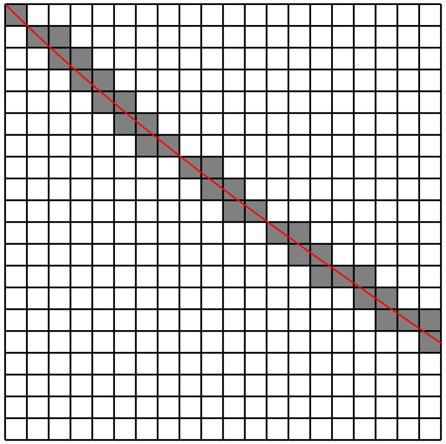
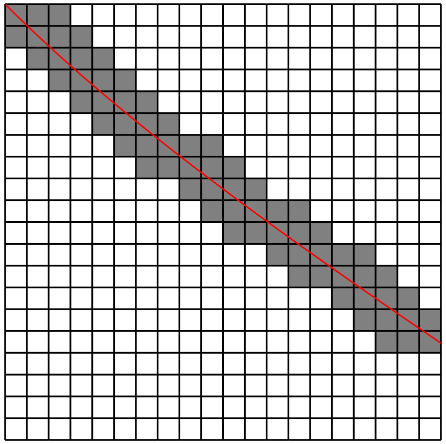
- Evaluate proposed mapping method
- Compare to the frequently used injection method



(i) Injection



(j) Proposed

Note shaded cells indicate influence from trajectory

- Consider a particle transport case (Vary particle mass loading)
- Observe solution residuals for both methods

Simple coupling case

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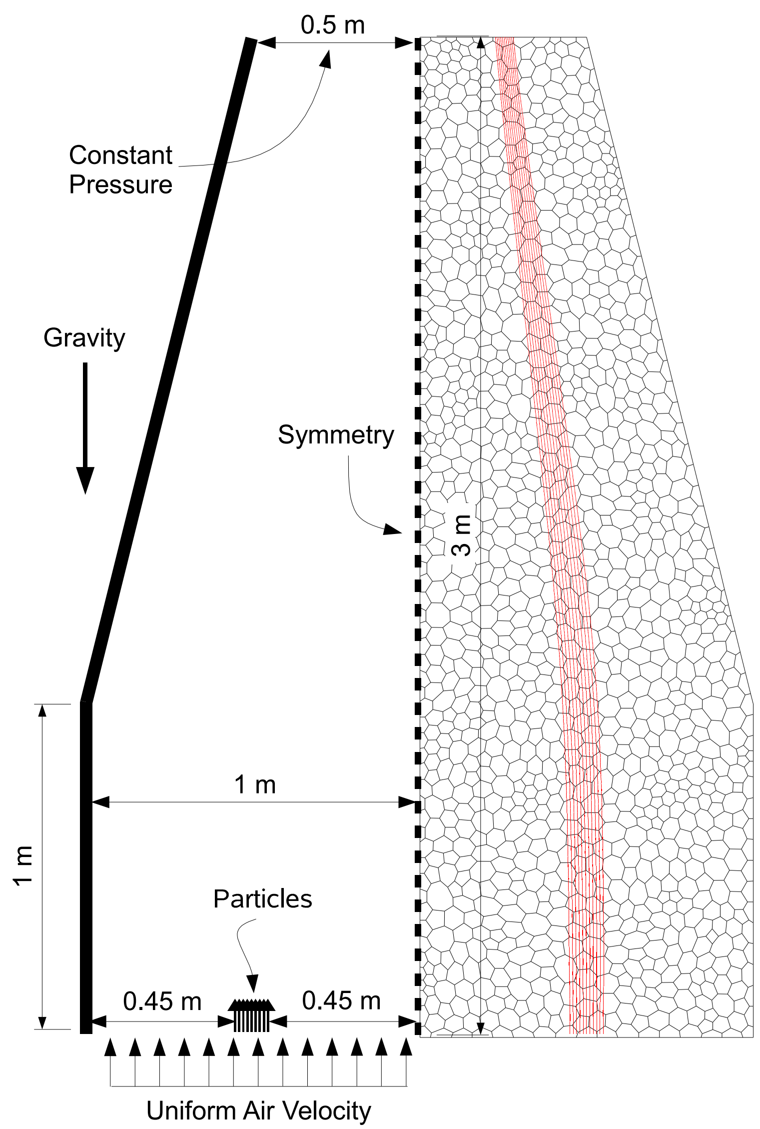

Residuals for 0.2 kg/s case

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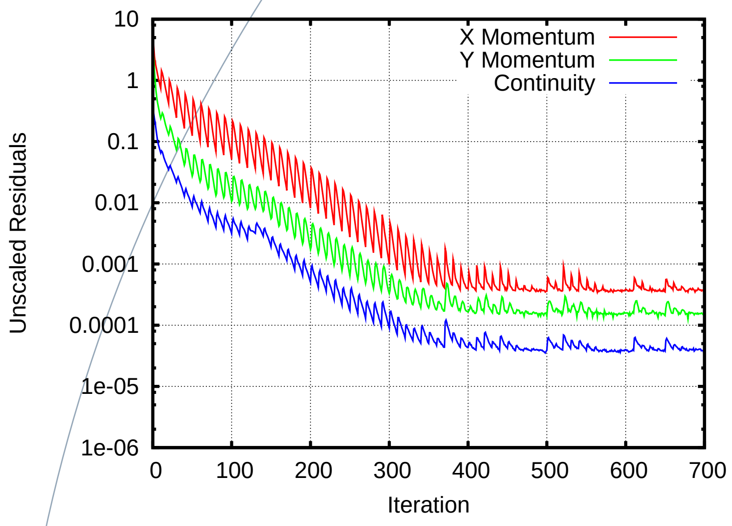
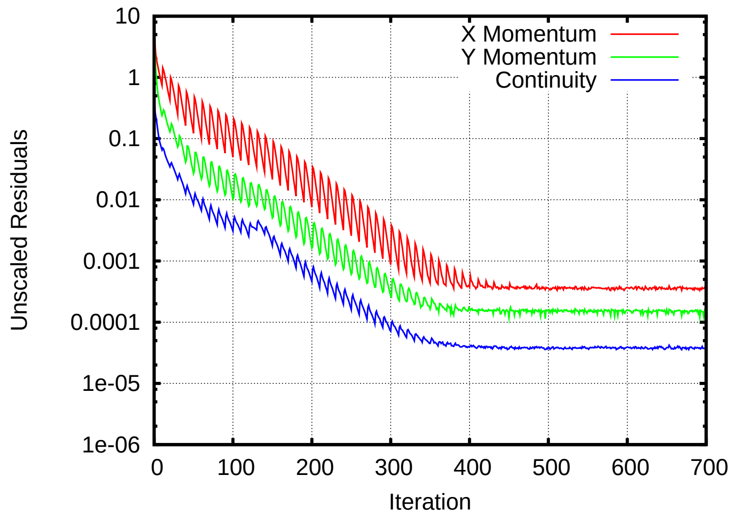


(k) Injection Method



(l) Proposed method

Residuals for 0.5 kg/s case

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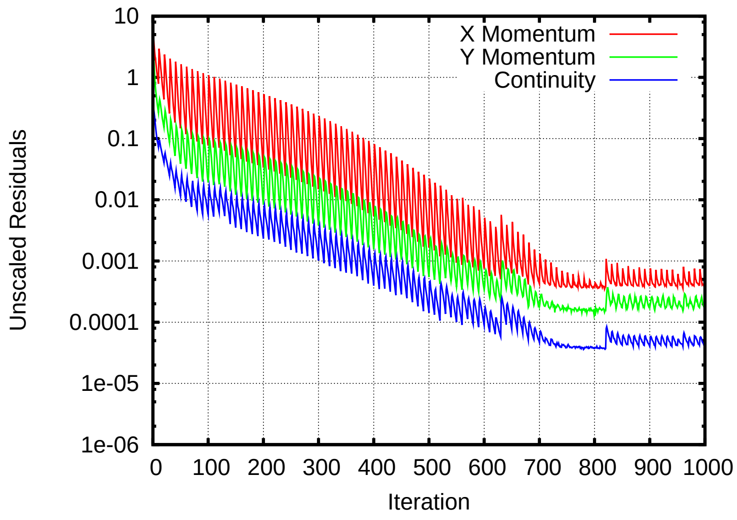
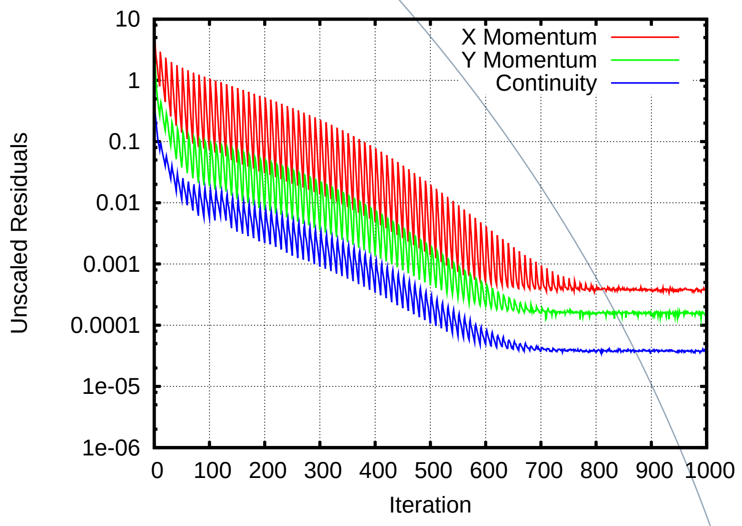


(m) Injection Method



(n) Proposed method

Residuals for 1.0 kg/s case

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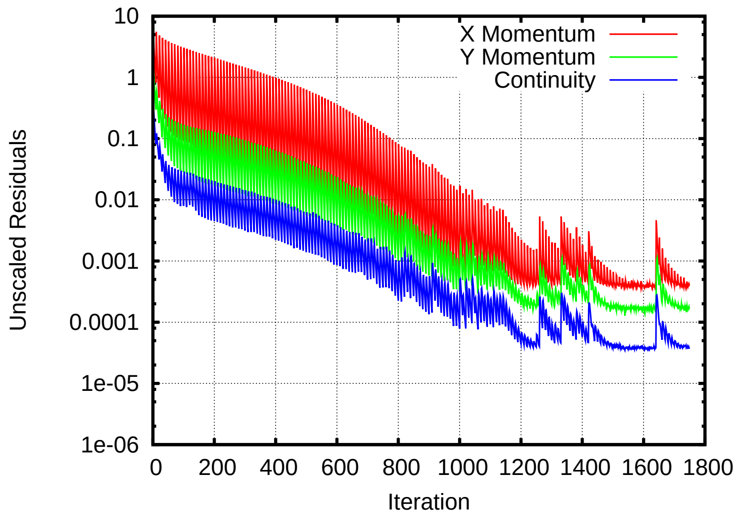
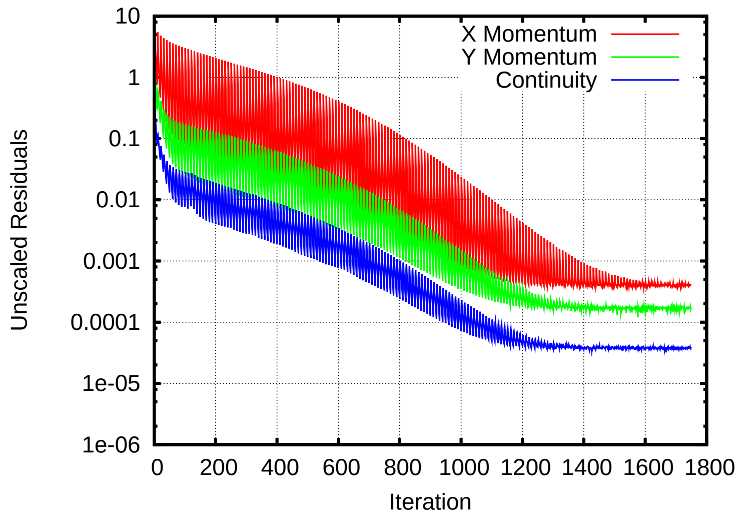


(o) Injection Method



(p) Proposed method

Residuals for 1.5 kg/s case

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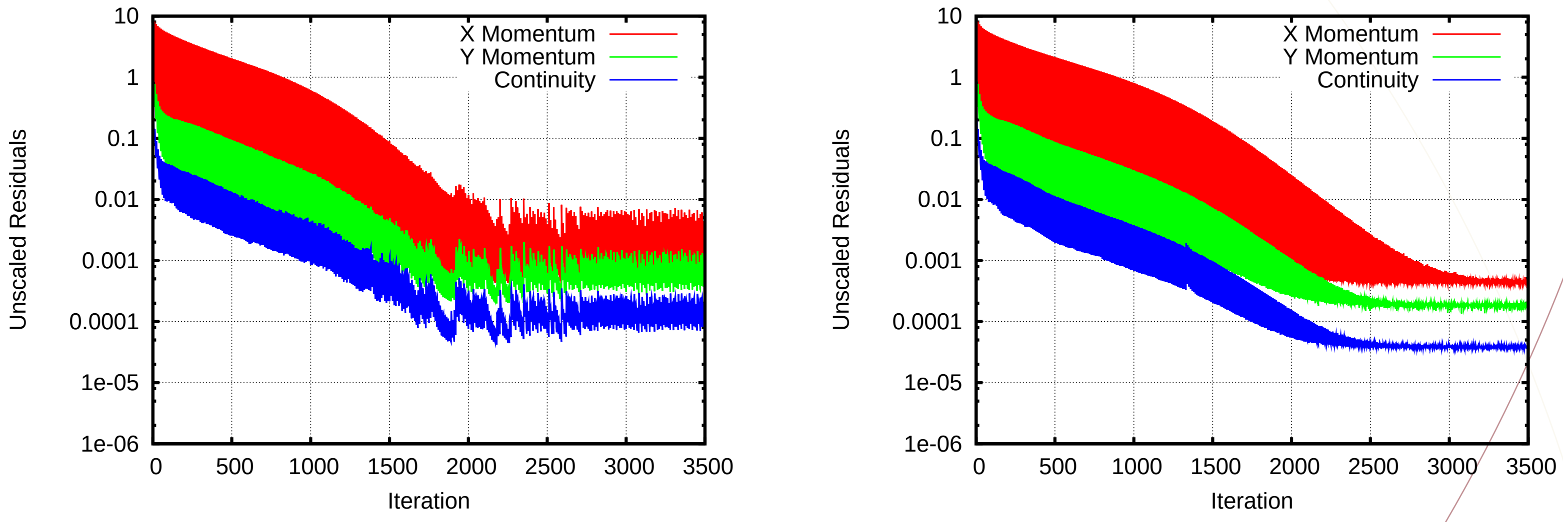

(q) Injection Method

(r) Proposed method

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❖ Eulerian Residuals 1/100 Particle Loading

❖ Eulerian Residuals 1/10 Particle Loading

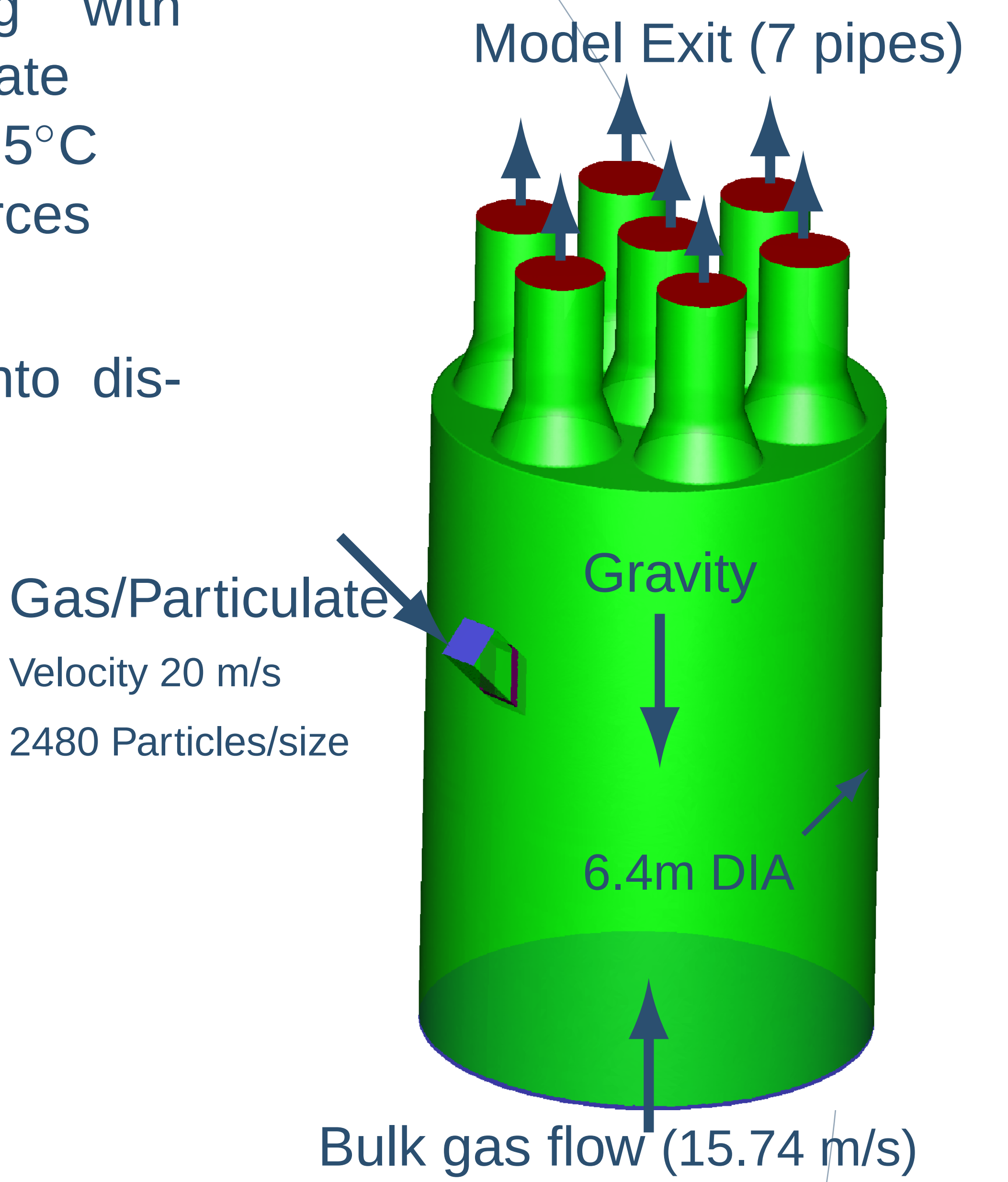
❖ Eulerian Residuals Full Particle Loading

❖ Comprehensive application summary

Summary

- Bulk gas mixing with dense gas/particulate
- Bulk gas - air at 175°C
- Strong coupling forces
  - Chute inlet
  - Acceleration into distribution pipes

Size μm	mdot (kg/s)
1.14	6.9
2.66	6.9
4.05	6.9
5.80	6.9
8.66	6.9
15.16	6.9
27.19	6.9
42.24	6.9
64.86	6.9
127.37	6.9
Sum	69.



Industrial Polyhedral Mesh

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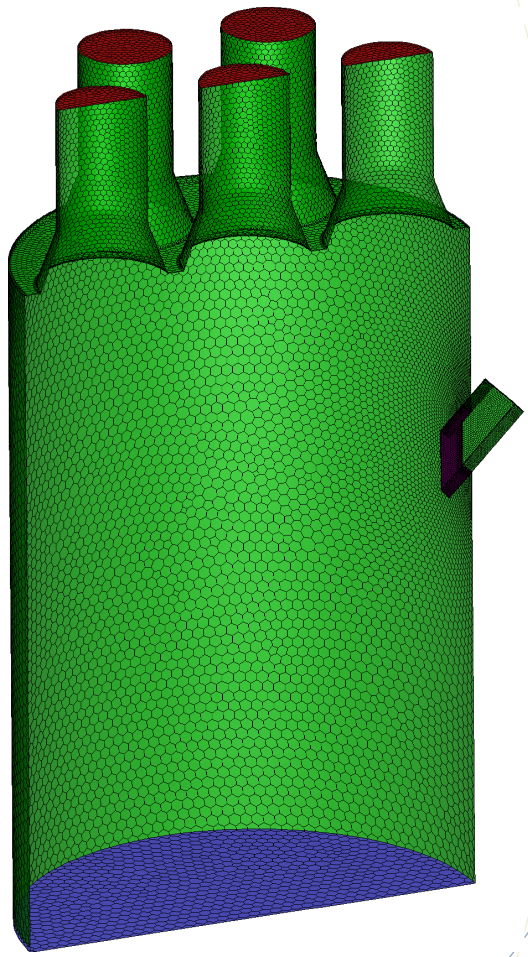
❖ Eulerian Residuals 1/10 Particle Loading

❖ Eulerian Residuals Full Particle Loading

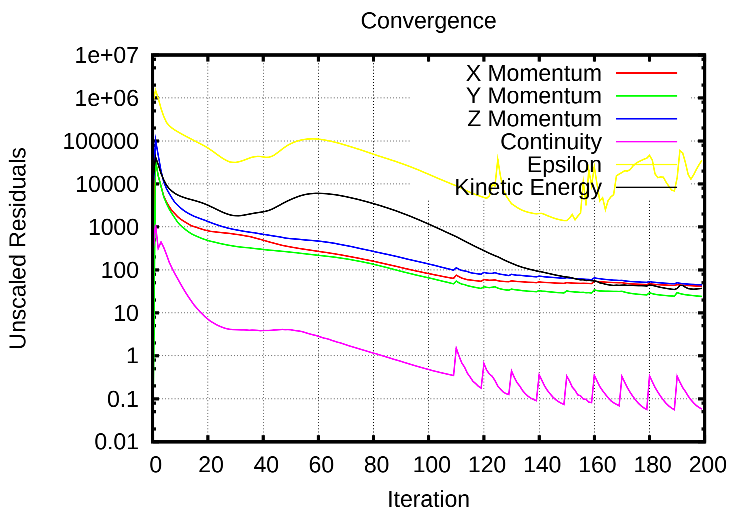
❖ Comprehensive application summary

Summary

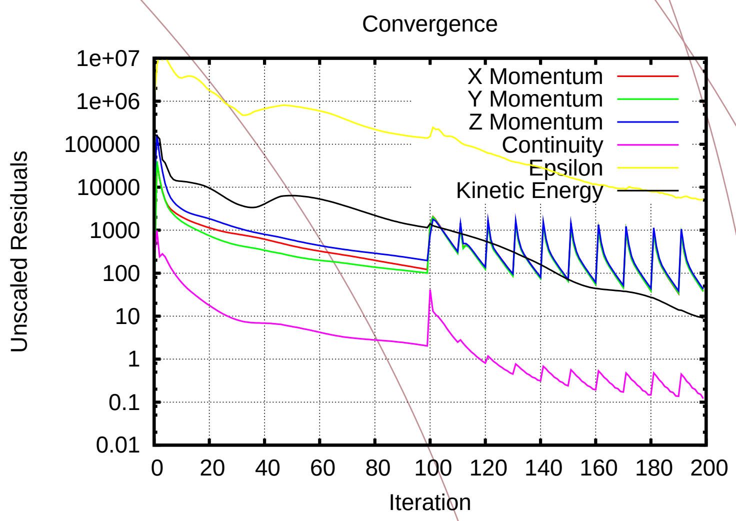
- Reduce model domain
  - Taking advantage of symmetry
- Polyhedral mesh topology
  - 235,537 control volumes
  - 6 to 33 faces per control volume
  - 1,563,705 interior faces
  - 32,362 exterior faces
  - Internal thin walls
- Interpolation sub mesh
  - 8,225,983 tetrahedra



Eulerian Residuals 1/100 Particle Loading



(s) Commercial



(t) Proposed method

- First 100 iterations fluid solution only
- Second 100 iterations particle coupling

Eulerian Residuals 1/10 Particle Loading

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❖ Industrial Polyhedral Mesh

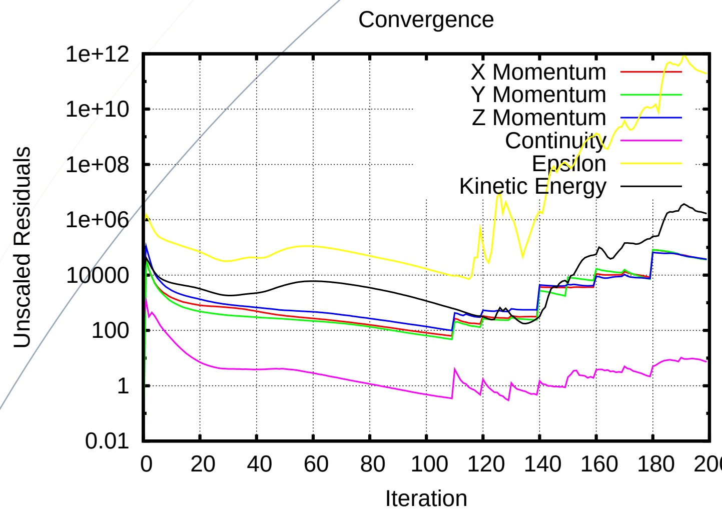
❖ Eulerian Residuals 1/100 Particle Loading

❖ Eulerian Residuals 1/10 Particle Loading

❖ Eulerian Residuals Full Particle Loading

❖ Comprehensive application summary

Summary



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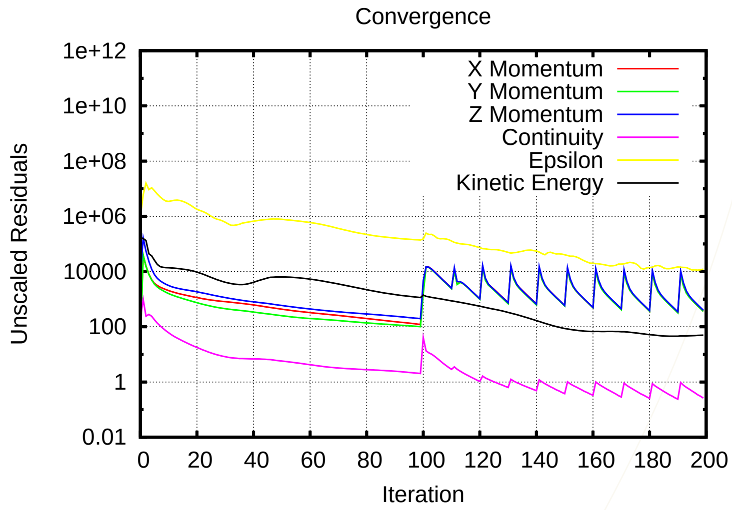

(v) Proposed method

Eulerian Residuals Full Particle Loading

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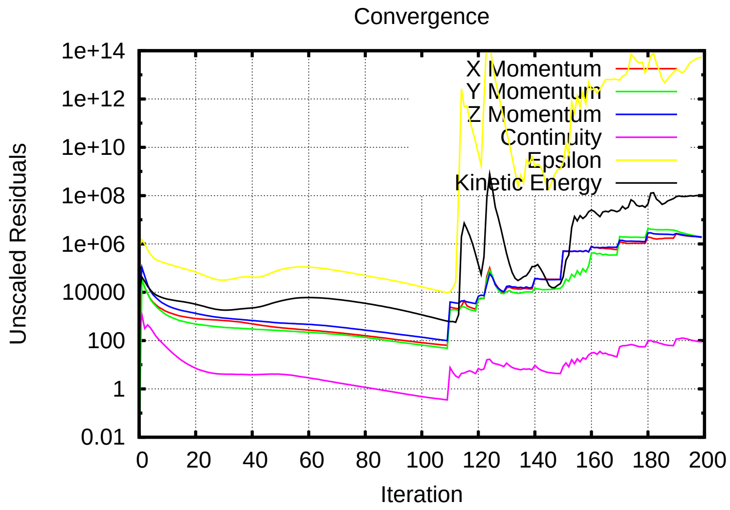
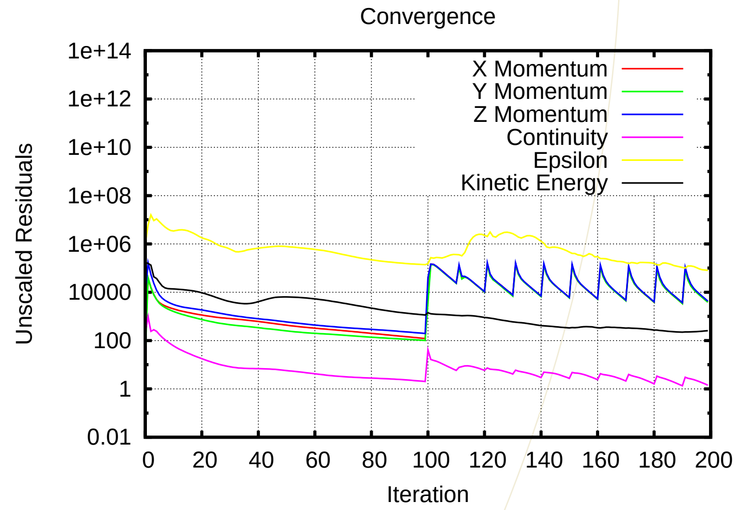
Summary



(w) Commercial



(x) Proposed method

Comprehensive application summary

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❖ Comprehensive application summary

Summary

A full comprehensive application of the algorithm showed the following

- Improved solution stability
- Applicable to polyhedral mesh topologies
- Larger and more complex problem typical for industrial applications
- Able to simulate a situation which previously could not be performed

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- A new foundation for the Lagrangian/Eulerian method has been proposed
- Three limitations have been investigated/improvements have been proposed
- All methods are applicable to polyhedral/polygonal mesh topologies
- Evaluated three limitations in isolation
- Applied the comprehensive algorithm to a more complex industrial problem

Questions