Rheological behavior of dense assemblies of granular materials: Experiments and Simulations

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Research Assistant: Mehrdad Kheiripour Langroudi

DoE Workshop on Granular Matter

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Collaborators

<u>Principal Investigator:</u>

- **Prof. Sankaran Sundaresan (Princeton University) - Simulation**

<u>Co-principal Investigators:</u>

- **Prof. Gabriel I. Tardos (The City College of the City University of New York) - Experiments**
- **Prof. Shankar Subramaniam (Iowa State University) - Simulation**

<u>Goal for Experimentation:</u> Provide precise and detailed experimental results in simple enough geometries to validate simulations.

<u>Papers</u>

1. M. Kheiripour Langroudi, S. Turek, A. Ouazzi and G. I. Tardos, “An investigation of frictional and collisional powder flows using a unified constitutive equation”, Powder Technology, Vol. 119, August, (2009).
2. M. Kheripour-Langrudi, J. Sun, S. Sundaresan and G. I. Tardos, “Transmission of stresses in static and sheared granular beds: the influence of particle size, shearing rate, layer thickness and sensor size”, in press, NETL Special Issue Journal (2010).

Goals

1. Take into account Industrial (DoE) needs
   - Design and operate powder processes using scientific approaches
   - Avoid endless scale-up and substitute by computations
2. Study industrially (DoE) relevant powders
3. Measure and Predict flow-fields, stresses and forces on objects in flowing powders in industrially important geometries.

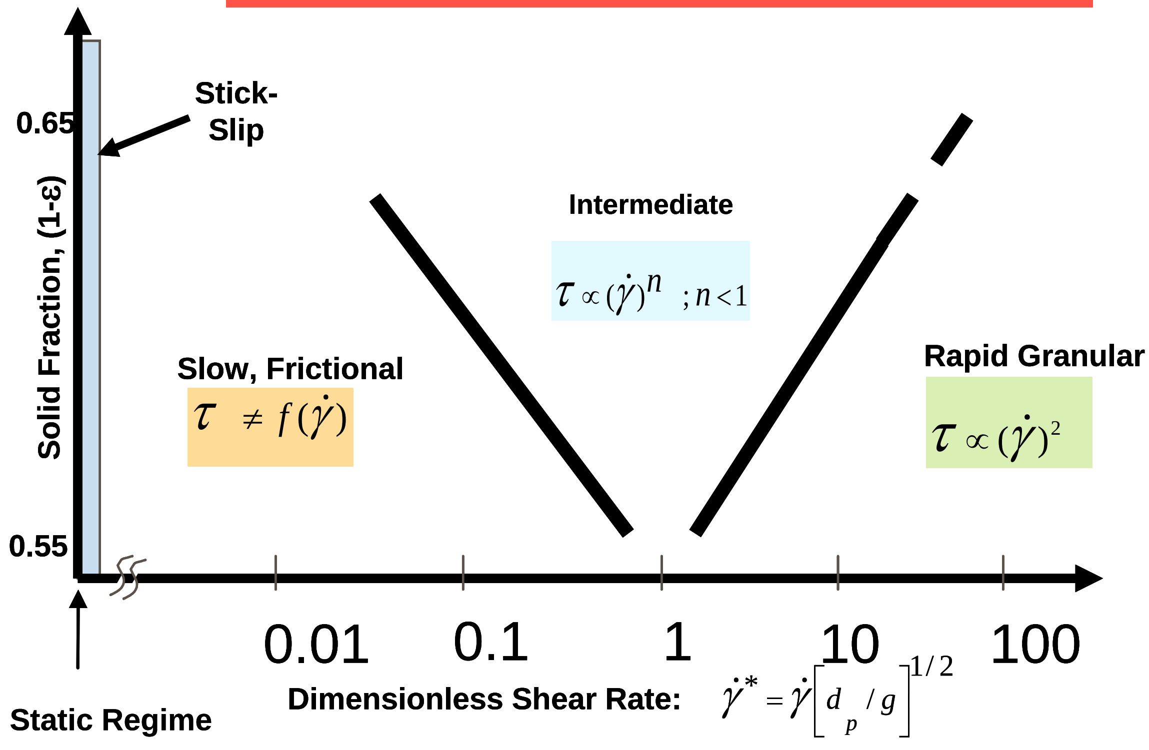
Problems to overcome

1. Measure appropriate powder properties that characterize flows
2. Identify and test instrumentation to measure these characteristics
3. Find and test numerical schemes that can bring Powder Mechanics in line with Computational Fluid Mechanics (CFM)

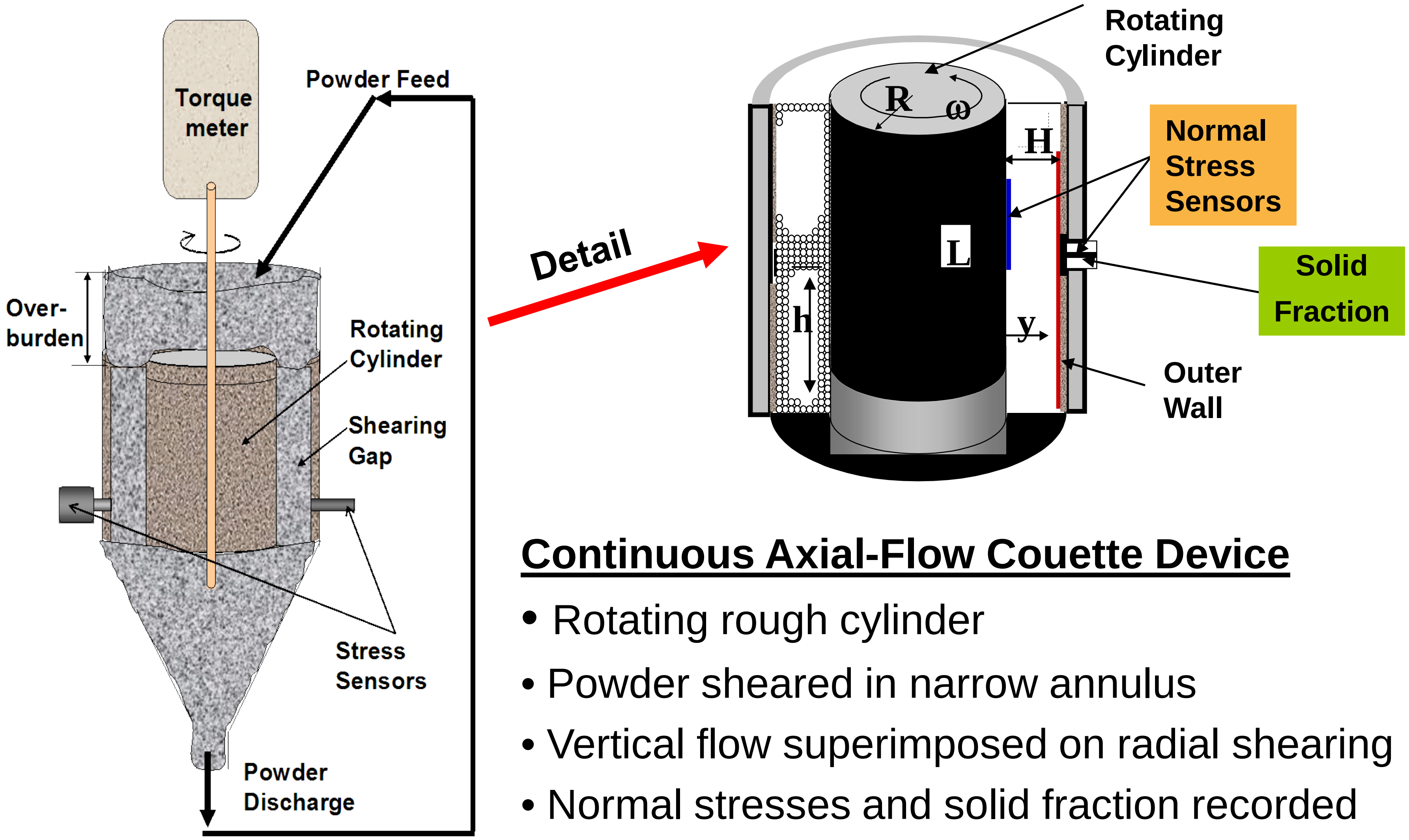
The Intermediate Regime of Flow

- Coexistence of <u>collisions</u> & <u>enduring</u> contacts in Dense Flows
- Characteristic of industrial processes
- Power-law dependency of shear stress to shear rate: $\tau \propto \dot{\gamma}^n$ (n < 2)

Map of Powder Flow Regimes



Measure Powder Characteristics in Annular Couette



Continuous Axial-Flow Couette Device

- Rotating rough cylinder
- Powder sheared in narrow annulus
- Vertical flow superimposed on radial shearing
- Normal stresses and solid fraction recorded
- Torque measured

Couette Device with Normal Sensors

Side view with wall
normal stress sensor

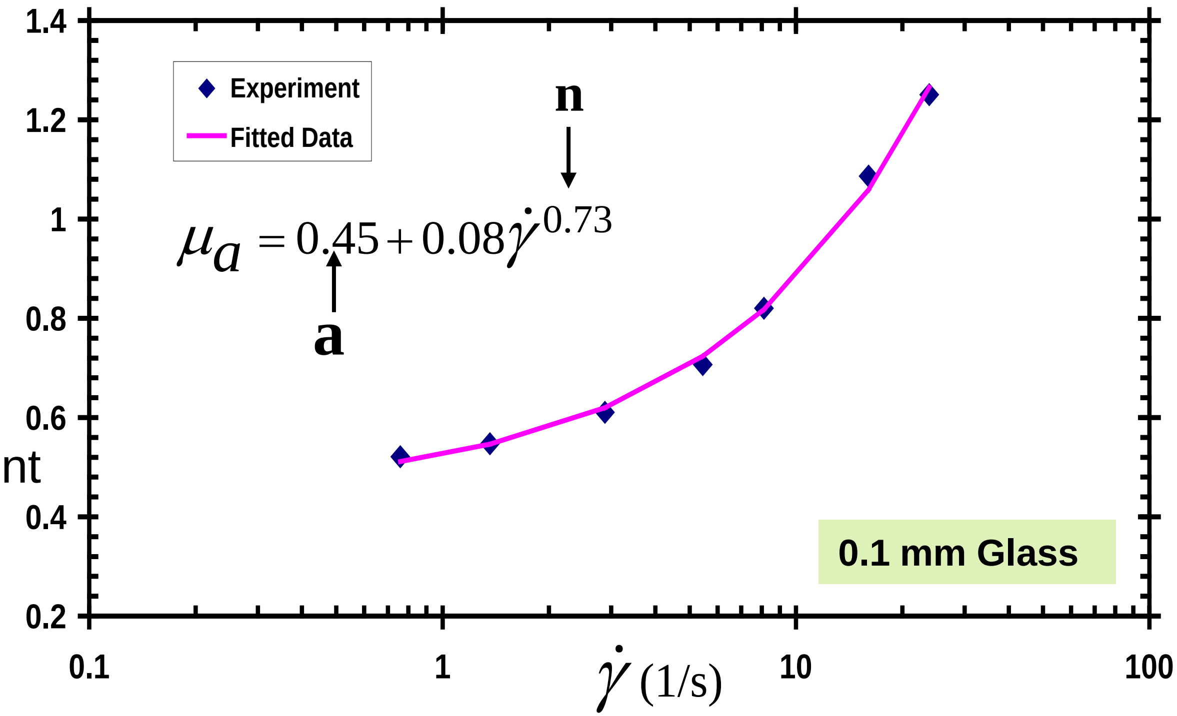
Remote normal stress sensors
on inner rotating cylinder

Friction Coefficient from Experiment in Couette Device

From Torque

$$\mu_a = \frac{\tau}{\sigma}$$

From Normal Stress Measurement



- ✓ When Shear rates ⟶ 0 : $\mu_a \rightarrow \tan(\phi)$ (ϕ : internal angle of friction)
- ✓ At higher shear rates: $\mu_a \propto \dot{\gamma}^n$ ⟶ Liquid-like behavior

Constitutive equation

$$T_{ij} = p\delta_{ij} + \sqrt{2}\,p\sin(\phi)\frac{\dot{\gamma}_{ij}}{\left|\dot{\gamma}_{ij}\right|} + \sqrt{2}\,ap\cos(\phi)\frac{\dot{\gamma}_{ij}}{\left|\dot{\gamma}_{ij}\right|}\dot{\gamma}_{ij}^{\,n}$$

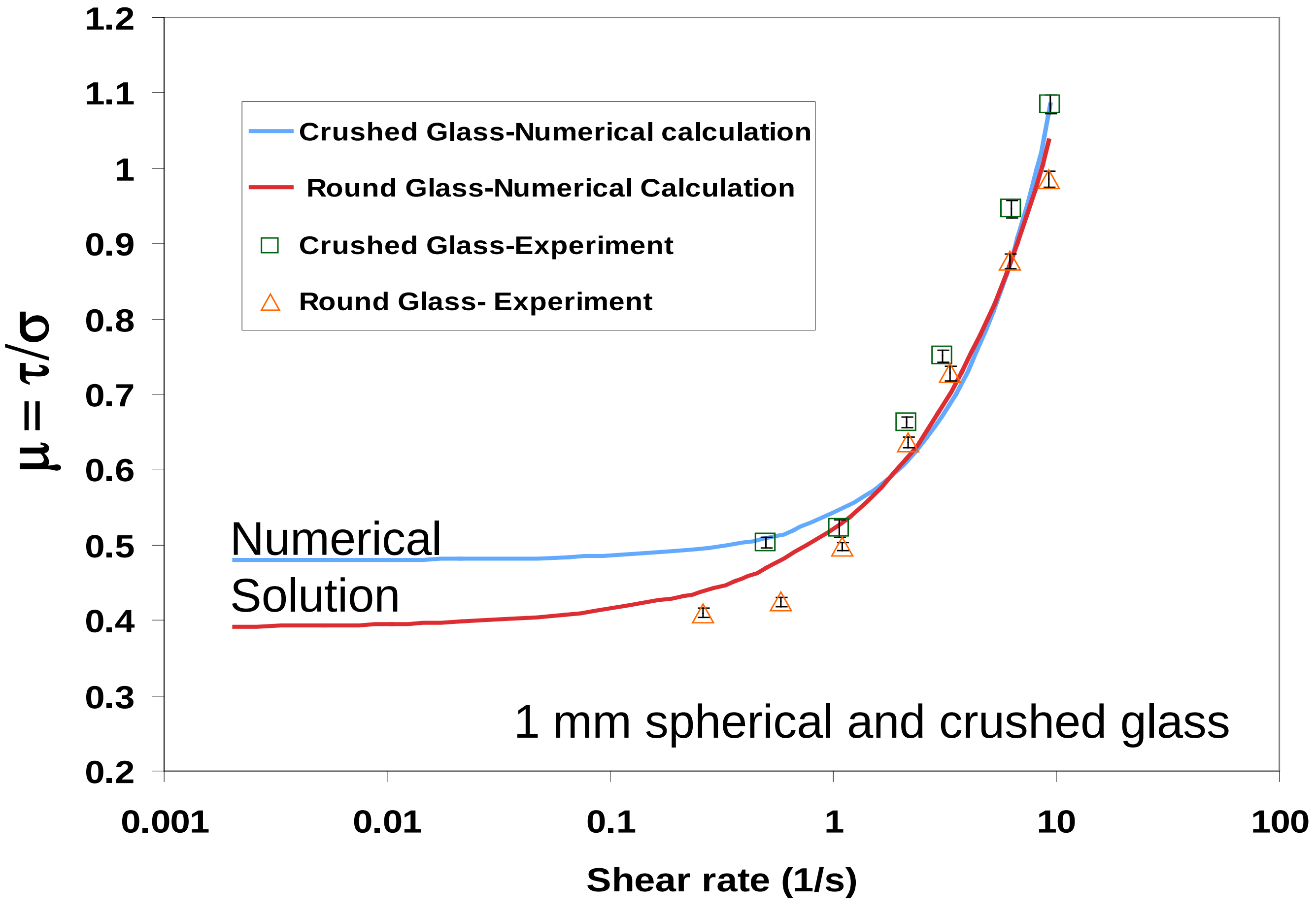
Schaeffer Equation

Liquid-like behavior

Modeling Couette flow with FE Methods

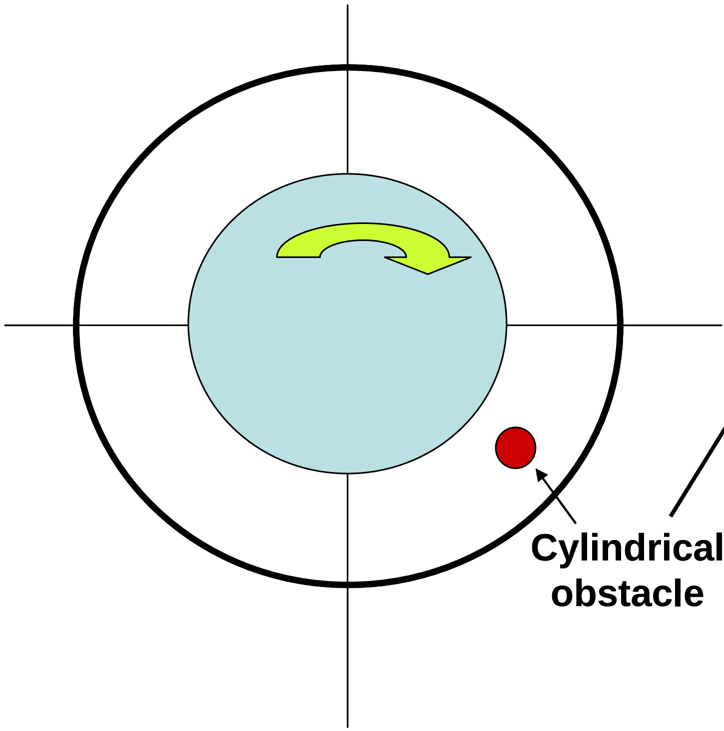
Finite Element Method:

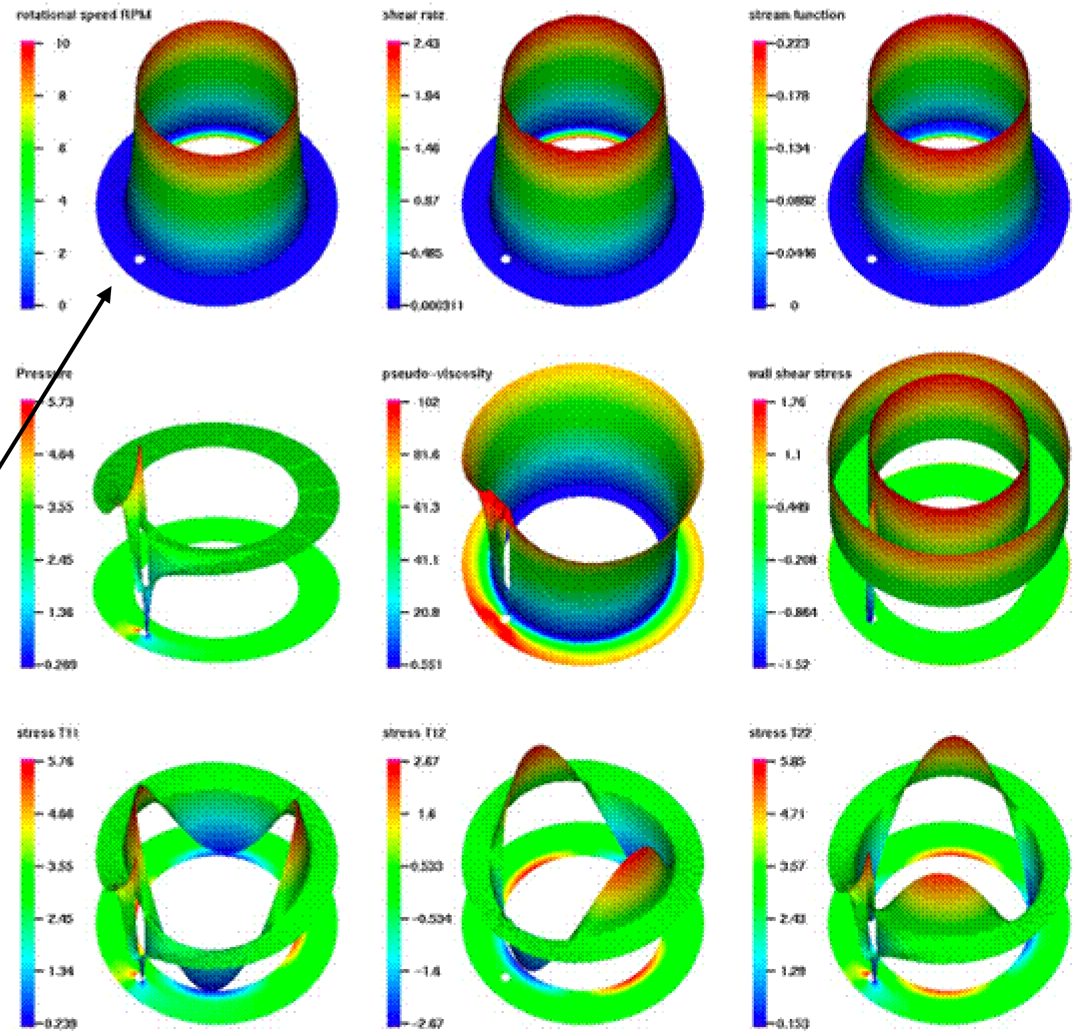
- FleatFlow, collaboration with Professor S. Turek, Univ. of Dortmund, Germany
- Navier-Stokes type approach using Constitutive equation
- Use of a stabilizing parameter to avoid ill-posedness.



Modeling Couette flow with FE Methods

Couette with Static “obstacle”



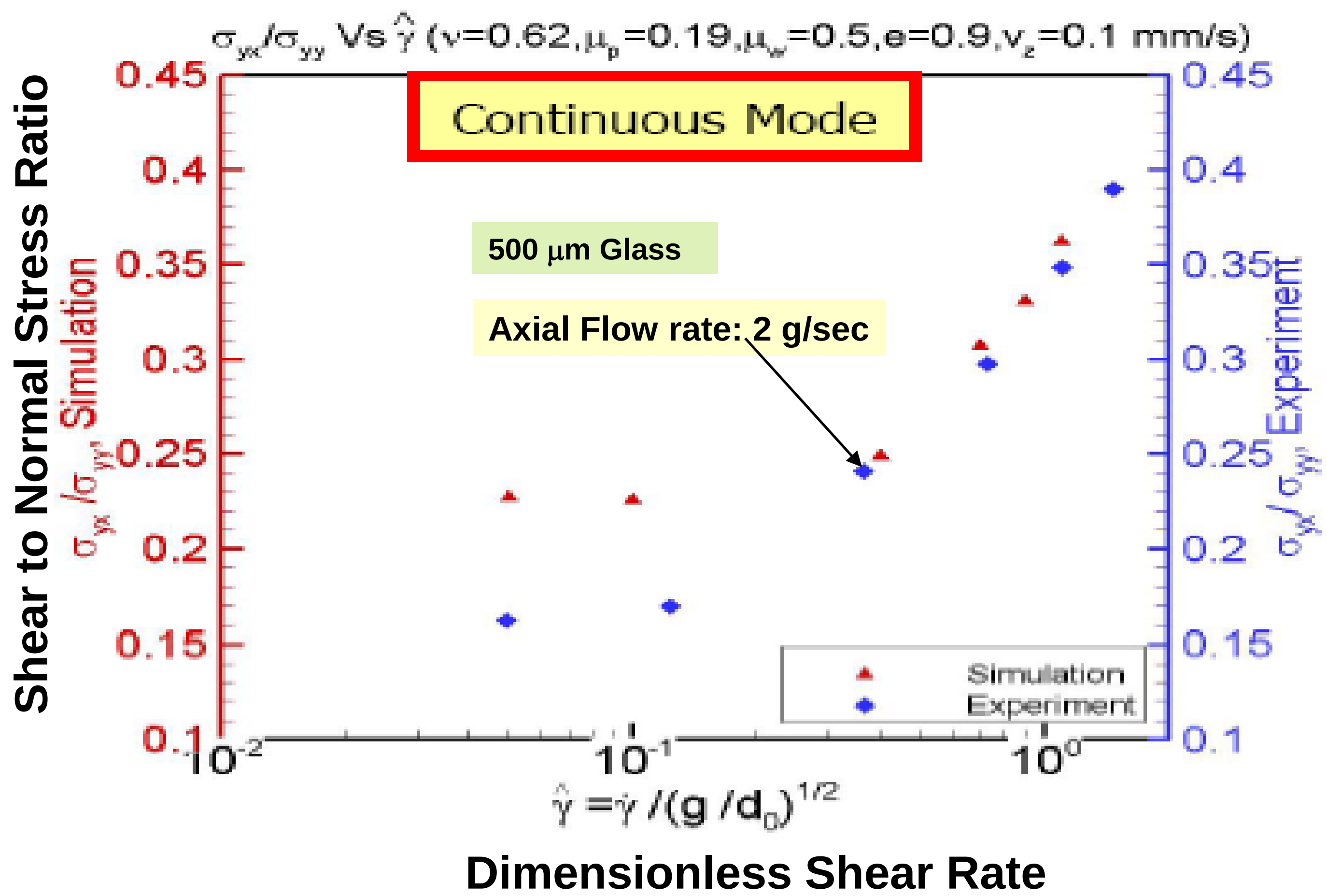


Numerical solution of the generalized Navier-Stokes equations for a powder in the intermediate regime. Concentric Couette with a cylindrical obstacle in the middle of the sharing gap.

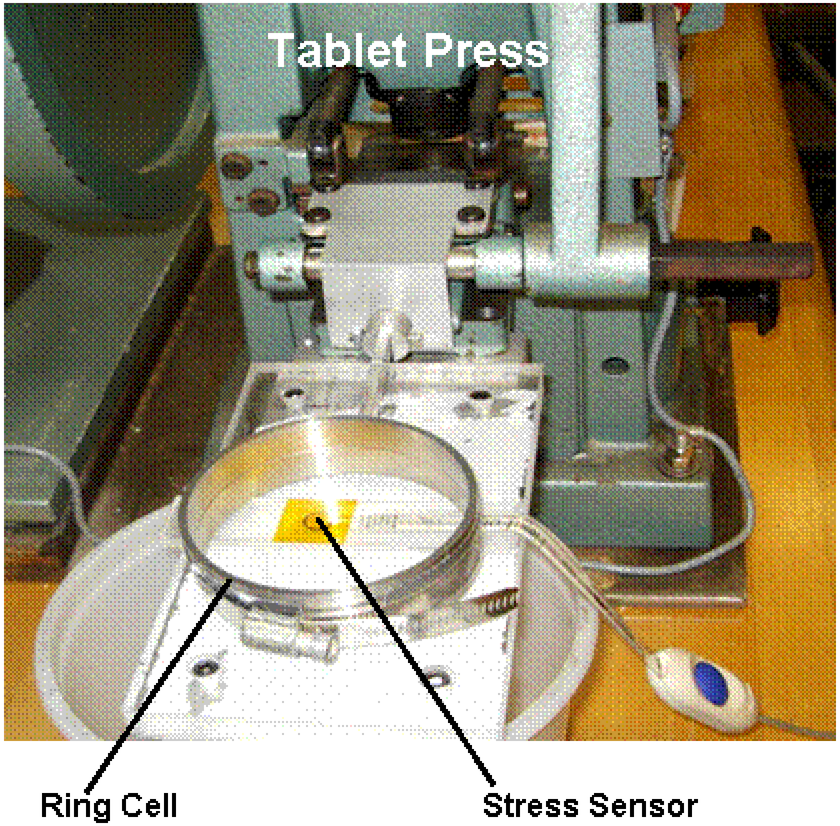
Modeling Couette flow with "Order Parameter" (OP) Model

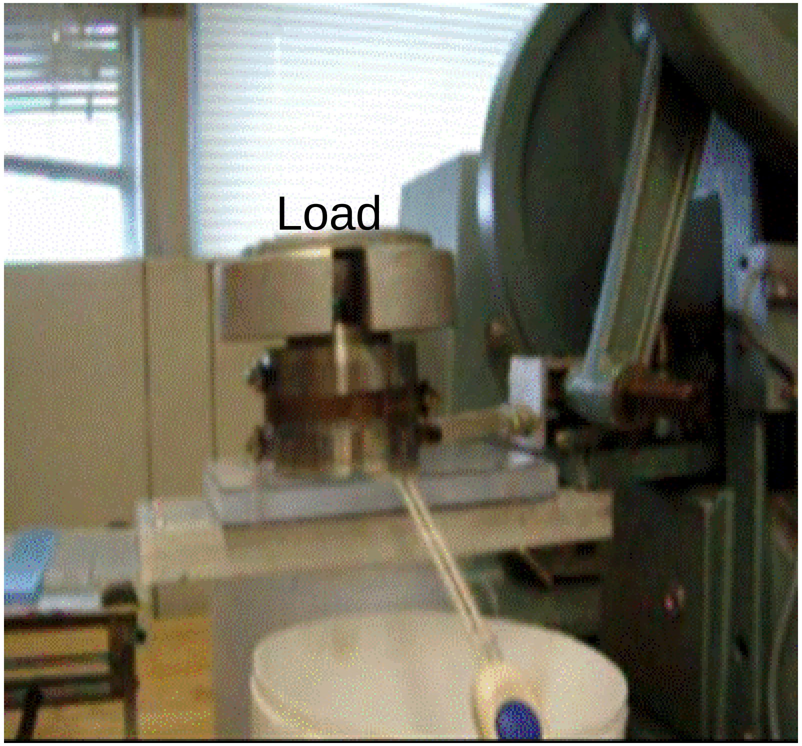
Continuous Model:

- Collaboration with Professor S. Subramaniam of Iowa State University
- Combination of Rapid Granular Flow model with Quasi-Static Flow Theory



The “fast” Shearing Jenike cell
Comparison of Experiment and Simulations





Loaded Jenike Cell

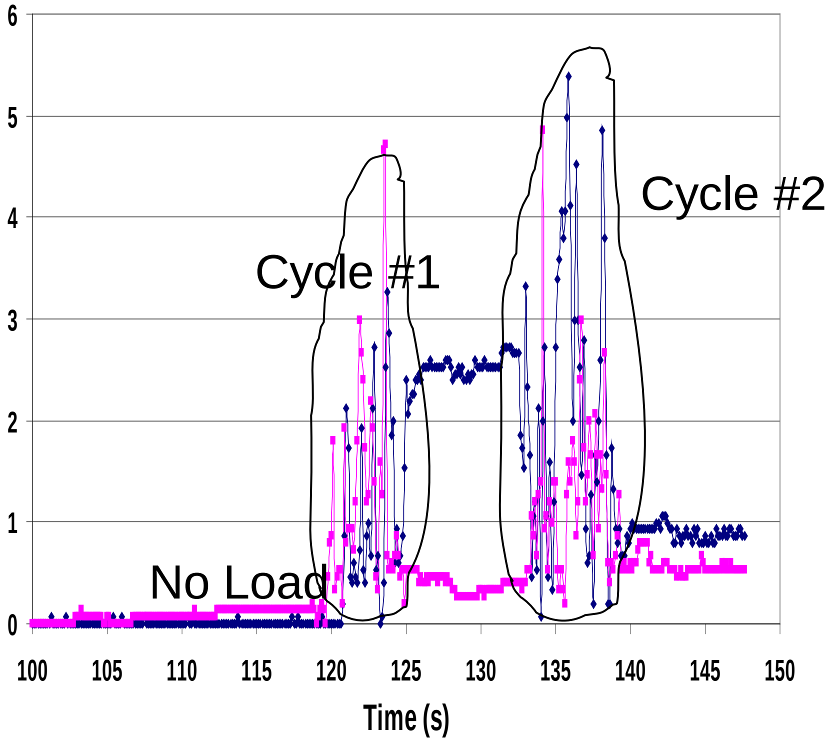
- **Up to 500 times the Shear Rate in the Jenike cell**
- **Multiple sensors at different locations**

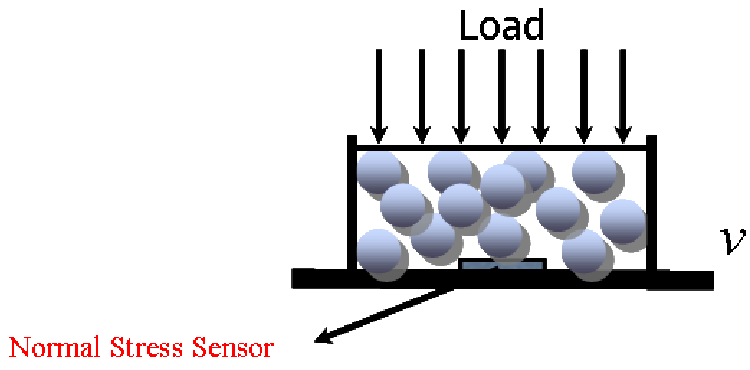
Comparison of Experiment and Simulations

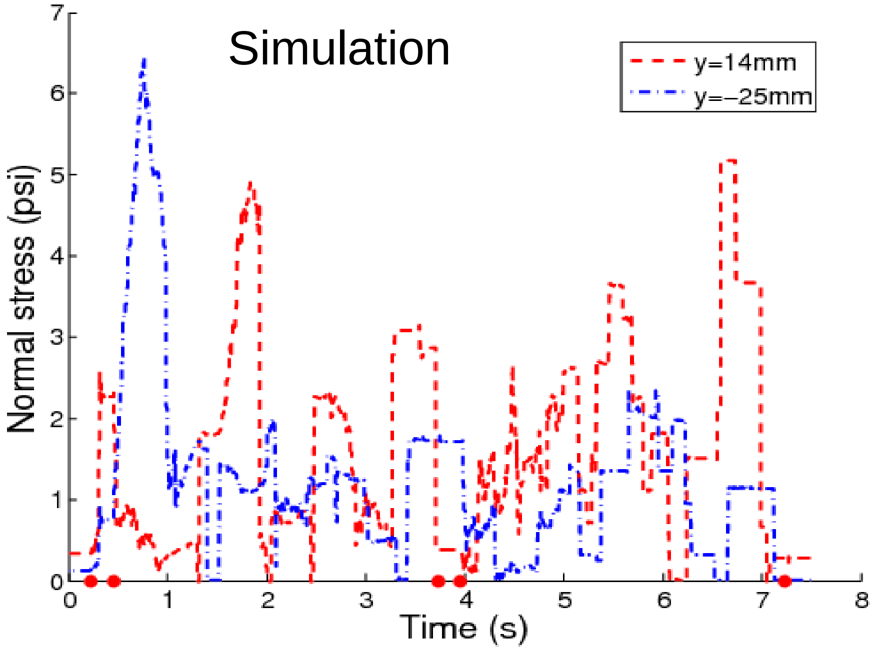
DEM Model:

- Collaboration with Professor S. Sundaresan of Princeton University

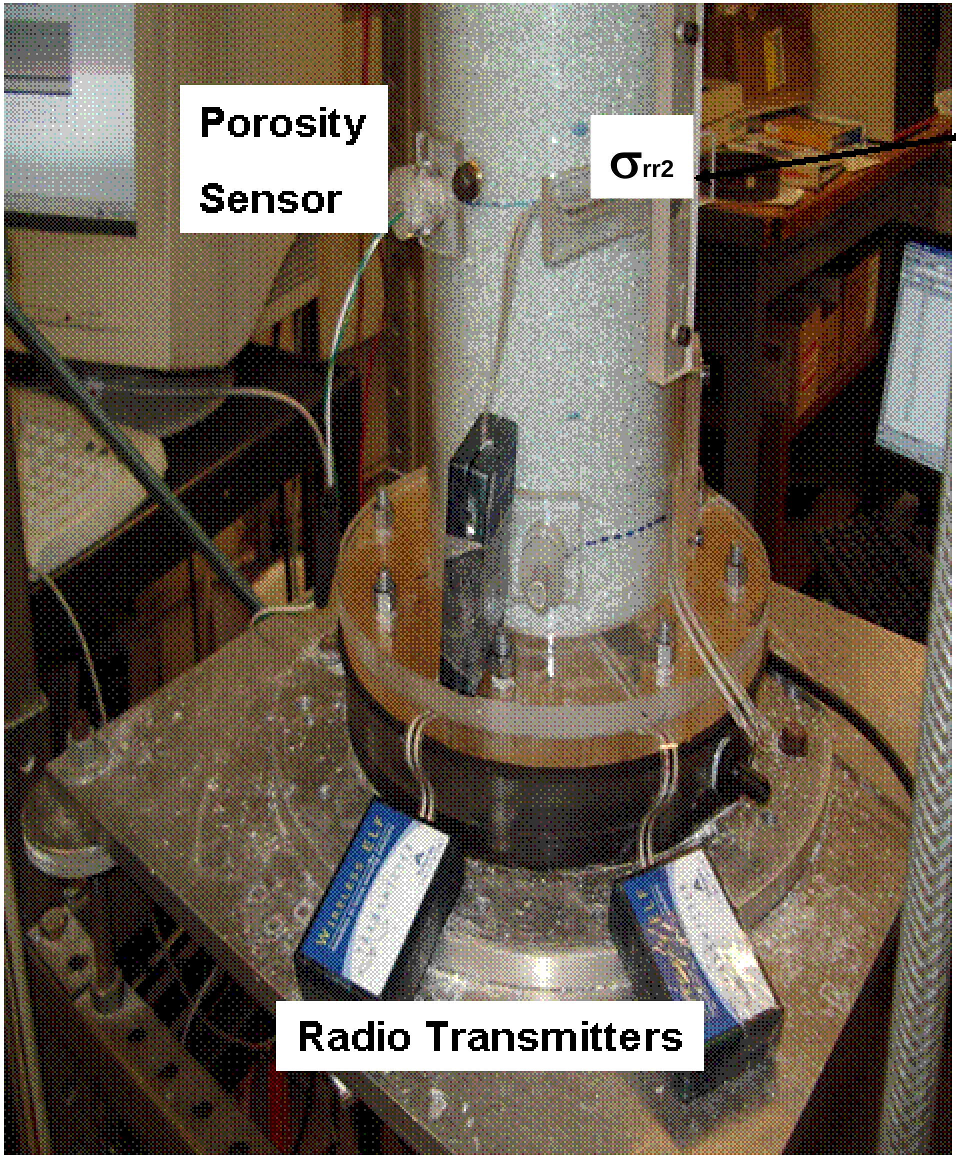
Experiment



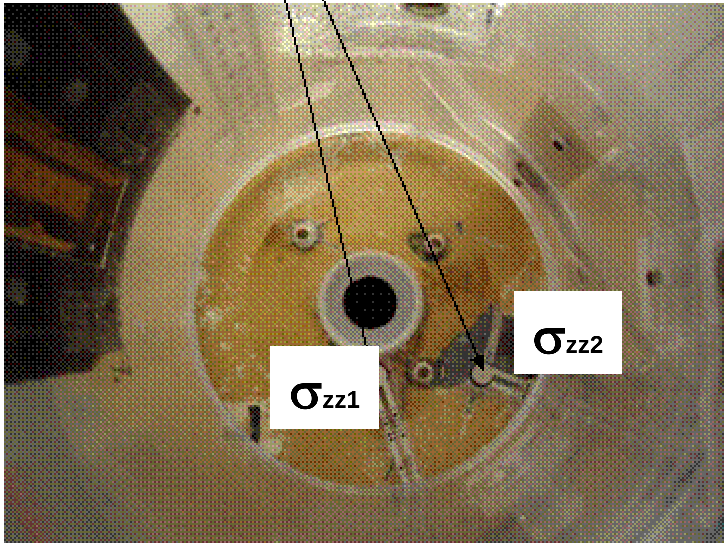




Experiments with a flat-bottom silo



Stress Sensors

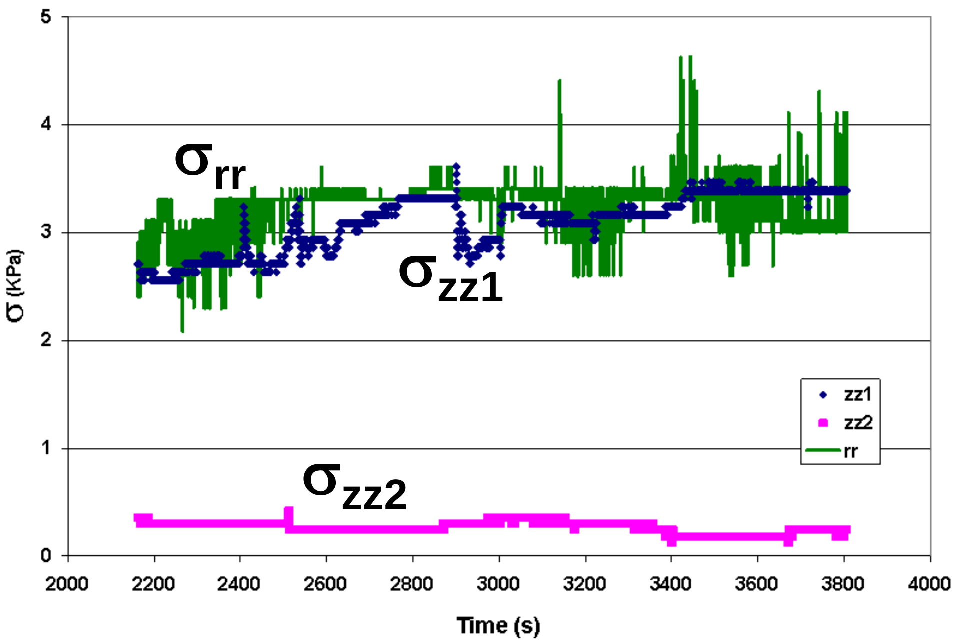


• Bottom Plate: view from above

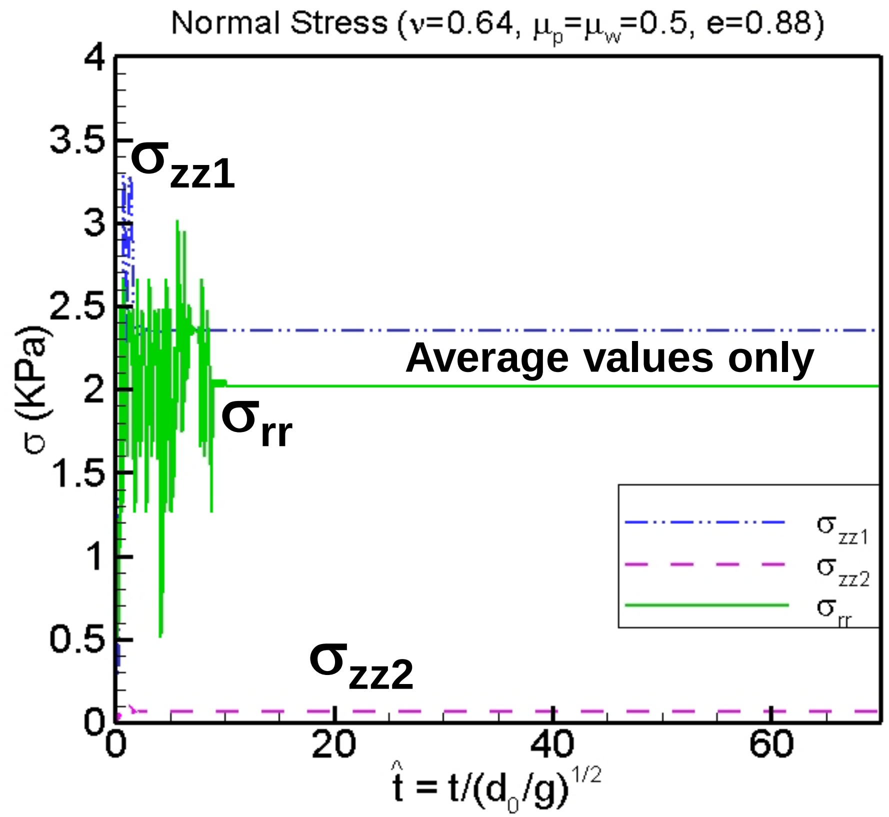
Flat-bottom silo
Comparison of experiment and theory

DEM Simulation:

- Collaboration with Professor S. Subramaniam of Iowa State University

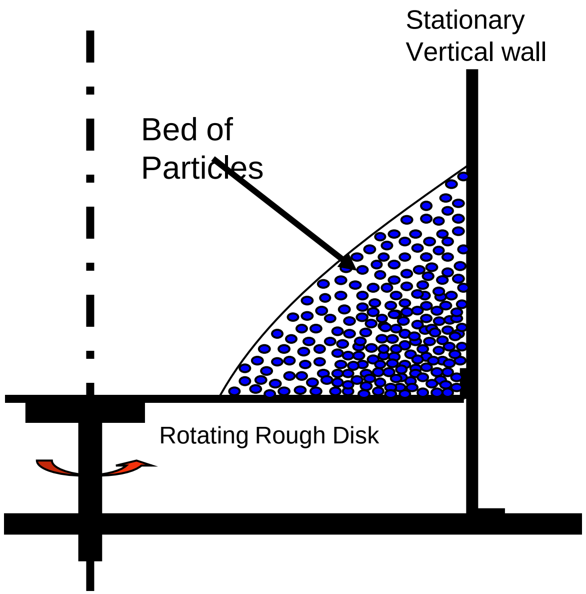


Experiments



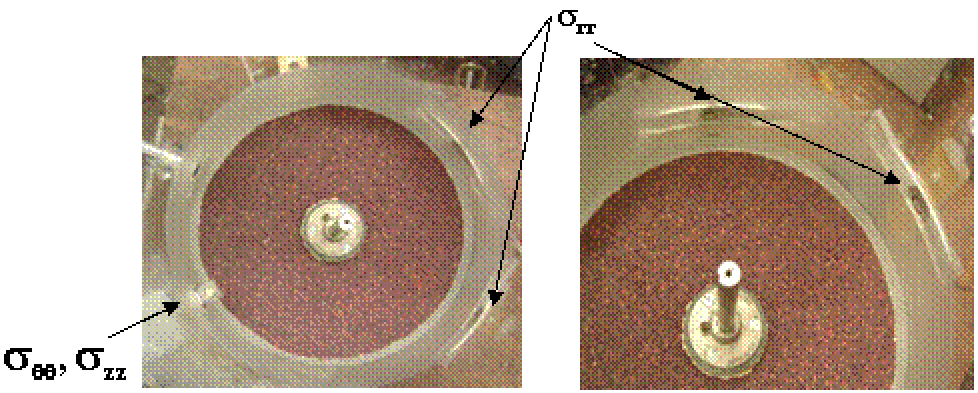
DEM Simulation

Centripetal Flow in a Spheronizer

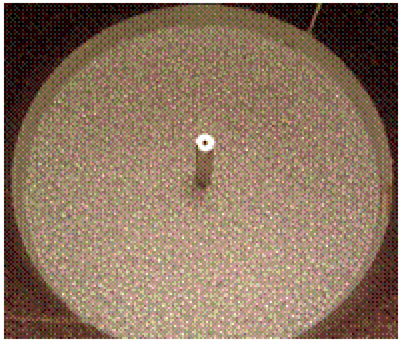


- **Spheronizer: a fixed cylinder and a rapidly rotated bottom surface**
- **Inertial effects become important as the centripetal force on a grain becomes comparable to the gravitational (hydrostatic) force**
- **Change in topology is observed at high rotation rates: formation and deepening of empty volume in the center of the rotating disc**

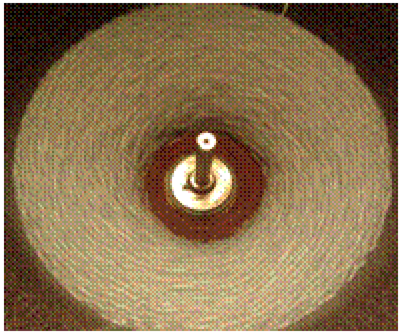
View from above



At rest

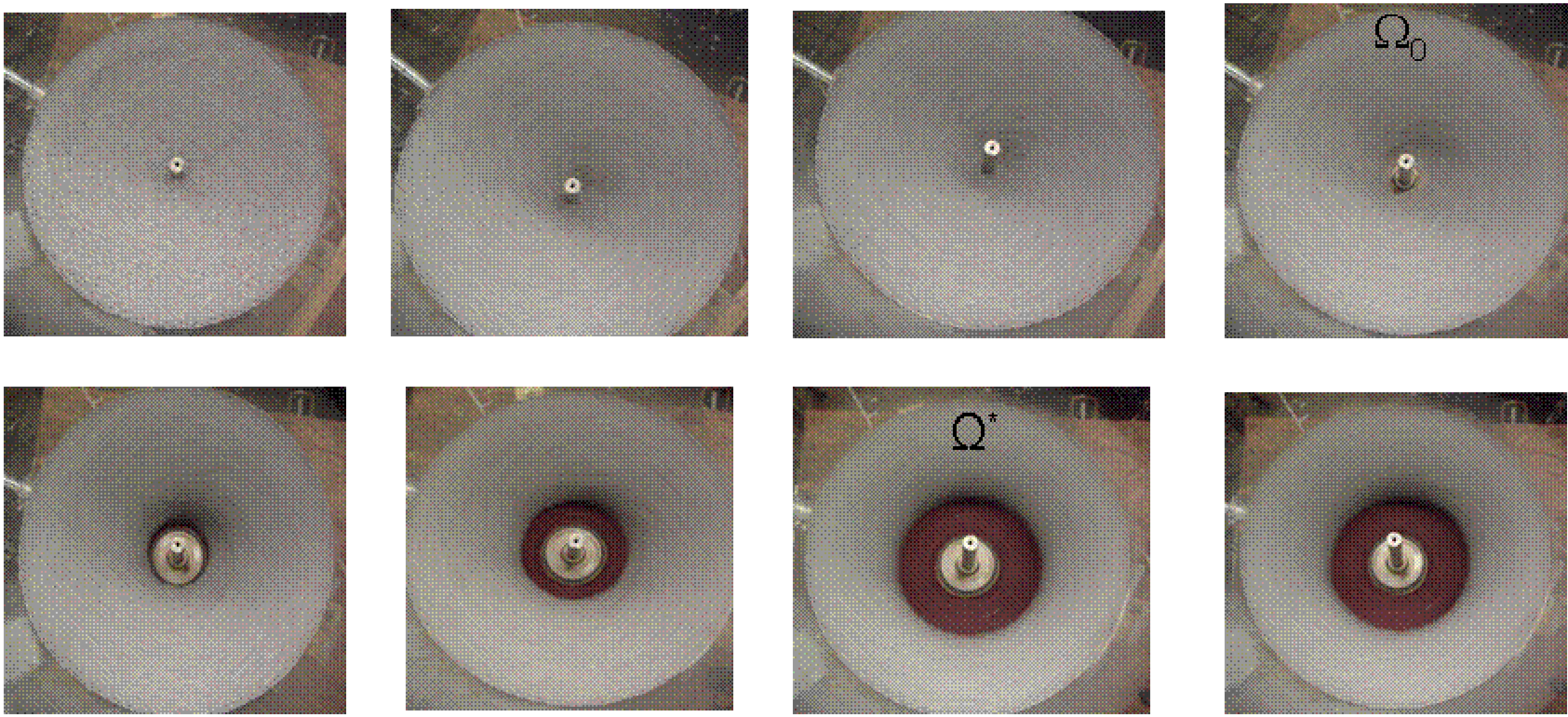


Rope Flow



Solid Body Rotation to Rope-like Motion by increasing Ω

3 mm Glass beads in Spheronizer, Increasing → Ω



- **Ω_0: Start of the rope-like motion-Empty hole forming in the middle**
- **$\Omega > \Omega^*$: Shape of free surface and height of material on the outer wall do not change**
  - σ_{rr}, $\sigma_{\theta\theta}$, and ν (solid fraction) remain constant (Experiments)
  - Average value of azimuthal velocity (v_θ) and convective velocity ($\sqrt{v_r^2 + v_z^2}$) remain constant

E. Corwin, Physical Review E 77, 031308 (2008)

A CFD Approach for Particulate Flows with Free Surface

- **Collaboration with Dr. Mehrdad Shahnam of DoE**

  - Axis-symmetric problem: $\partial / \partial\theta = 0$
  - Difficulties in computation:
    - Non-zero velocities in r, z and θ directions
    - Non-linearity of inertial terms in momentum equations
    - Presence of free surface: solving equations for 2 phases
  - Continuity Eq.:

$$\frac{\partial}{\partial t}(\upsilon\rho) + \nabla \bullet (\upsilon\rho\vec{V}) = 0 \qquad \upsilon : \text{volume fraction}$$

  - General Navier Stokes Eq.

$$\frac{D(\rho u)}{Dt} = -\nabla P + \sqrt{2}\sin(\phi)\nabla.((\frac{P}{|\dot{\gamma}_{ij}|} + a\cot(\phi)|\dot{\gamma}_{ij}|^{n})\dot{\gamma}_{ij}) + \rho g \qquad \dot{\gamma}_{\theta r} = \frac{v_\theta}{r} - \frac{dv_\theta}{dr} \qquad \dot{\gamma}_{\theta z} = -\frac{dv_\theta}{dz}$$

  - Conservation of momentum in azimuthal (swirl) direction:

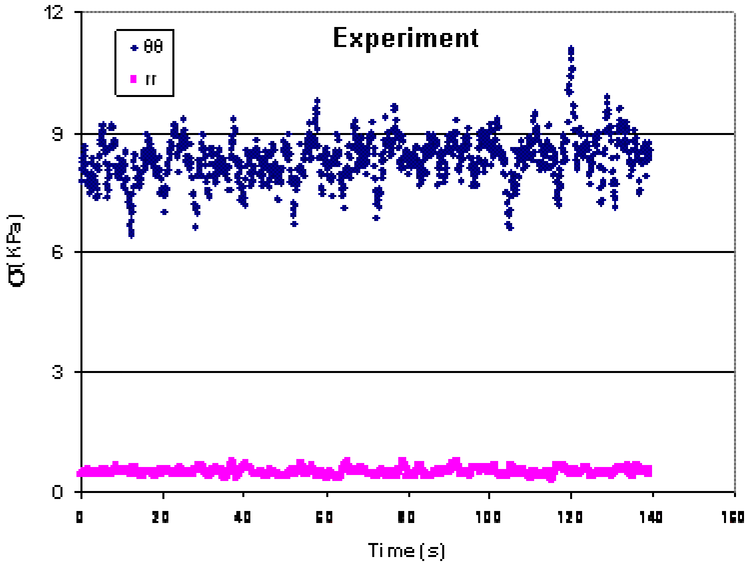
$$\frac{\partial}{\partial t}(\rho v_\theta) + \frac{1}{r}\frac{\partial}{\partial z}(r\rho v_r v_\theta) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r v_\theta) = \frac{1}{r}\frac{\partial}{\partial z}\left[r\eta(\dot{\gamma}_{II}, P)\frac{\partial v_\theta}{\partial z}\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^3\eta(\dot{\gamma}_{II}, P)\frac{\partial}{\partial r}(\frac{v_\theta}{r})\right] - \frac{\rho v_r v_\theta}{r}$$

A CFD Approach for Particulate Flows with Free Surface

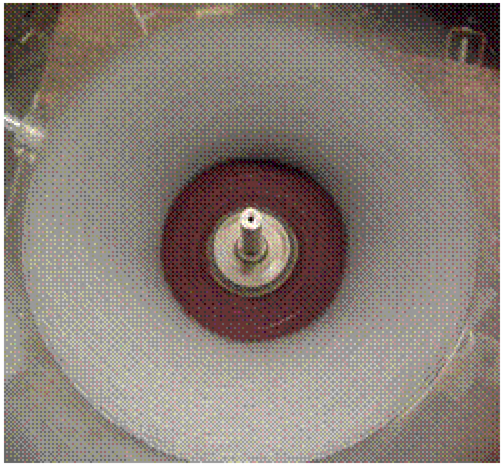
ANSYS FLUENT Capable of solving non linear inertial terms in momentum equation

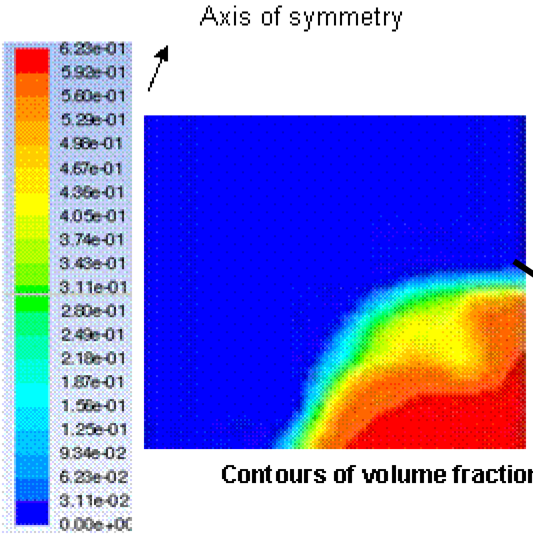
- **Eulerian method to solve for 2 phase flow (phase I: air, Phase II: granular)**
  - **Allows for the modeling of multiple separate, yet interacting phases**
  - **Momentum and continuity equations are solved for each phase**
- **Select 2-D axis-symmetric option**
- **Insert material properties like size (3 mm) and internal angle of friction (30^0)**
- **Use constitutive equation obtained in Couette geometry (rheological parameters a=0.44, n=0.75)**
  - **Defining the shear-stress-shear-rate ratio obtained in Couette experiment by writing UDF (User Defined Functions)**
- **Compare results of simulation to experiment:**
  - **Height of materials and Shape of the free surface as a function of Ω**
  - **Normal stresses in radial and swirling directions**
  - **Solid volume fraction at different positions**

Preliminary Results from Experiments and Simulation



Ω= 25 rad/s



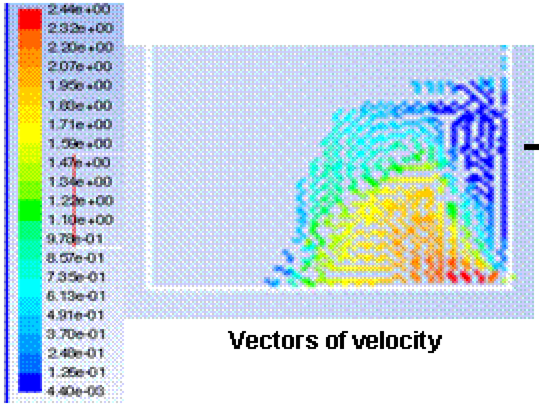


Contours of volume fraction

Simulation by Dr. Shahnam

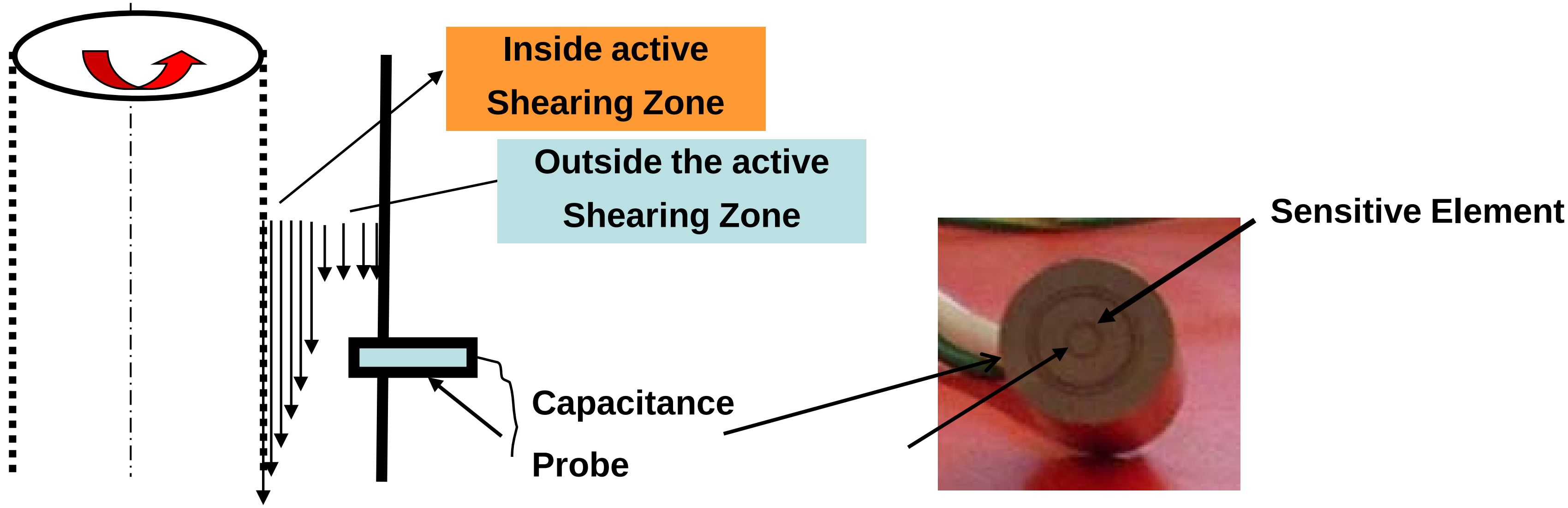
Experiment & simulations matches:

- Normal stresses in θ and r directions
- Height of material on the stationary wall
- Shape of free surface & size of empty hole
- Solid volume fraction on the stationary wall

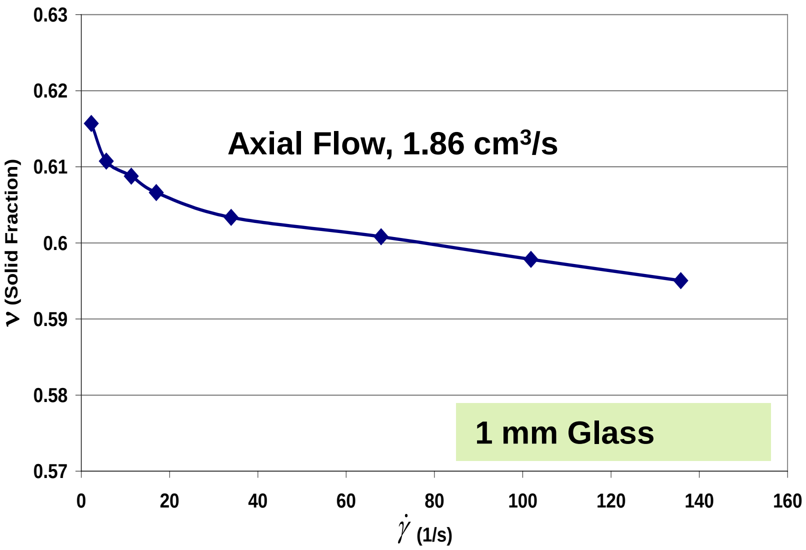


Vectors of velocity

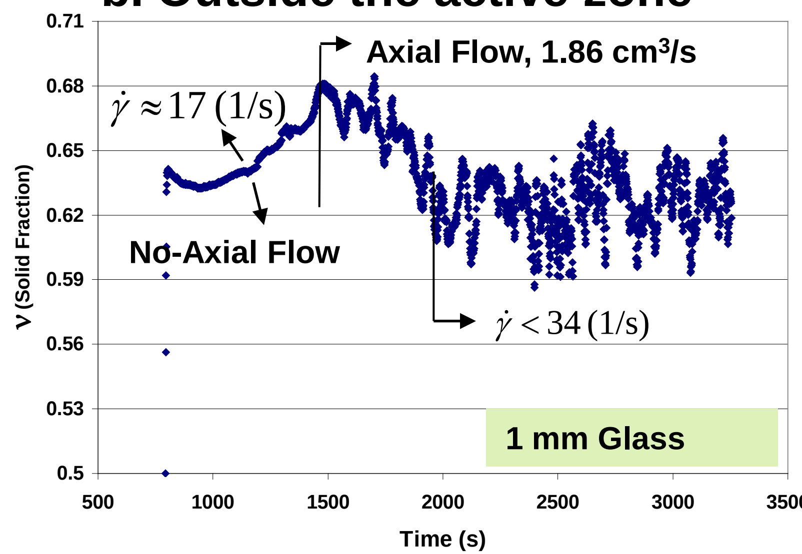
Solid Fraction Measurements using a Capacitance Probe



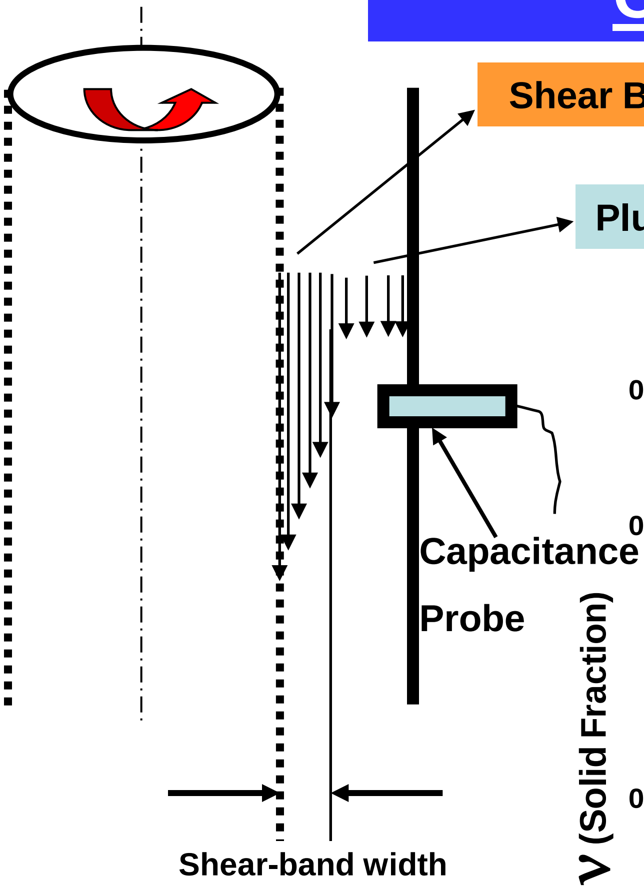
a. Inside the active zone



b. Outside the active zone

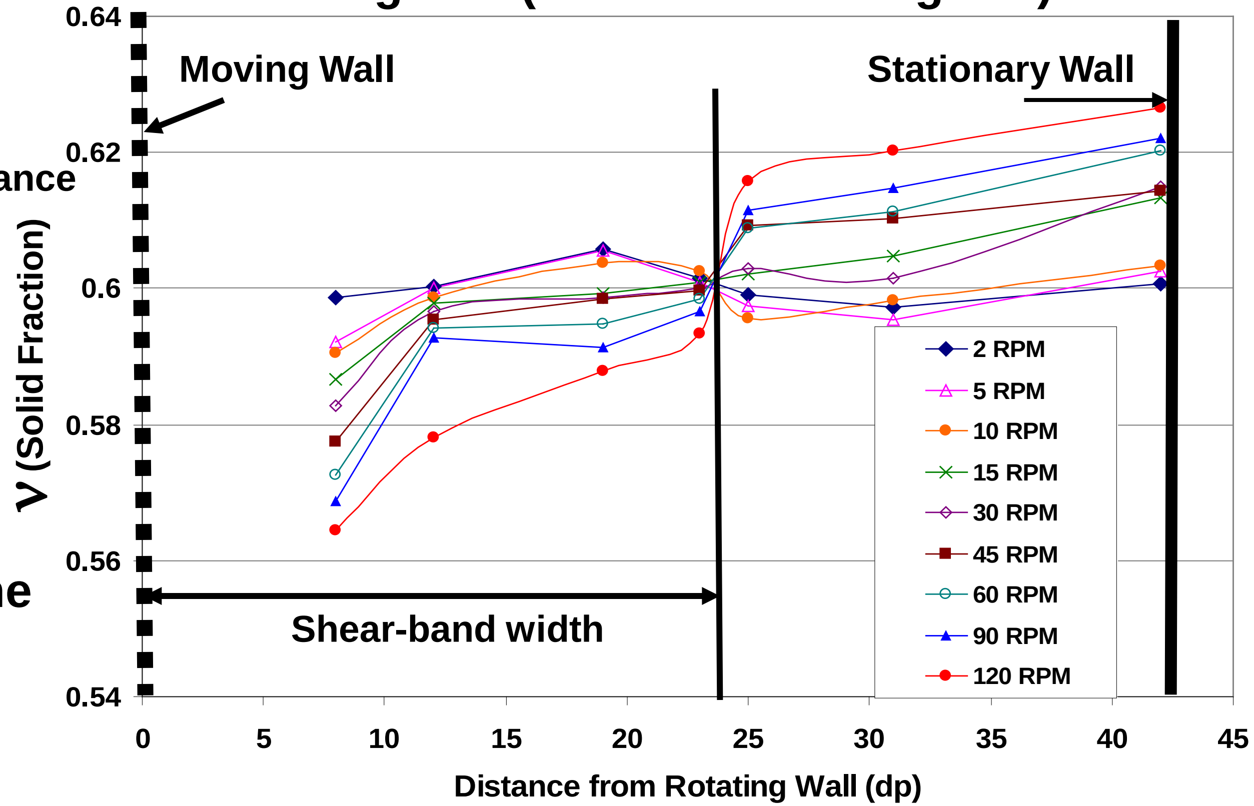


Width of Shear-Band in Axially Flowing Couette using Solid Fraction

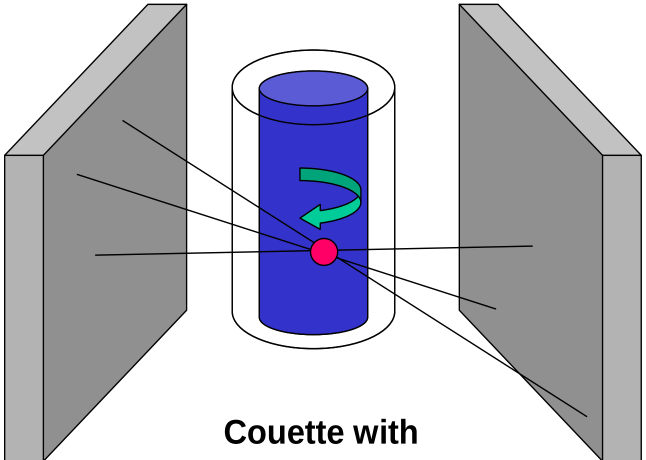


Schematic of shearing zone

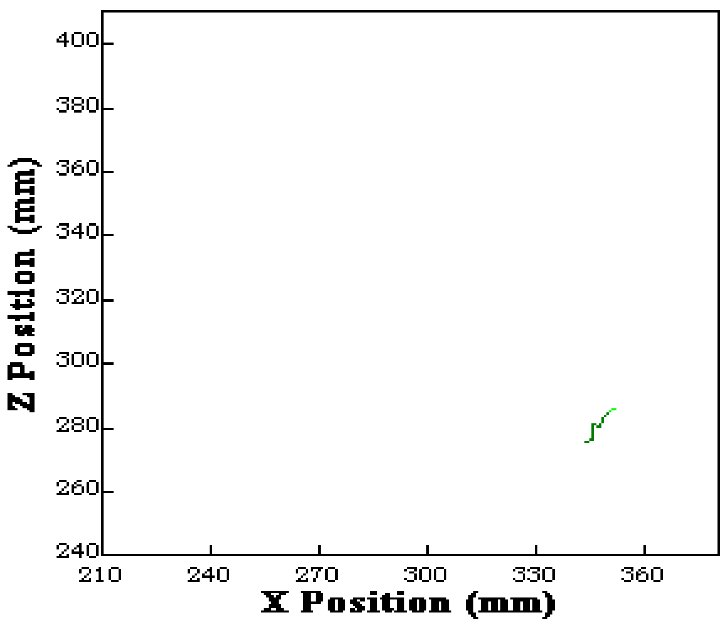
Solid fraction at different distances from rotating wall (0.6 mm crushed glass)



Width of Shear-Band in Batch Couette using Positron Emission Particle Tracking [PEPT]

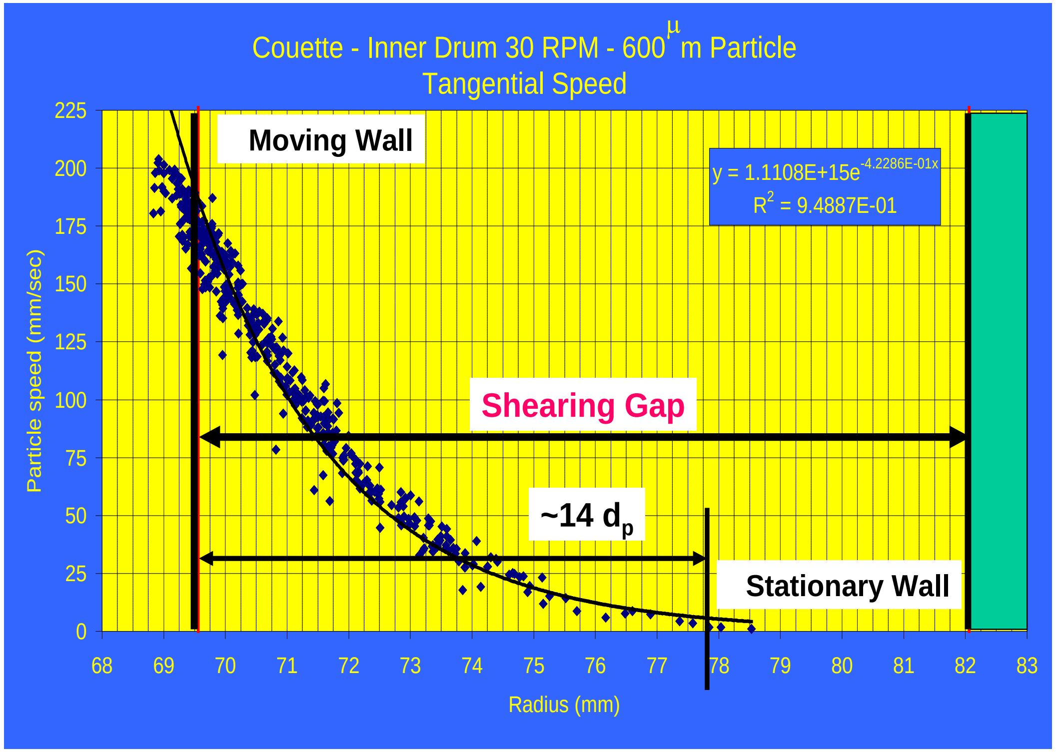


Couette with tagged particle



Experiment:

• Collaboration with J.P.K. Seville and A. Ingram, Univ. of Birmingham, UK



Conclusions

- Ratio of Shear Stress to Shear rate is constant at low and increases at higher shear rates – experimental correlation can be used as “constitutive equation”.
- Solid Fraction measurements showed that the bed has to increase its porosity for the transition to the intermediate regime
- Simulation from, FEM, OP, DEM and FLUENT favorably compare to results from Couette, “Fast” Jenike cell and Spheronizer geometries.