

# Applying Uncertainty Quantification to Multiphase Flow CFDs

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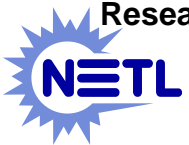
**Aytekin Gel<sup>2</sup>**

ALPEMI Consulting, LLC /  
National Energy Technology Laboratory

## Acknowledgment: The CCSI Technical Team

<sup>1</sup> This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract No. DE-AC52-07NA27344

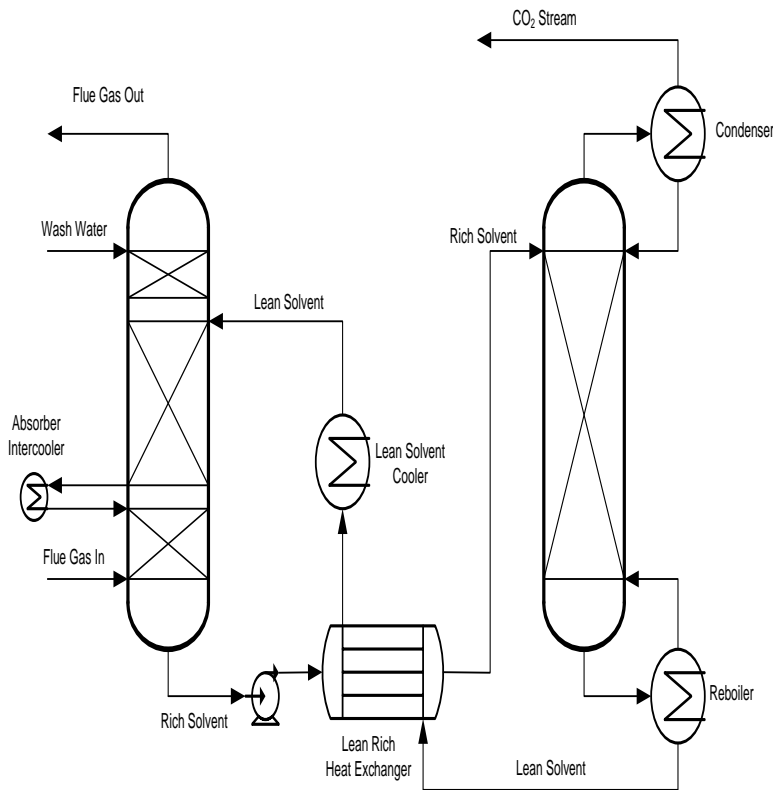
<sup>2</sup> This work was performed in support of the National Energy Technology Laboratory's ongoing Research in advanced multiphase numerical simulation of multiphase flows under the RES contract DE-FE0004000



# Presentation Outline

- Brief Introduction to Uncertainty Quantification & Analysis
- Introduction to UQ Toolkit, PSUADE
- Preliminary Results for Demonstration of Non-intrusive UQ Analysis for MFIX Simulations:
  - Gasification
  - DES Fluidized Bed
- Summary

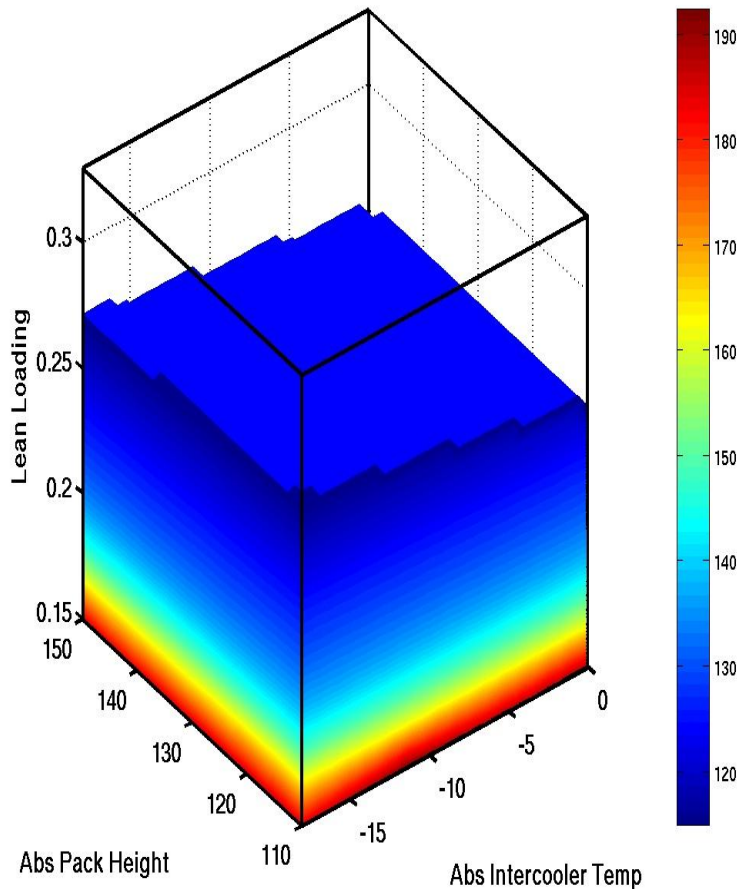
# Let's use an example to illustrate the need for UQ



Variable	Min	Max	Units
Lean Loading	0.15	0.33	mol CO <sub>2</sub> /mol MEA
Lean Solvent Feed Temperature	110	150	°F
Rich Solvent Feed Temperature	205	214	°F
Absorber Packing Height	15	40	ft
Absorber Intercooler Temperature Change	-18	0	°F
Regenerator Packing Height	10	30	ft
Regenerator Condenser Pressure	20	24	psia
Regenerator Condenser Temperature	110	150	°F
Compressor Intercooler 1 Temperature	110	140	°F
Compressor Intercooler 2 Temperature	110	140	°F

**Objective: minimize the levelized cost of electricity while keeping carbon capture at above 90%**

# Optimization results using surrogate models



## Optimal solution:

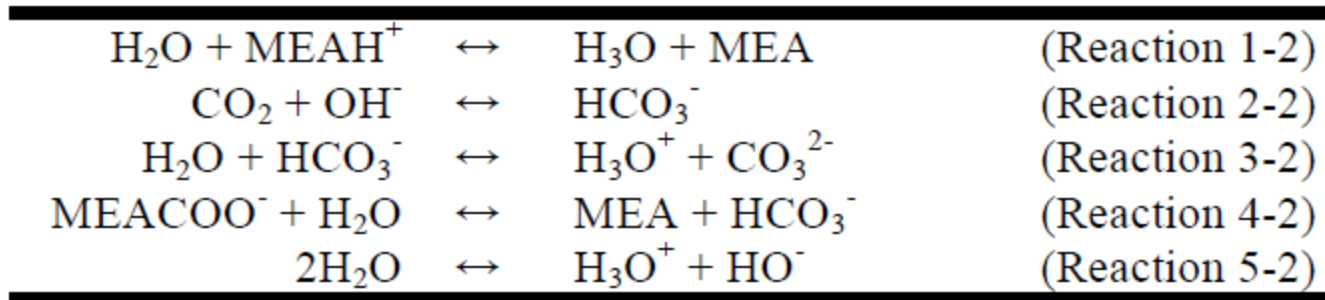
Lean solution	=	2.66e-01
Abs packing height	=	2.78e+01
Regen packing height	=	1.95e+01
Abs Intercooler delta T	=	-1.11e+01
Lean Solvent Feed T	=	1.29e+02
Rich Solvent Feed T	=	2.14e+02
Regen Condenser P	=	2.00e+01
Regen Condenser T	=	1.29e+02

Which gives 90% CO<sub>2</sub> capture  
and LCOE ~ 113

However, it is known that some of the parameters in the model are uncertain: for example, reaction parameters



- Scope: MEA equilibrium reactions

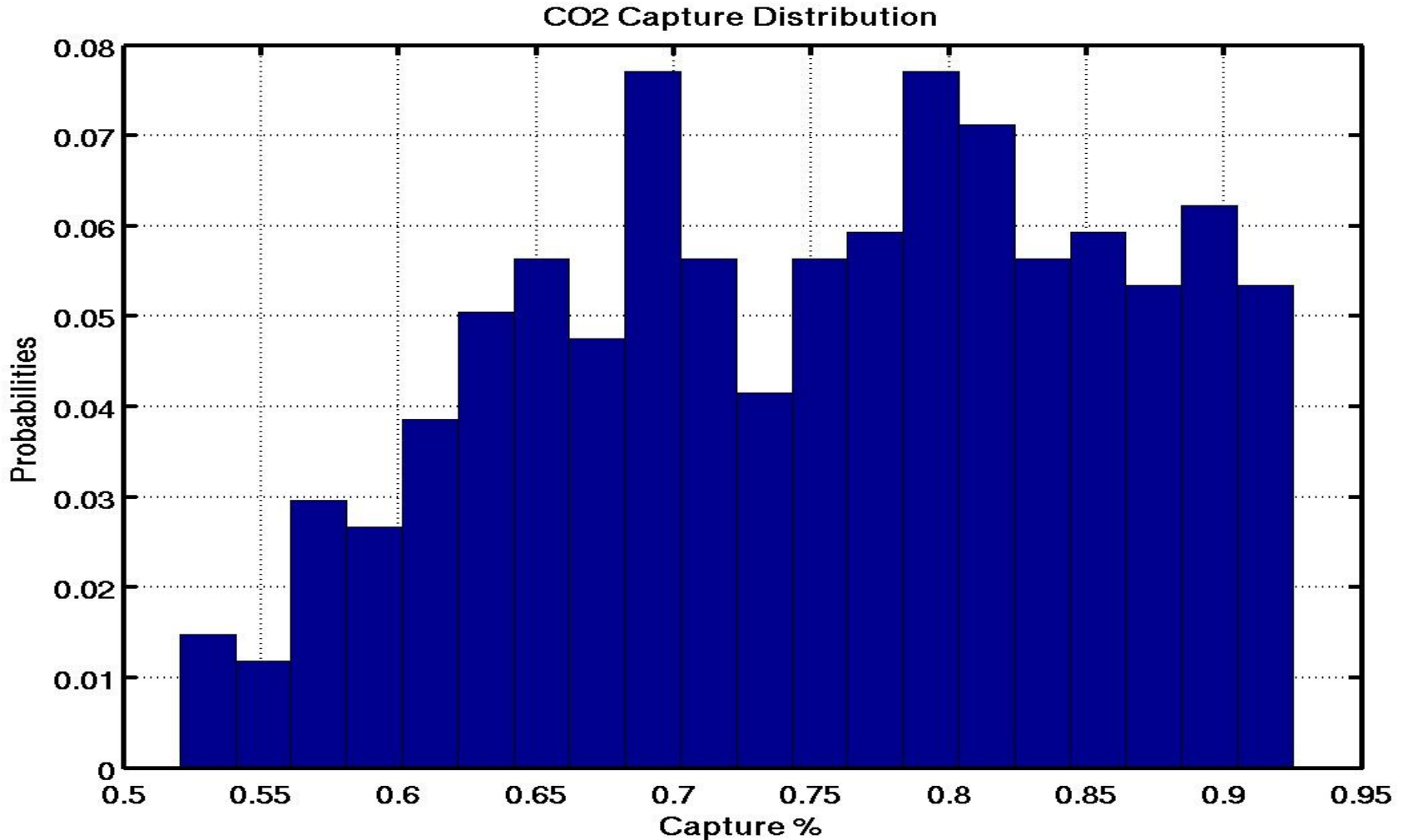


$$\ln K_{eq} = A + \frac{B}{T} + C \ln T + DT$$

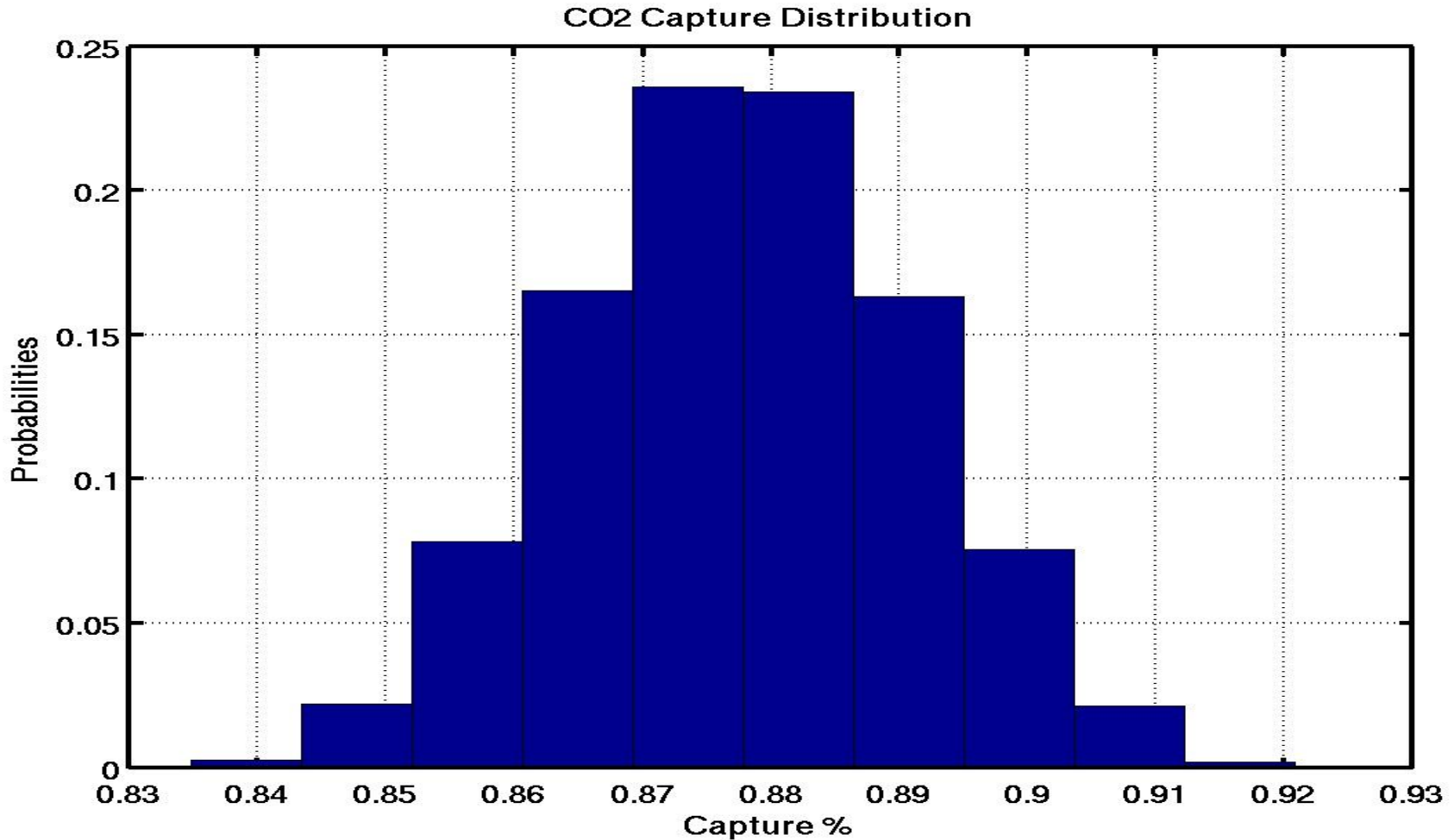
Reaction	A	B	C	D
1-2	0.7996	-8094.81	0.0	-0.007484
2-2	98.566	1353.8	-14.3043	0.0
3-2	216.049	-12431.7	-35.4819	0.0
4-2	1.282562	-3456.179	0.0	0.0
5-2	132.899	-13445.9	-22.4773	0.0

Other sources of uncertainties: mass transfer, equilibrium model, Flue gas composition, boundary conditions, ...

# As a result of parametric uncertainties, we have an uncertainty distribution for the CO2 capture %



The distribution may be unrealistic due to loose prescription of uncertainty bounds, adding data into the analysis, for example, gives



## Questions we may ask about these uncertainties

- What is the uncertainty of the CO<sub>2</sub> capture % as a result of these uncertainties?
- What other parameters in the systems are uncertain?
- Which parameters have the most effect on the output uncertainties?
- If I have more data, how much do they help in narrowing the output uncertainties?
- As a result of uncertainties, what is the probability that the CO<sub>2</sub> capture falls below 90%?
- I am using an approximate process model, what is the effect of approximation on the accuracy of the solution?
- How do uncertainties affect the system design?

Welcome to the world of UQ





# What is uncertainty quantification? One possible definition

Uncertainty quantification is the

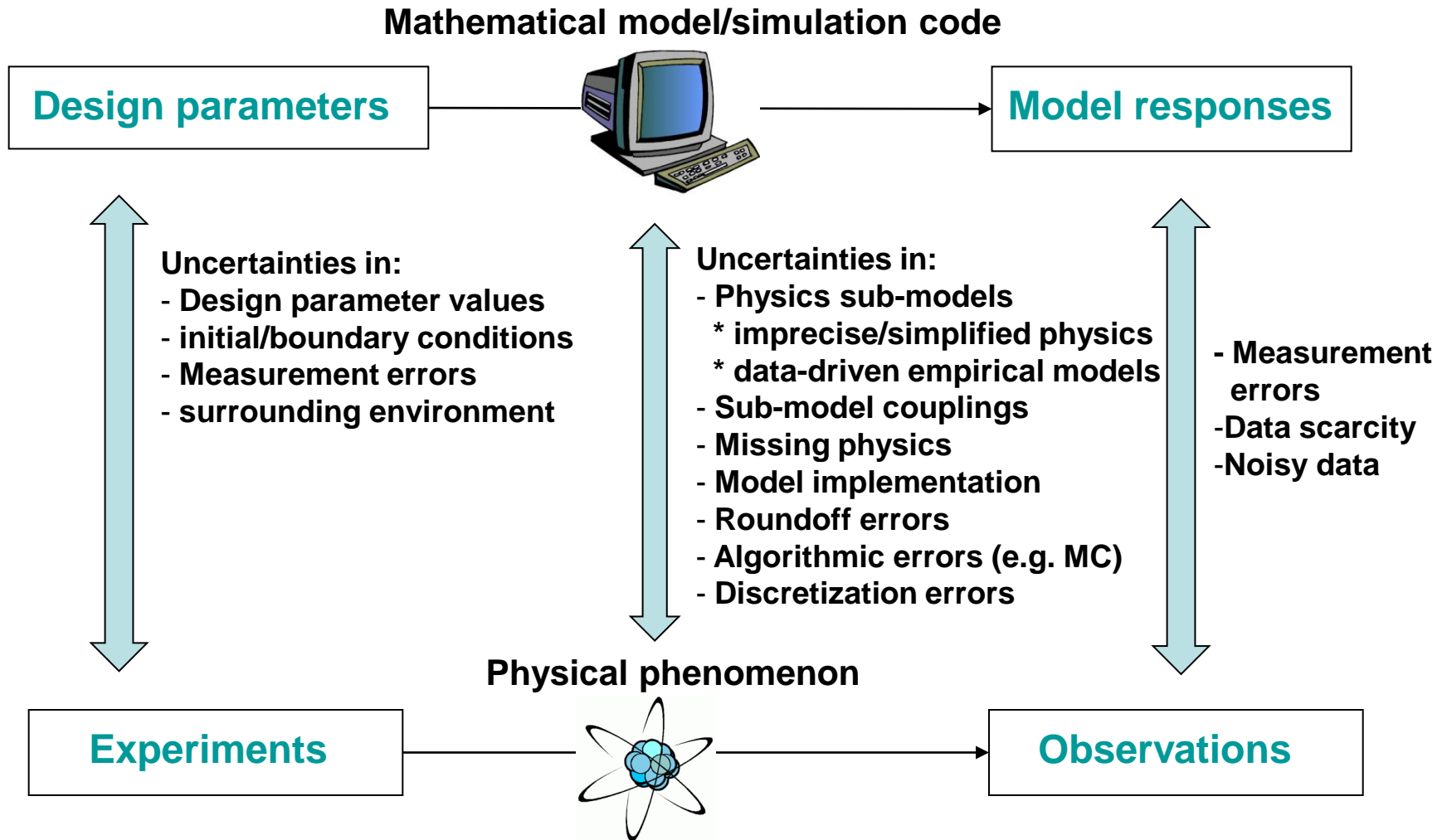
- **identification** (where the uncertainties are),
  - Physics model, boundary conditions, data, ...
- **characterization** (what form they are),
  - Parametric (bounds, PDF, beliefs), model form
- **propagation** (how they evolve, forward/inverse),
- **analysis** (what are the impacts, quantitative) , and
  - Sensitivity analysis, risk analysis, ...
- **reduction**

of uncertainties (all?) in simulation models.

## How do we put these into practice? → a UQ process

1. Define the objective of the UQ study (e.g. quantify risk)
2. Problem specification (model, assumptions, QOI, data)
3. Preliminary parameter **identification** and selection
4. **Characterize** parameter uncertainties (literature, expert)
5. Integrate data into models (Data Fusion Methodology)
6. Parameter screening (Dimension Reduction Methodology)
7. Build surrogates (Response Surface Methodology)
8. **Uncertainty/Sensitivity analysis (Global SA methodology)**
9. **Sensitivity/Risk analysis and predictability assessment**
10. Expert reviews, documentation

# Identifying relevant sources of uncertainties is a very important first step in a UQ study

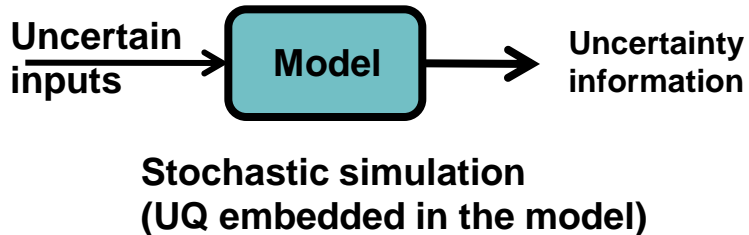


# Proper characterization of uncertainties is key to accurate propagation of uncertainties

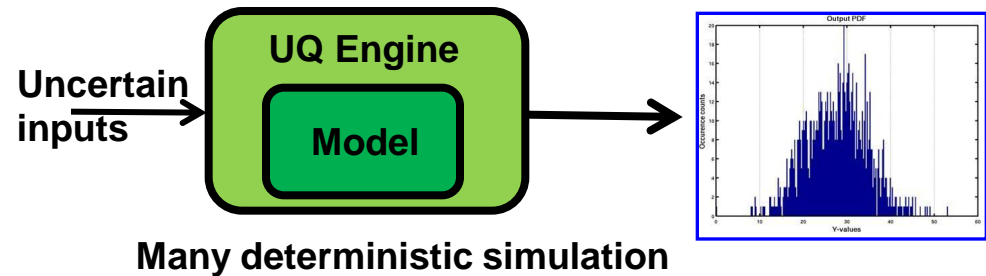
- **Aleatoric (known probability distributions)**
- **Epistemic**
  - unknown probability distributions
  - use intervals or belief functions
  - missing physics (will give systematic errors)
- **Mixed aleatoric/epistemic**
  - known pdfs, unknown means and/or standard deviations
- **Model form uncertainties**
  - many possible equations to represent the submodels
  - each sub-model may have its own aleatoric/epistemic uncertainties
- **Errors (considered as uncertainties?)**
  - discretization errors, roundoff errors, algorithmic errors

# Different approaches to propagate uncertainties

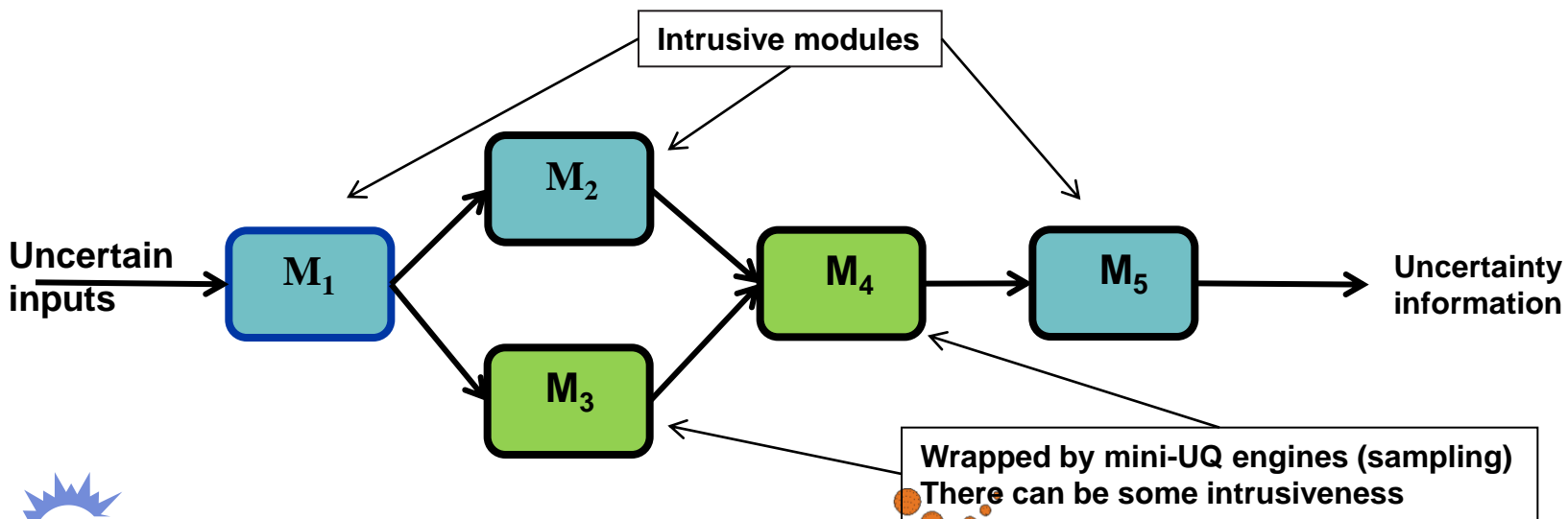
## ▪ Intrusive approach



## ▪ Non-intrusive approach



## ▪ hybrid approach for multi-physics (one scenario)

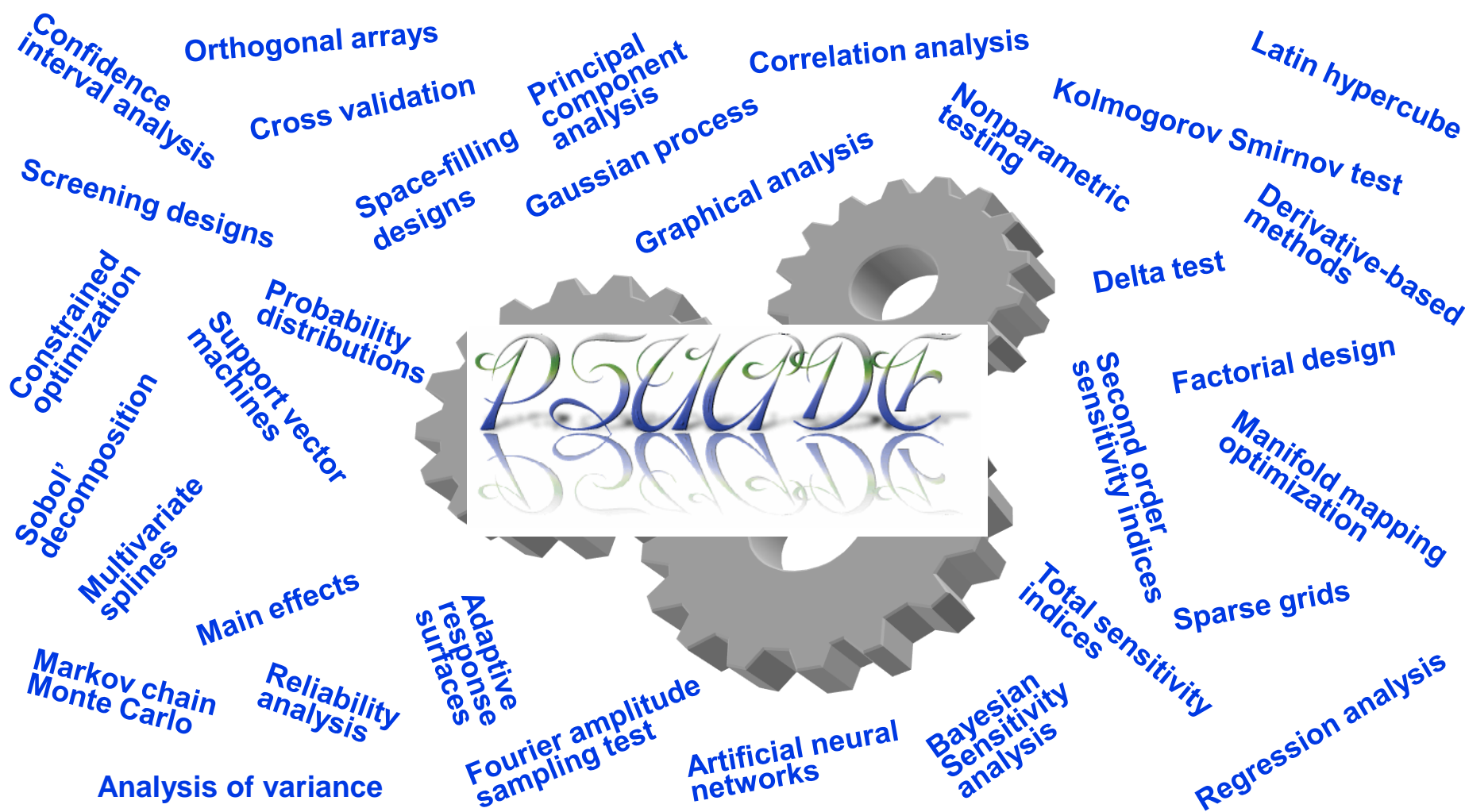


# Uncertainty propagation can be challenging for complex physics models due to

- Models may be expensive to evaluate (hours on many processors)
- Nonlinear (may be discontinuous) input-output relationships
- High-dimensionality of the uncertain parameters (10's -100's)
- Complex correlation between uncertain parameters
- Mostly epistemic uncertainties (maybe mixed aleatoric/epistemic)
- Model form (structural) uncertainties
- Different types of data at different physics modules/subsystems
- Data scarcity
- Model operating at different regime than experiments (extrapolation)
- Uncertainties mixed with numerical errors in operator splitting
- Unknown unknowns (unknown processes, unknown couplings)



# PSUADE (A Problem Solving environment of Uncertainty Analysis and Design Exploration) is a software library of UQ tools



# PSUADE is a toolkit for facilitating different UQ methodologies

- **Methodologies/methods: (arbitrary input inequality constraints)**
  - several dimension reduction methods
  - classical uncertainty analysis methods
  - many response surface methods (including adaptive)
  - several global sensitivity analysis methods
  - some basic risk assessment methods
  - numerical/stochastic optimization methods
  - hypothesis testing, principal component analysis
- **A job execution environment (to support automation)**
  - synchronous and asynchronous modes
  - dependency and chain modes (suitable for psub/moab)
  - multiple single-processor, multiple multiple-processor (intrusive)
- **An interactive user interface**
  - many ways of visualizing uncertainties

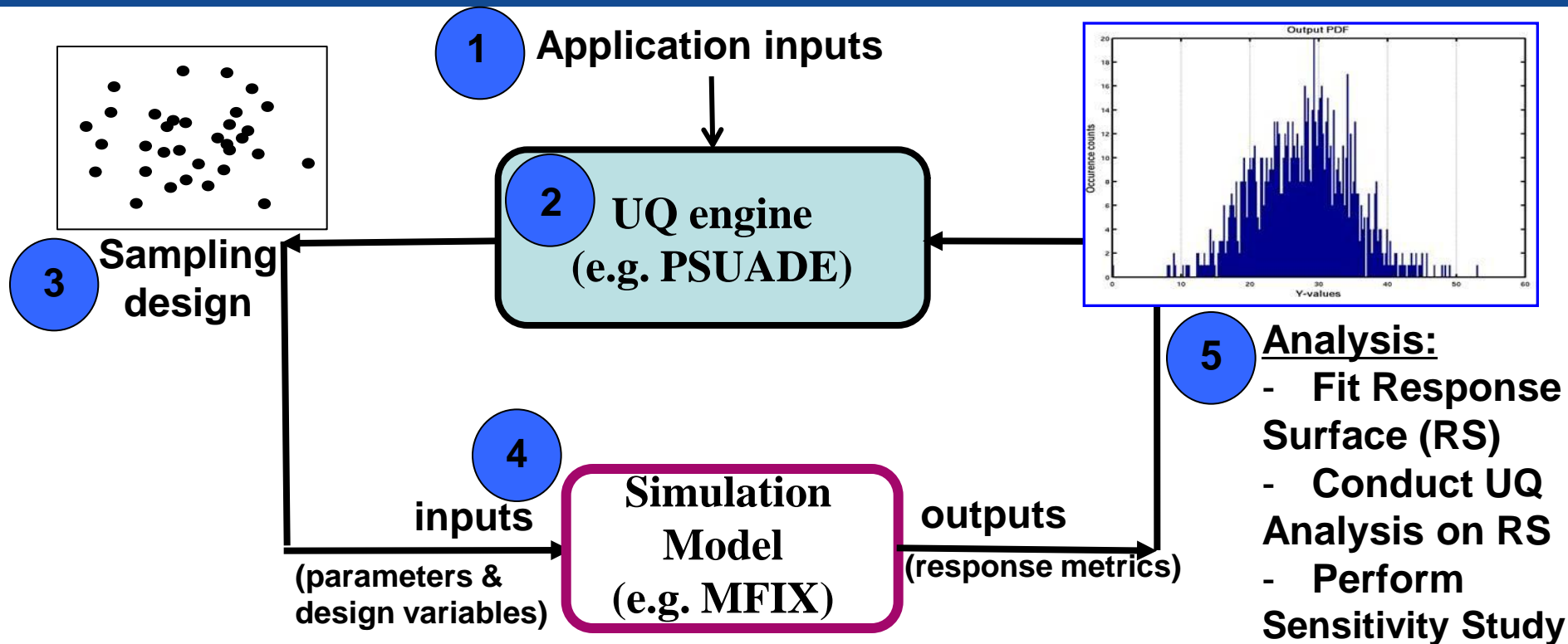


# Preliminary Results for Demonstration of Non-intrusive Uncertainty Quantification Study with MFIX Simulations:

- Sample Problem # 1: DES Fluidized Bed
- Sample Problem # 2: Gasification

- What impact do parameter/model uncertainties have on model outputs? Establish confidence levels & quantitative quality assessment in simulation results.
- Which parameters cause the most output uncertainties? [\[Sensitivity Analysis\]](#)
- How do output uncertainties affect input uncertainties? [\[Inverse UQ\]](#)
- How to use observed data to calibrate system parameters? [\[Calibration\]](#)
- In view of uncertainty, how to quantify risk?

# Non-intrusive Uncertainty Quantification



- No need to modify simulation models: “black boxes”
- No need for analysis of the mathematical structures in the model
- May require large sample size for sufficient accuracy

# MFIX, Open Source Multiphase Flow Code

Mass conservation for phase m (m=g for gas and s for solids)

$$\frac{\partial}{\partial t} (\varepsilon_m \rho_m) + \nabla \cdot (\varepsilon_m \rho_m \vec{v}_m) = \sum_{l=1}^{N_m} R_{ml}$$

Momentum conservation

$$\frac{\partial}{\partial t} (\varepsilon_m \rho_m \vec{v}_m) + \nabla \cdot (\varepsilon_m \rho_m \vec{v}_m \vec{v}_m) = \nabla \cdot \bar{\bar{S}}_m + \varepsilon_m \rho_m \vec{g} + \sum_n \vec{I}_{mn}$$

Granular energy conservation (m ≠ g)

$$\frac{3}{2} \varepsilon_m \rho_m \left( \frac{\partial \Theta_m}{\partial t} + \vec{v}_m \cdot \nabla \Theta_m \right) = \nabla \cdot \vec{q}_{\Theta_m} + \bar{\bar{S}}_m : \nabla \vec{v}_m - \varepsilon_m \rho_m J_m + \Pi_{\Theta_m}$$

Energy conservation

$$\varepsilon_m \rho_m C_{pm} \left( \frac{\partial T_m}{\partial t} + \vec{v}_m \cdot \nabla T_m \right) = -\nabla \cdot \vec{q}_m + \sum_n \gamma_{mn} (\rho_n - T_m) - \Delta H_{rm}$$

Species mass conservation

$$\frac{\partial}{\partial t} (\varepsilon_m \rho_m X_{ml}) + \nabla \cdot (\varepsilon_m \rho_m X_{ml} \vec{v}_m) = R_{ml}$$

**MFIX**  
<http://mfix.netl.doe.gov>



R&D100  
Award 2007



Excellence in  
Technology Transfer  
Award 2008 for

C3M

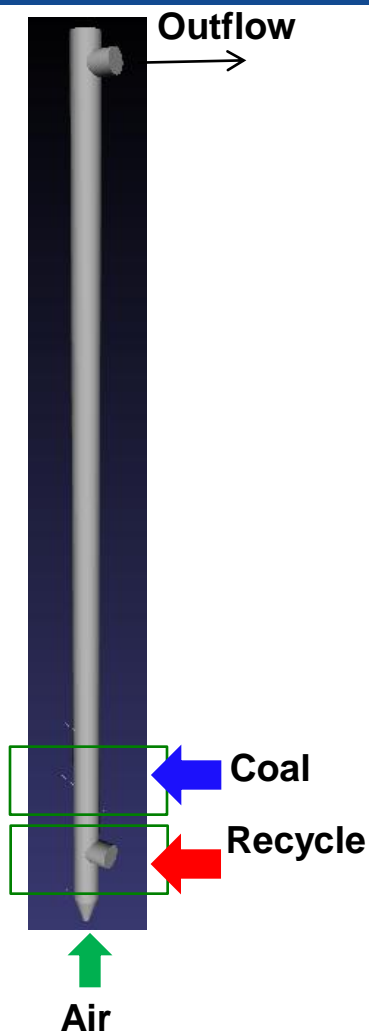


Tech-Transfer  
Award 2006

- Syamlal et al. "MFIX Documentation, Theory Guide," DOE/METC-94/1004, NTIS/DE94000087 (1993)
- Benyahia et al. "Summary of MFIX Equations 2005-4", From URL <http://www.mfix.org/documentation/MfixEquations2005-4-3.pdf>, July 2007.



# Demonstration Problem for Parametric Non-Intrusive UQ: Gasification



## Problem Setup and Properties:

**Solids:** Rosebud coal with  $D_p = 0.01$  cm,  $\rho_p = 2.85$  g/cm<sup>3</sup>  
Coal flow rate: 1 g/s, Recycled char :100 g/s

**Gas:** Air flow rate: 2.76 g/s

**Geometric dimensions** = 10 cm x 200 cm

**Grid Resolution** = (10 x 200) cells (**2-D simulation**)

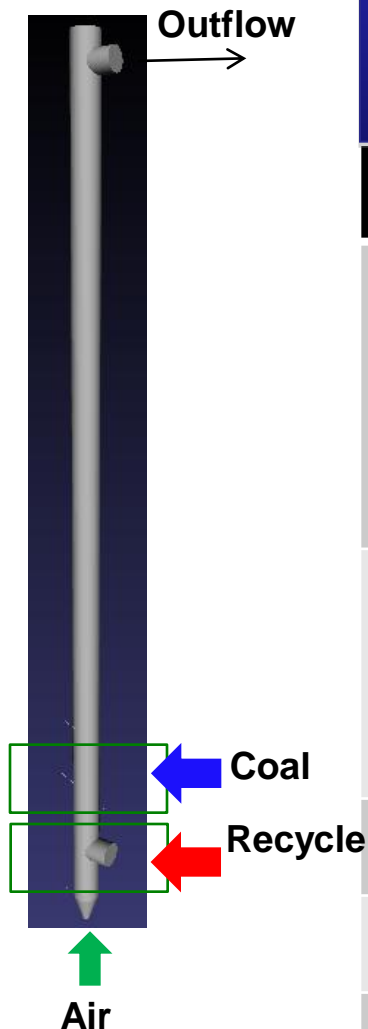
**Governing Physics & Models:** Multiphase flow hydrodynamics, heat transfer, chemical reactions.

**Numerical Scheme:** Spatial discr. : Upwind  
Temporal discr.: 1<sup>st</sup> order

Test problem provided by Dr.Tingwen Li



# Demonstration Problem for Parametric Non-Intrusive UQ: Gasification (con't)



**Objective: Determine the effect of uncertainty in reactions rates on the species mass composition at the outlet of the gasifier.**

## Uncertainty Quantification Study Properties:

### Input parameters with Uncertainty (min-max range):

- (1) Reaction rate constant for CO<sub>2</sub> gasification  
C(6) : 0.1 – 10,100,1000.0 [Uniform distribution]
- (2) Reaction rate constant for devolatilization  
C(8) : 0.1 – 10,100,1000.0 [Uniform distribution]

### Response Variables:

- (1) CO species mass fraction at the outlet
- (2) CH<sub>4</sub> species mass fraction at the outlet
- (3) H<sub>2</sub> species mass fraction at the outlet

**UQ Toolbox/Engine: PSUADE from LLNL**

**Sampling Method = LPTAU, Sample Size = 100, 1024**

**Computational Cost to simulate 40 seconds  
Per sample: 1 to 1.5 hrs wallclock on single core**





# Demonstration Problem for Parametric UQ Study: Gasification

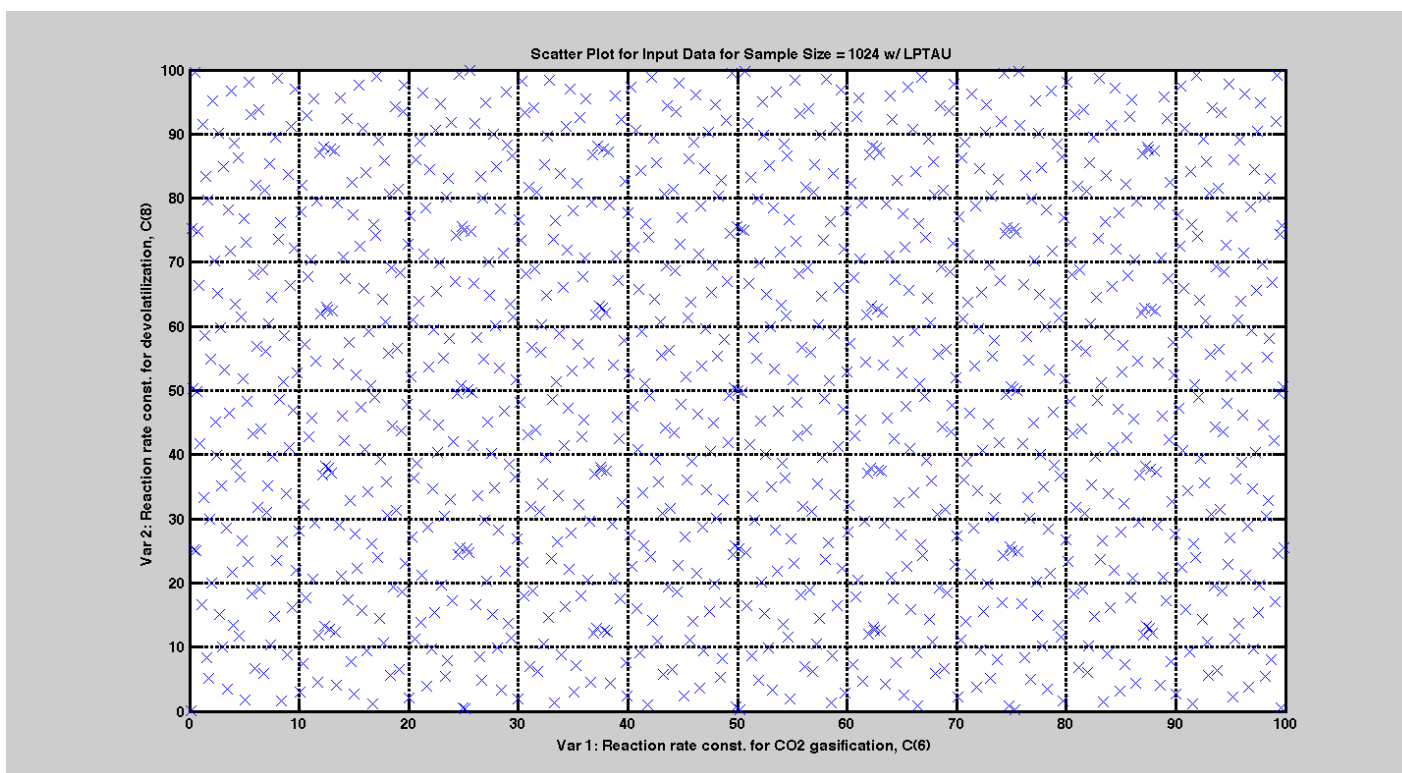
Sample size = 1024

Variables with uncertainty:

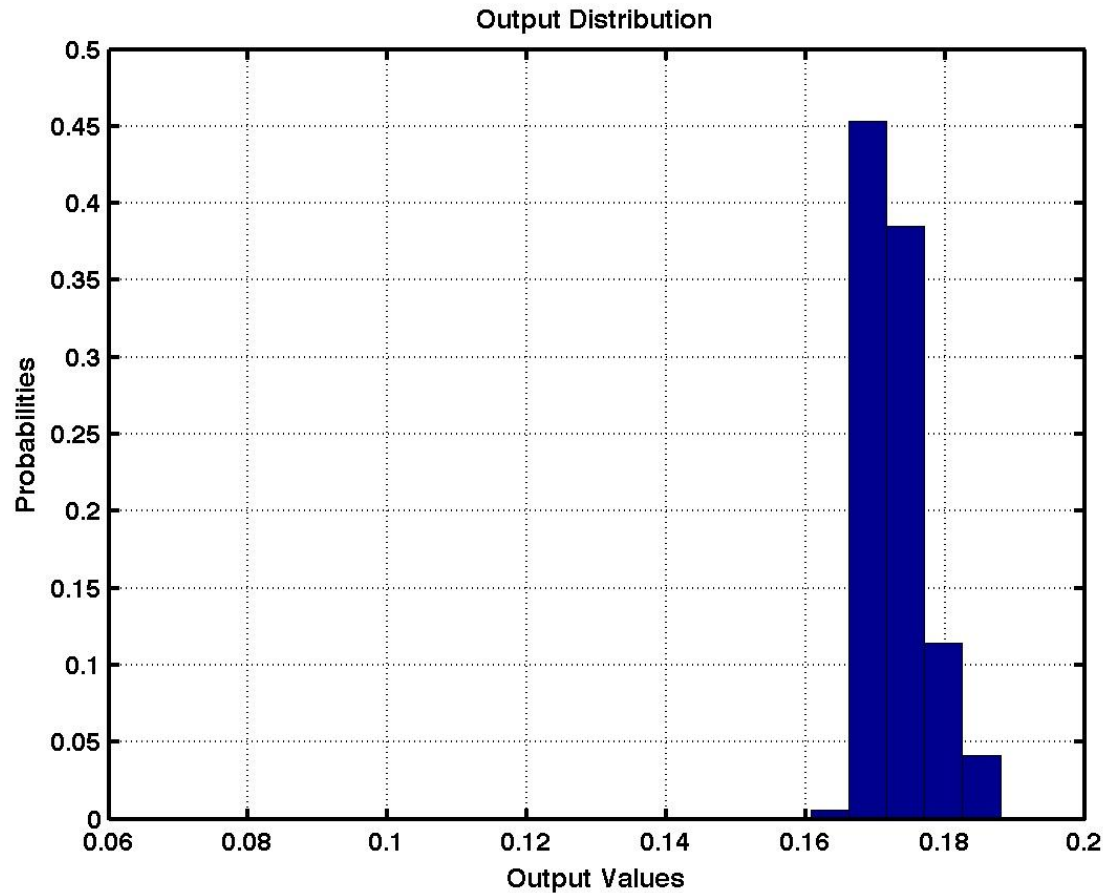
lower – upper bound

(1) Reaction rate constant for CO<sub>2</sub> gasification, C(6) : 0.1 - 100

(2) Reaction rate constant for devolatilization, C(8) : 0.1 - 100



# Histogram of Output 1 : CO mass fraction (Xg\_CO)



Sample mean = 1.7295e-01

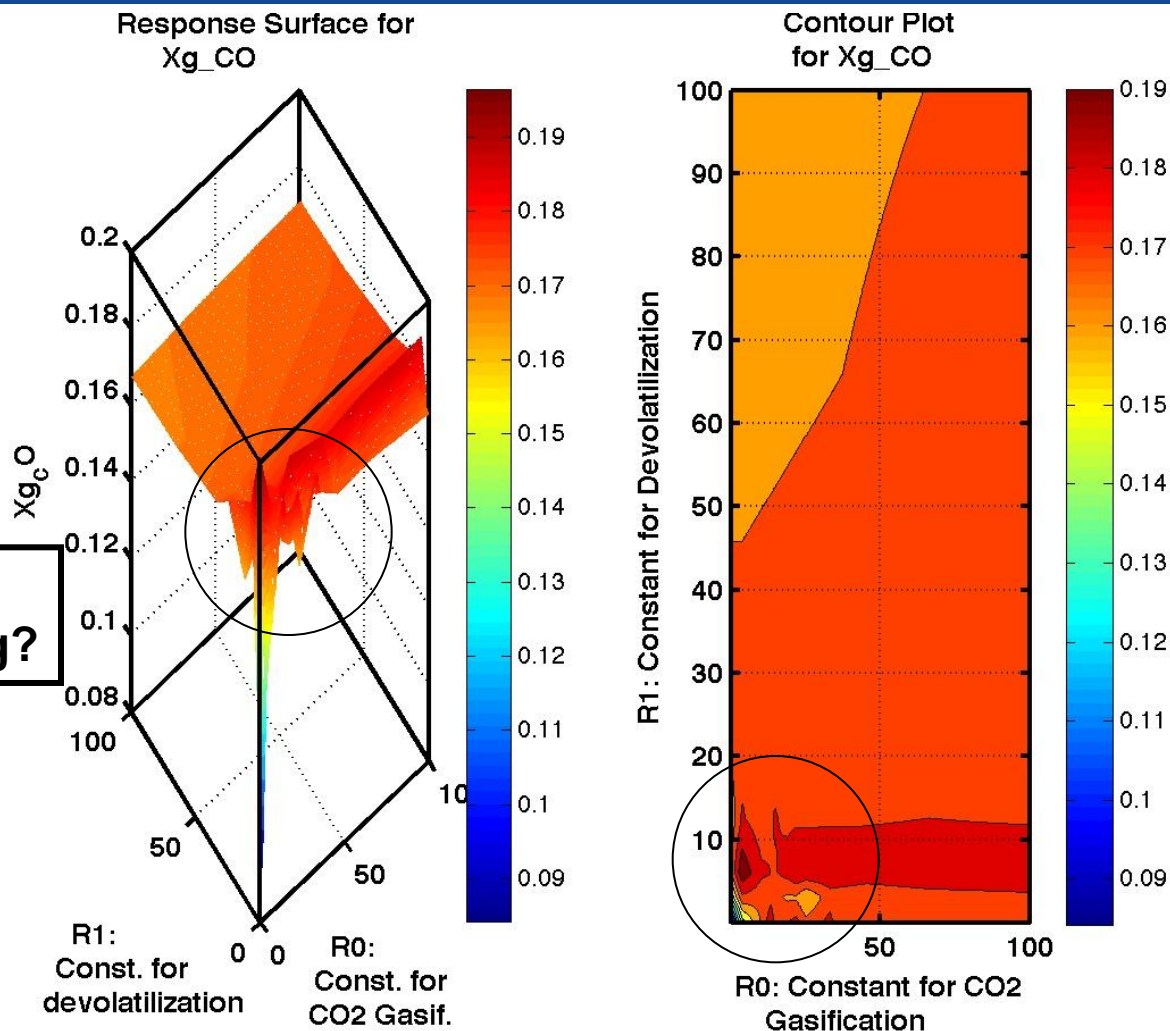
Sample std dev = 5.0652e-03



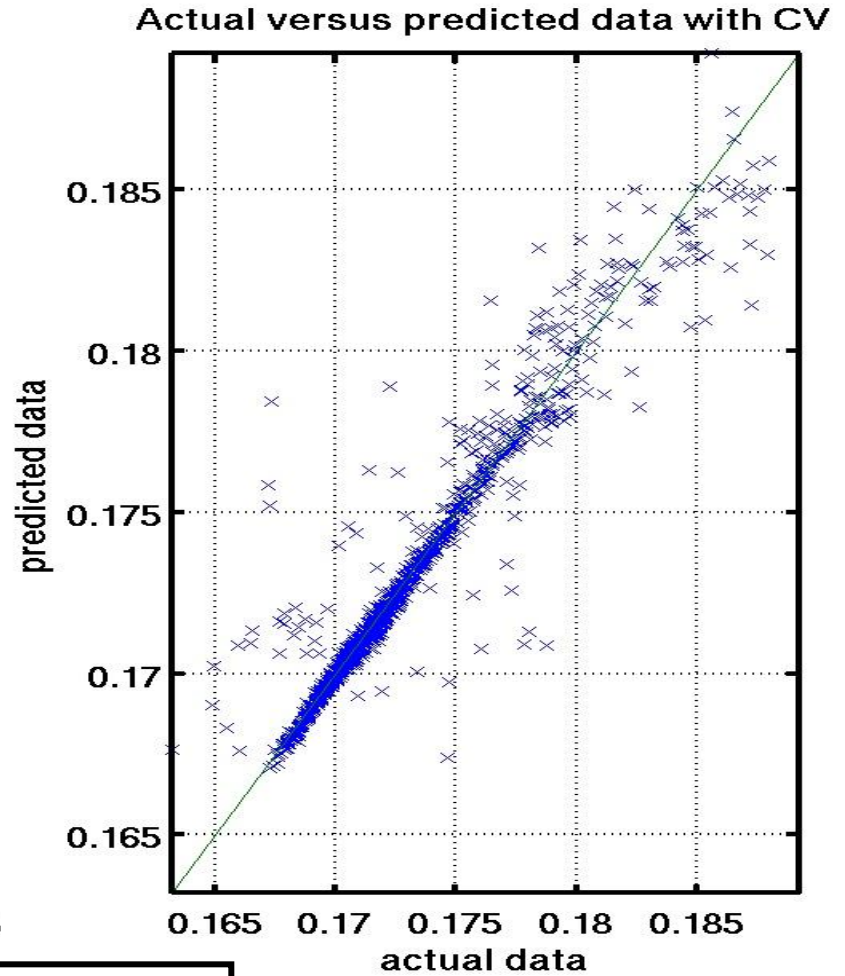
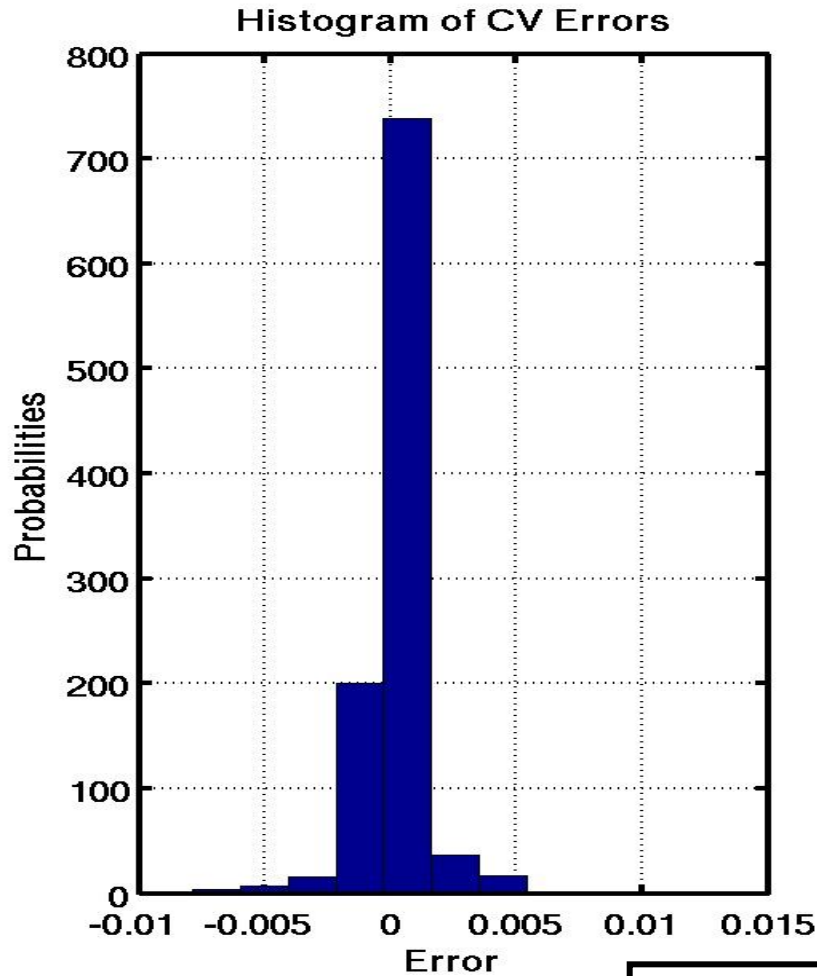


# Response Surface for CO mass fraction ( $X_{g\_CO}$ ): Cubic splines based method (MARS)

What is happening?



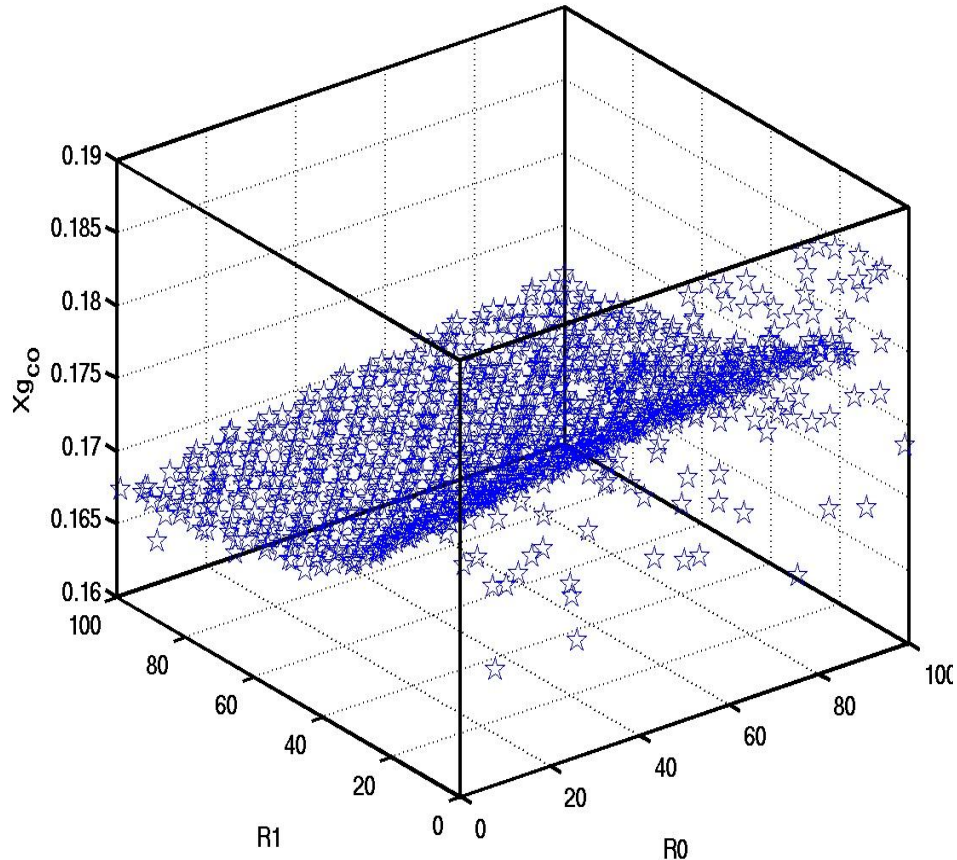
# Response Surface Analysis (Xg\_CO):



**Not acceptable!!**

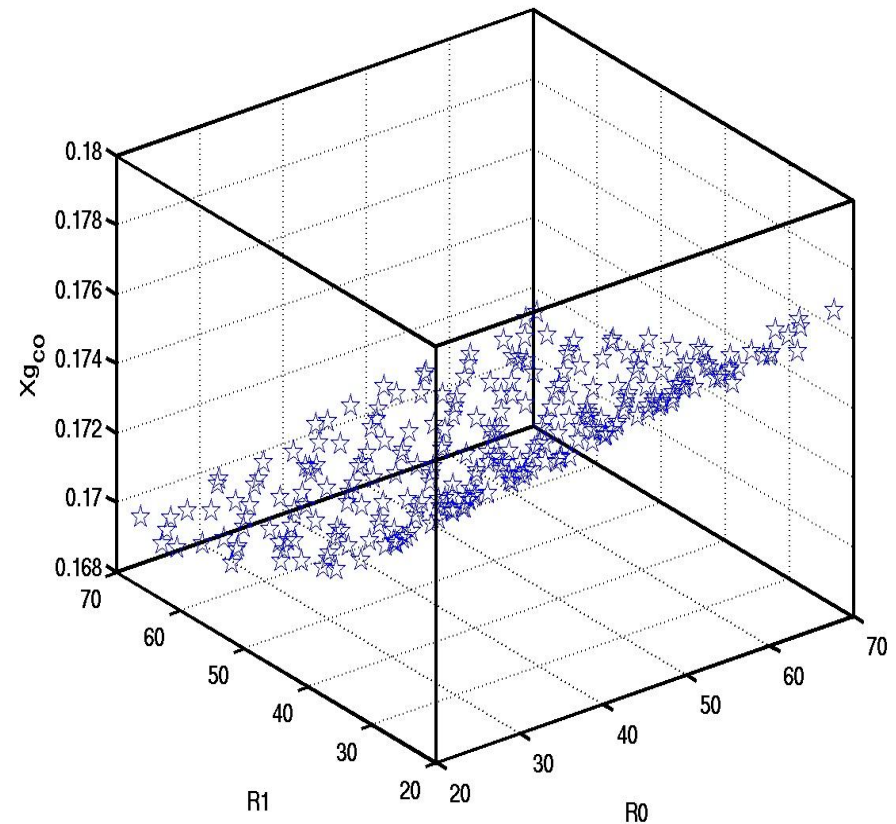
# What happened? Let's examine $X_{g\_CO}$ more closely.

3D 2-Input/1-Output Data Plot



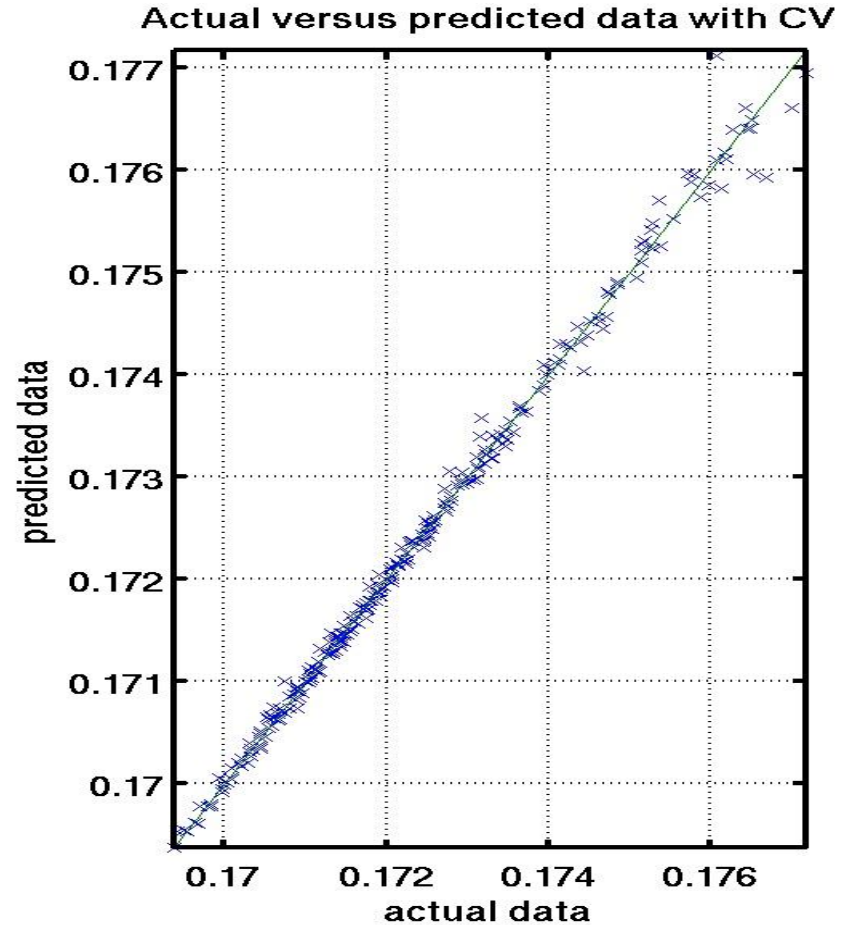
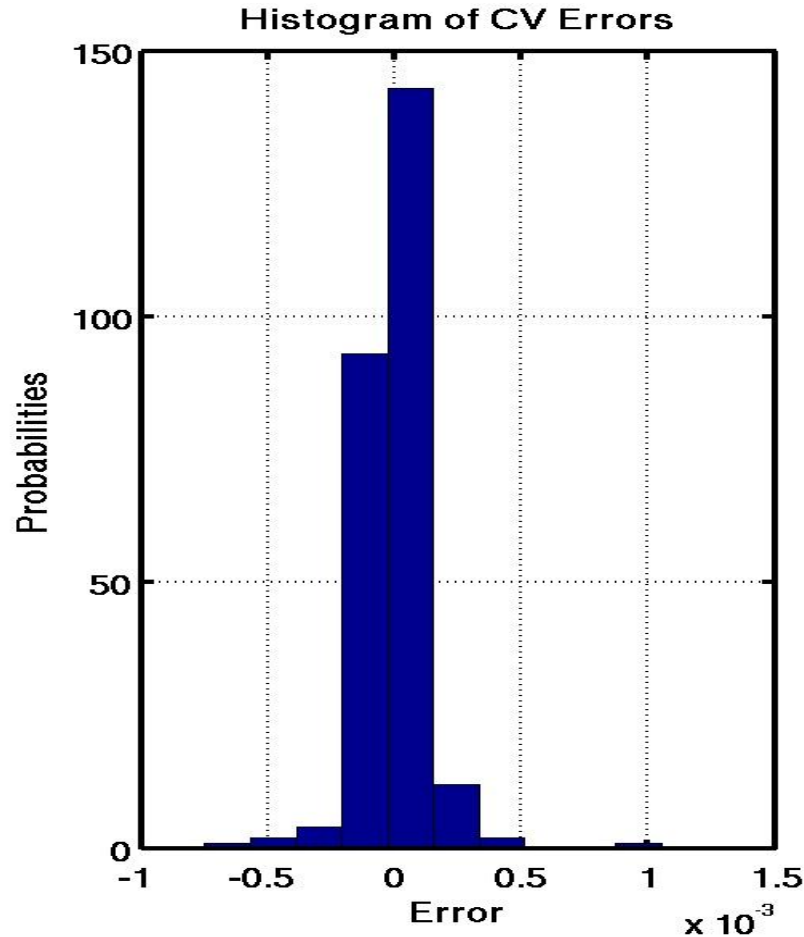
**Many outliers on the edge**

3D 2-Input/1-Output Data Plot



**Alter the input range**

# Response Surface Analysis (on Xg\_CO): on small range



**Much more acceptable!!**

## Sensitivity Analysis for CO mass fraction:

- Using the response surface for  $Xg\_CO$ , we compute the global sensitivity indices for both input variables.
- Assume uniform distributions for input uncertainties.
- The sample mean is 0.173
- The sample standard deviation is 0.005
- For this specific example problem % of variance from each input is determined as:
  - Input # 1: Reaction rate const. for CO<sub>2</sub> gasification ~ **10%**
  - Input # 2: Reaction rate constant for devolatilization ~ **90%**

# Summary

- **UQ activities recently started within Multiphase Flow Group, work in progress.**
- **Several challenges to perform UQ in multiphase reacting flows:**
  - Many uncertain parameters exist,
  - Highly nonlinear,
  - Transient behavior,
  - Computationally intensive simulations,
  - No assurance all samples will converge
- **The trade-off between sample size and non-intrusive UQ analysis accuracy due to computational cost per sample.**



# Thank you!

# Questions ?



CCSI  
Carbon Capture Simulation Initiative



# APPENDIX



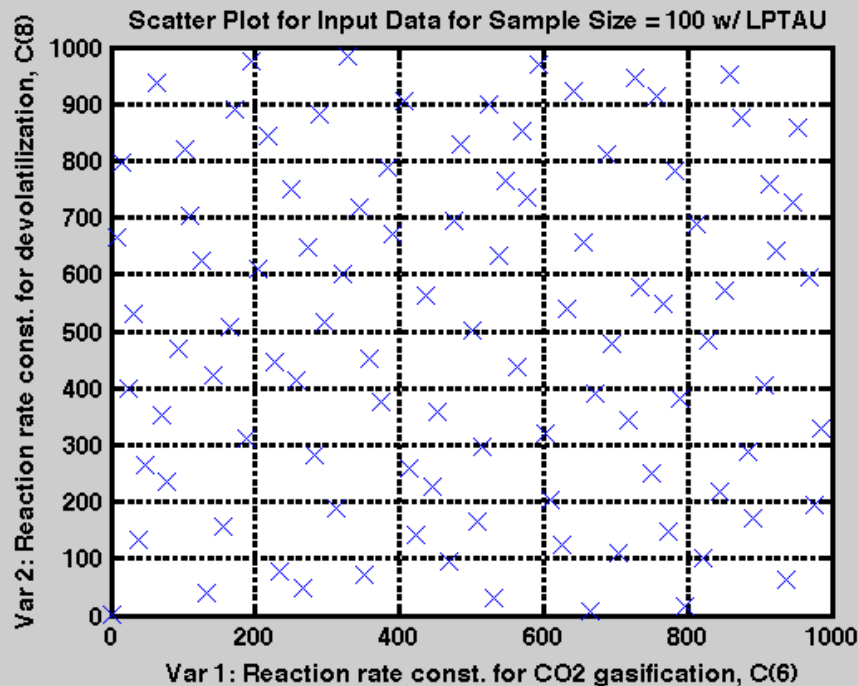
# Does the Sample Size Matter? A Comparison.

## Sampling Method: LPTAU

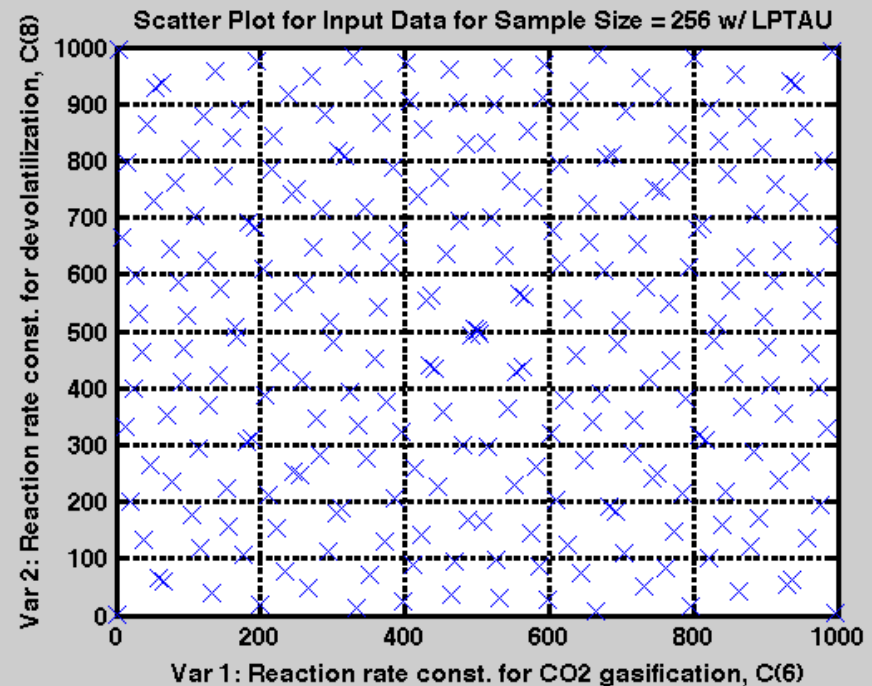
Variables with uncertainty:

- |   |     |   |      |
|---|-----|---|------|
| (1) Reaction rate constant for CO <sub>2</sub> gasification, C(6) : | 0.1 | - | 1000 |
| (2) Reaction rate constant for devolatilization, C(8) :             | 0.1 | - | 1000 |

lower – upper bound



Sample size = 100



Sample size = 256

# Does the Sampling Method Matter? A Comparison.

Sample size = 256

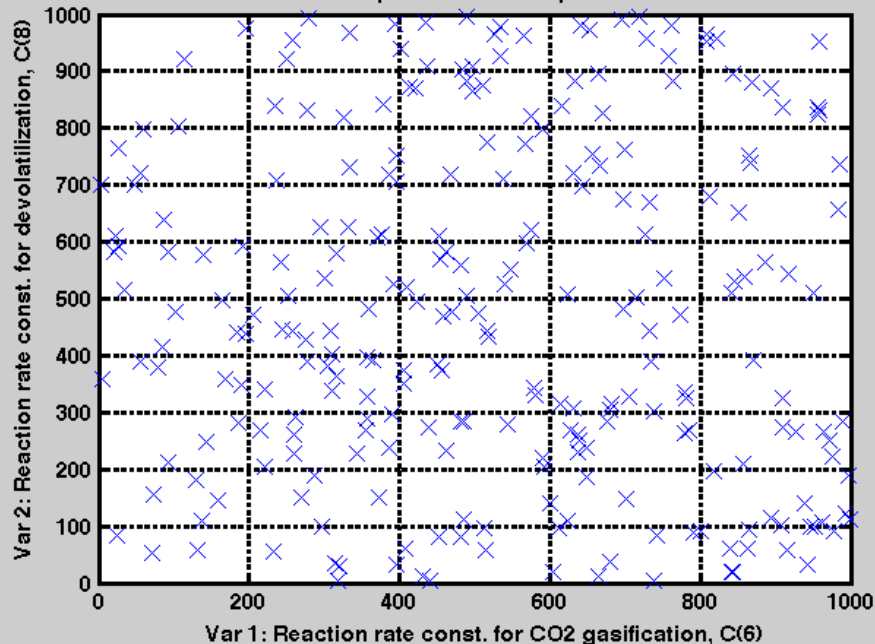
Variables with uncertainty:

(1) Reaction rate constant for CO<sub>2</sub> gasification, C(6) : 0.1 - 1000

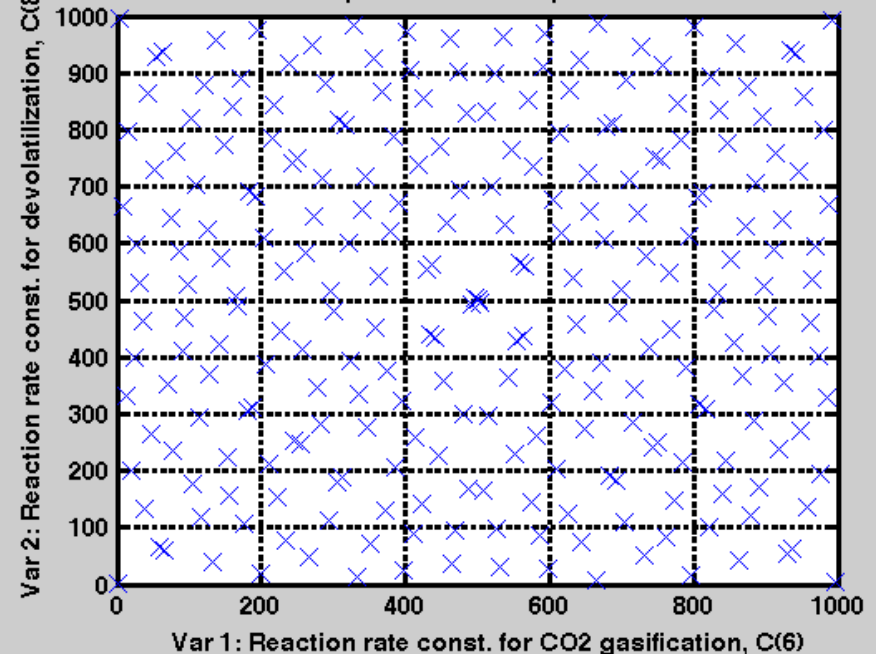
lower – upper bound

(2) Reaction rate constant for devolatilization, C(8) : 0.1 - 1000

Scatter Plot for Input Data for Sample Size = 256 w/ MC



Scatter Plot for Input Data for Sample Size = 256 w/ LPTAU



Monte Carlo Sampling (MC)

Quasi Random Sequence Generator  
Sampling (LPTAU)

# Does the Sampling Method Matter? A Comparison. (cont'd)

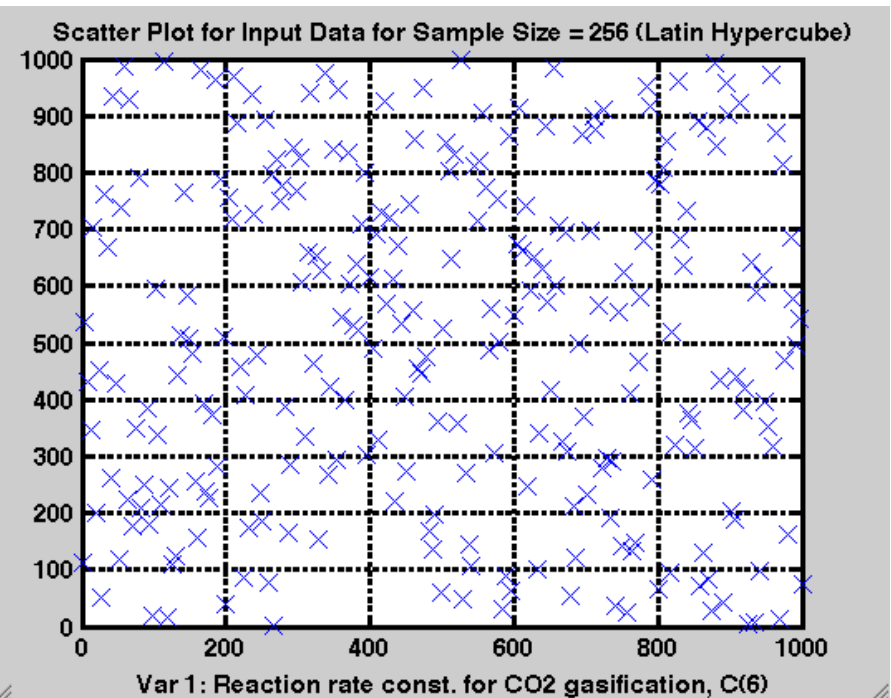
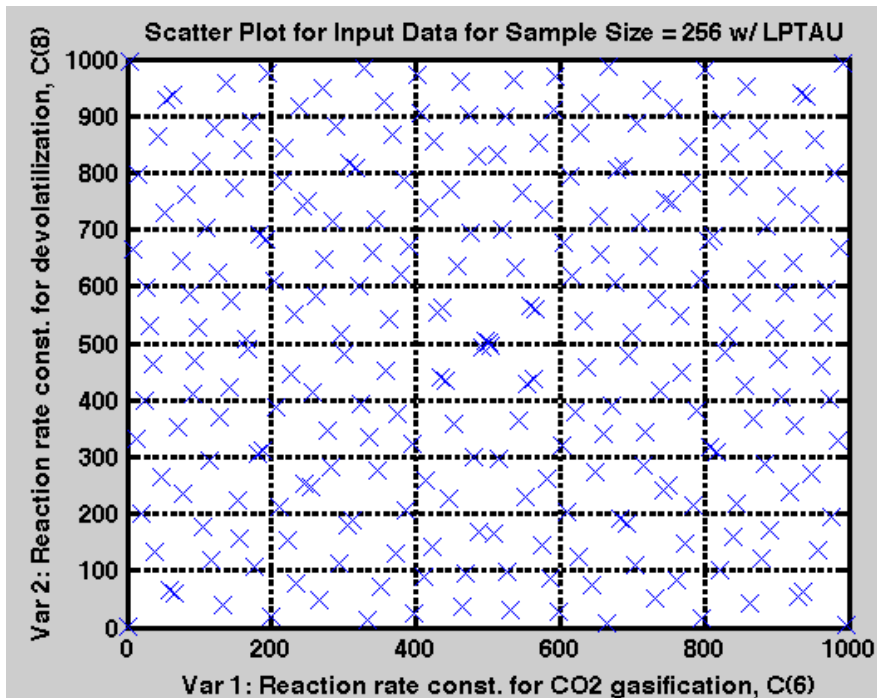
Sample size = 256

Variables with uncertainty:

(1) Reaction rate constant for CO<sub>2</sub> gasification, C(6) : 0.1 - 1000

lower – upper bound

(2) Reaction rate constant for devolatilization, C(8) : 0.1 - 1000



Quasi Random Sequence Generator  
Sampling (LPTAU)

Latin Hypercube Sampling (LH)

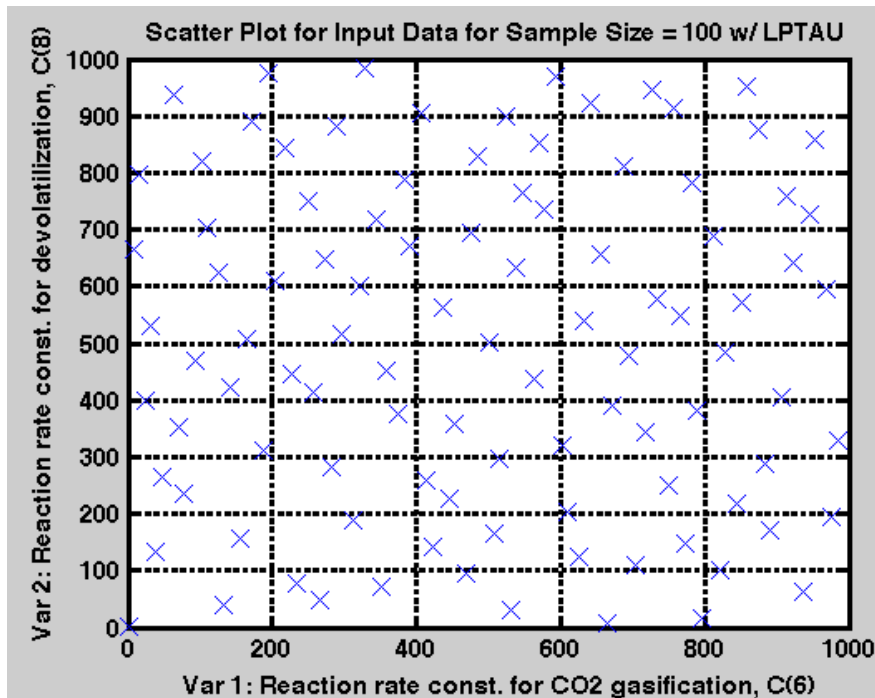
# Does the Sample Size Matter? A Comparison. (cont'd)

## Sampling Method: LPTAU

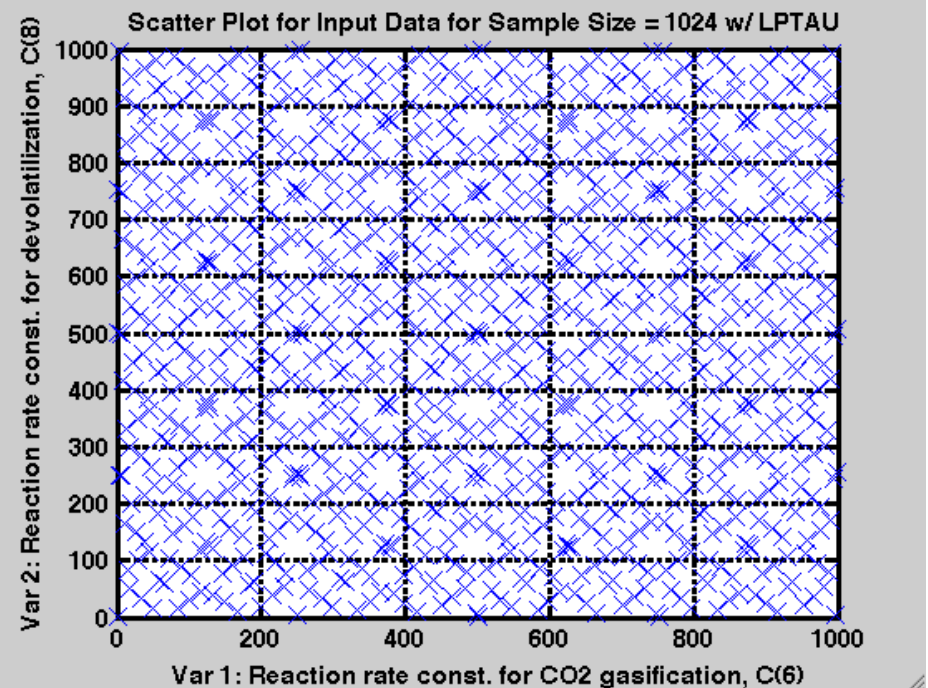
Variables with uncertainty:

- |   |     |   |      |
|---|-----|---|------|
| (1) Reaction rate constant for CO <sub>2</sub> gasification, C(6) : | 0.1 | - | 1000 |
| (2) Reaction rate constant for devolatilization, C(8) :             | 0.1 | - | 1000 |

lower – upper bound



Sample size = 100



Sample size = 1024

# Sample Problem # 2 for Parametric UQ Study: Gasification

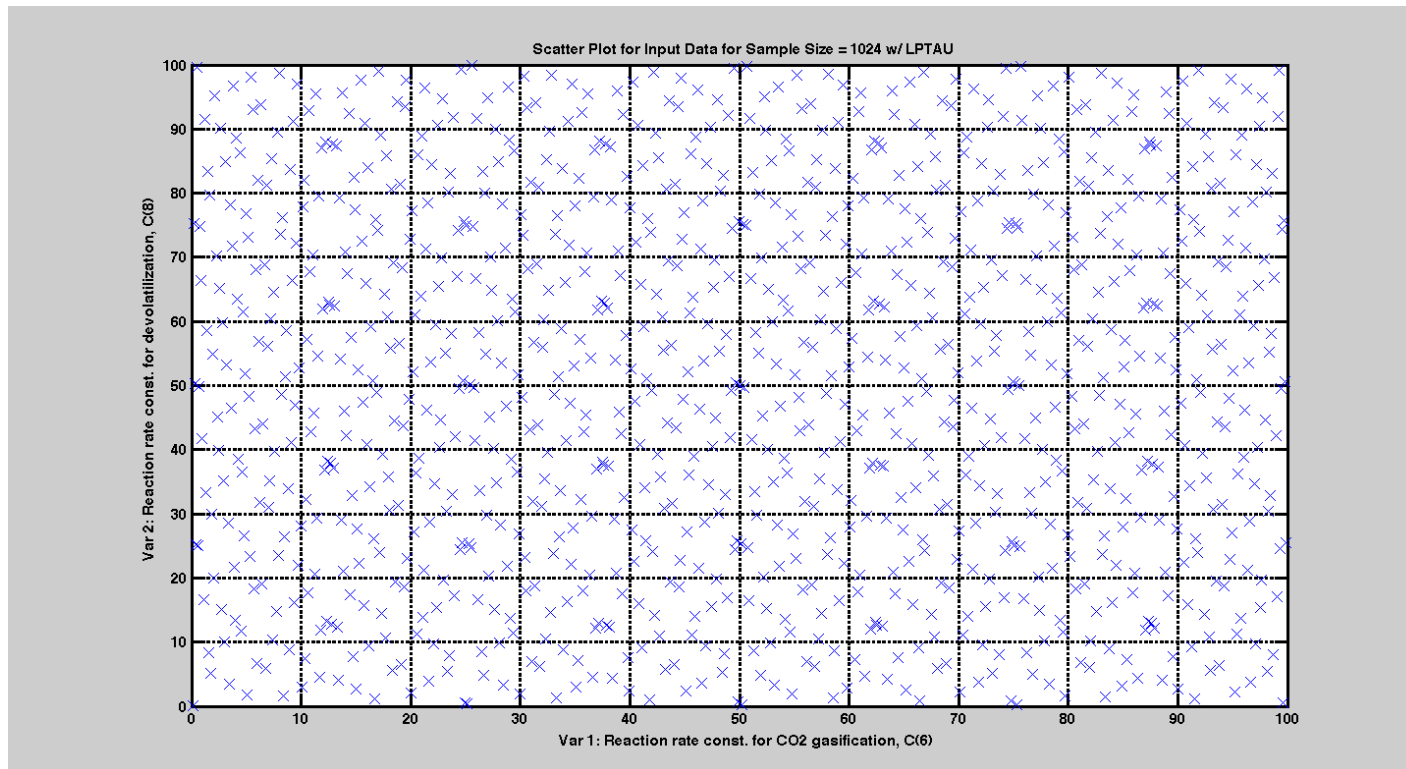
Sample size = 1024

Variables with uncertainty:

lower – upper bound

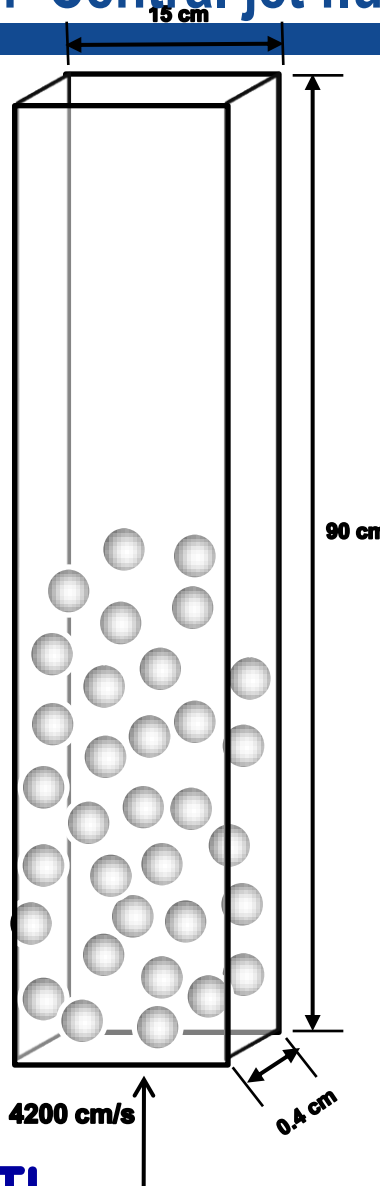
(1) Reaction rate constant for CO<sub>2</sub> gasification, C(6) : 0.1 - 100

(2) Reaction rate constant for devolatilization, C(8) : 0.1 - 100



# Sample Problem # 1 for Parametric Non-Intrusive UQ

## Study: Central jet fluidized bed



**Objective: Determine the effect of uncertainty in coefficients of restitution and friction on bed expansion and bubbling behavior.**

### Problem Setup and Properties:

**Solids:**  $D_p = 0.4 \text{ cm}$ ,  $\rho_p = 2.7 \text{ g/cm}^3$

Initial solid volume fraction: 0.4 up to height of 20 cm  
5 parcels per cell

**Gas:** Air at standard conditions

Fluidization velocity = 4200 cm/s with no slip BC at walls

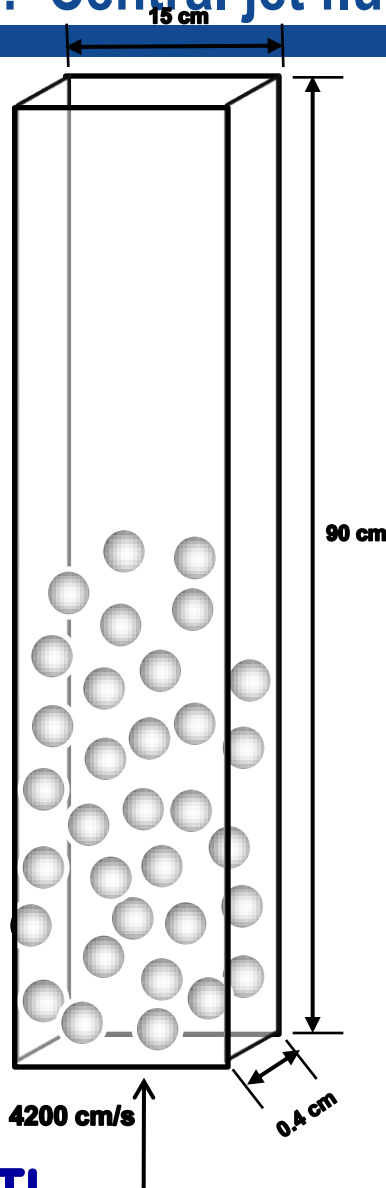
**Geometric dimensions** =  $(15 \times 90 \times 0.4) \text{ cm}^3$

**Grid Resolution** =  $(15 \times 45)$  cells **(2-D simulation)**

**Governing Physics & Models:** Multiphase flow hydrodynamics with DEM, Drag model: Wen & Yu/Ergun,

**Numerical Scheme:**  $\Delta t_{\max} = 1.E-03.$  , First order upwind

# Sample Problem # 1 for Parametric Non-Intrusive UQ Study: Central jet fluidized bed (cont'd)



**Objective: Determine the effect of uncertainty in coefficients of restitution and friction on bed expansion and bubbling behavior.**

## Uncertainty Quantification Study Properties:

### Input parameters with Uncertainty (min-max range):

- (1) Particle-particle coefficient of restitution  
 $e_n = 0.6 - 1.0$  [Uniform distribution]
- (2) Particle-wall coefficient of restitution  
 $e_n = 0.6 - 1.0$  [Uniform distribution]

### Response Variables:

- (1) Average bed expansion height (cm?)
- (2) Average pressure drop (??)

**UQ Toolbox/Engine: PSUADE from LLNL**

**Sampling Method = LPTAU, Sample Size = 24, 100**

**Computational Cost to simulate 20 seconds  
Per sample: 2 to 2.5 hrs wallclock on single core**



# Does Sample Size Matter?

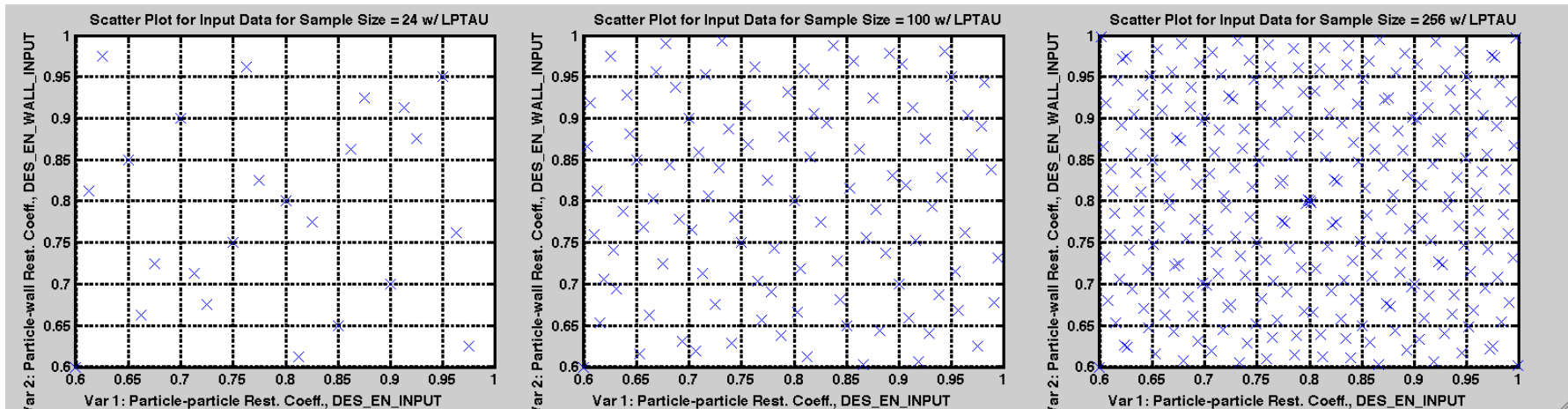
## Quasi Random Sequence Generator Sampling (LPTAU)

Variables with uncertainty:

lower – upper bound

(1) Particle-particle Restitution Coeff., DES\_EN\_INPUT : 0.6 - 1.0

(2) Particle-wall Restitution Coeff., DES\_EN\_WALL\_INPUT: 0.6 - 1.0



Sample size = 24

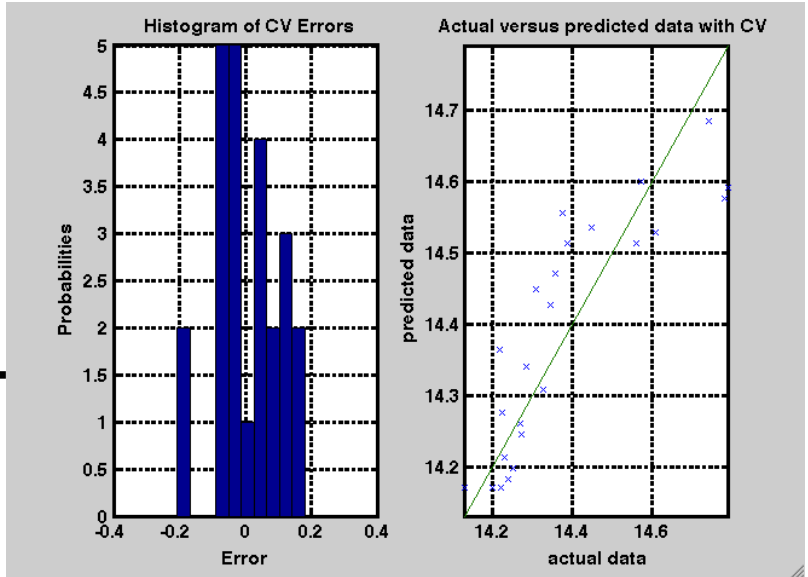
Sample size = 100

Sample size = 256

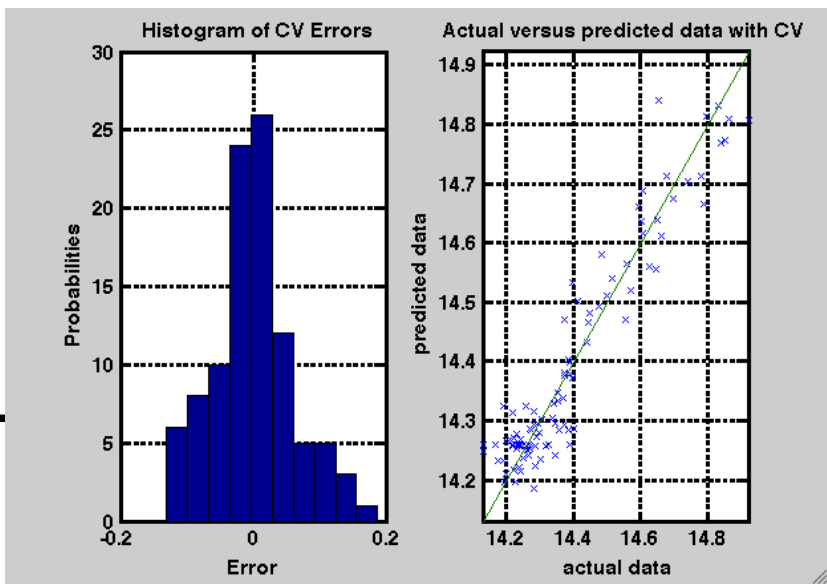


# Comparison of Fitness Quality of Response Surface for Output 1: Avg. Bed Height for different sample size runs:

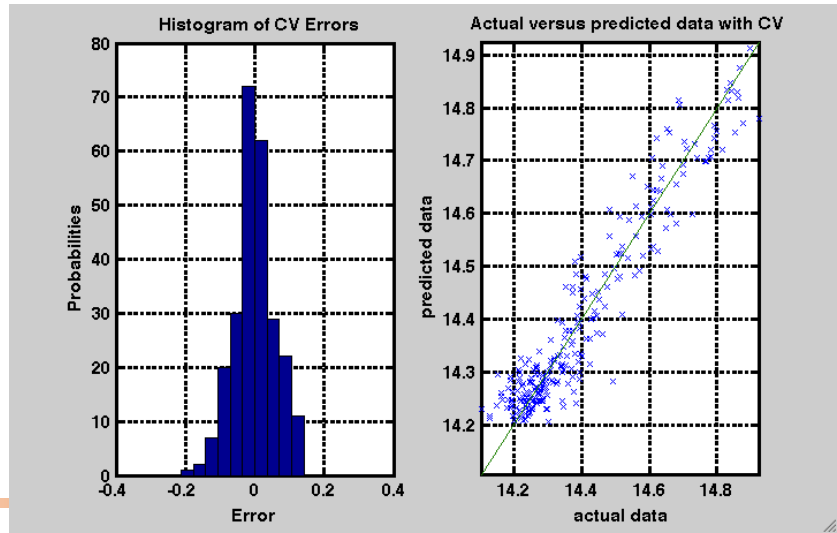
**Sample Size = 24**



**Sample Size = 100**



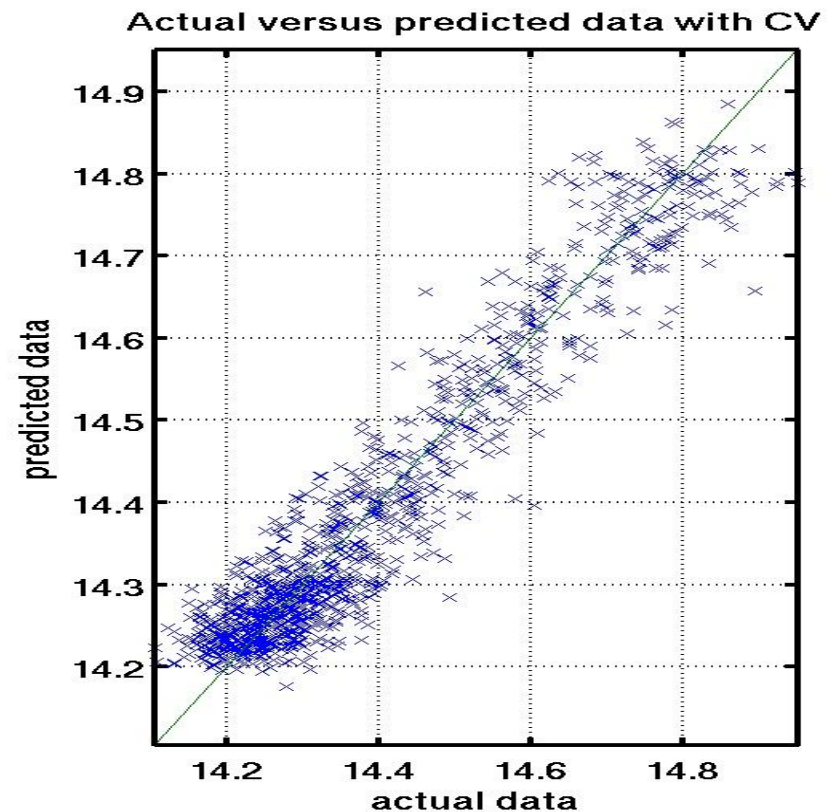
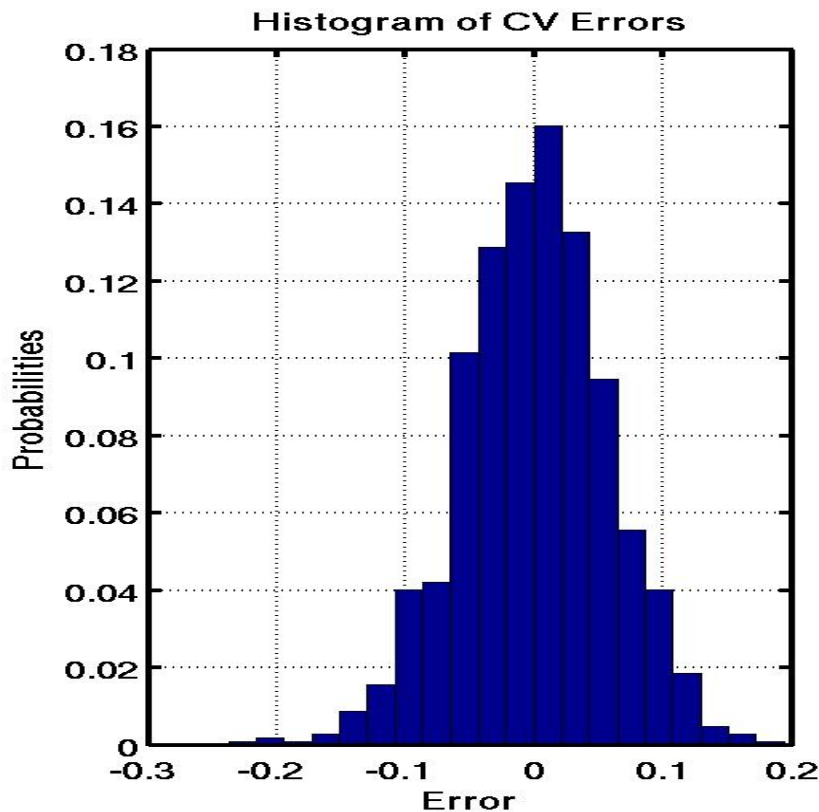
**Sample Size = 256**



**Response Surface Method: Multivariate Adaptive Regression Splines (MARS)**

# Fluidized Bed Data Analysis (Avg\_h)

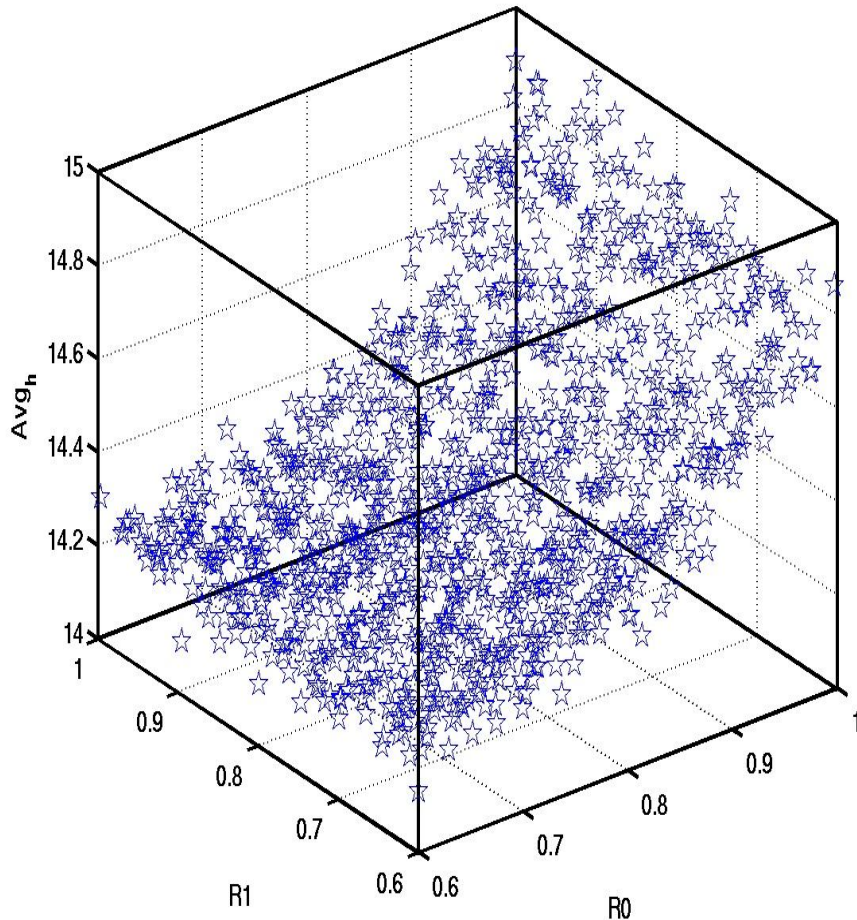
- response surface analysis using cubic splines gives



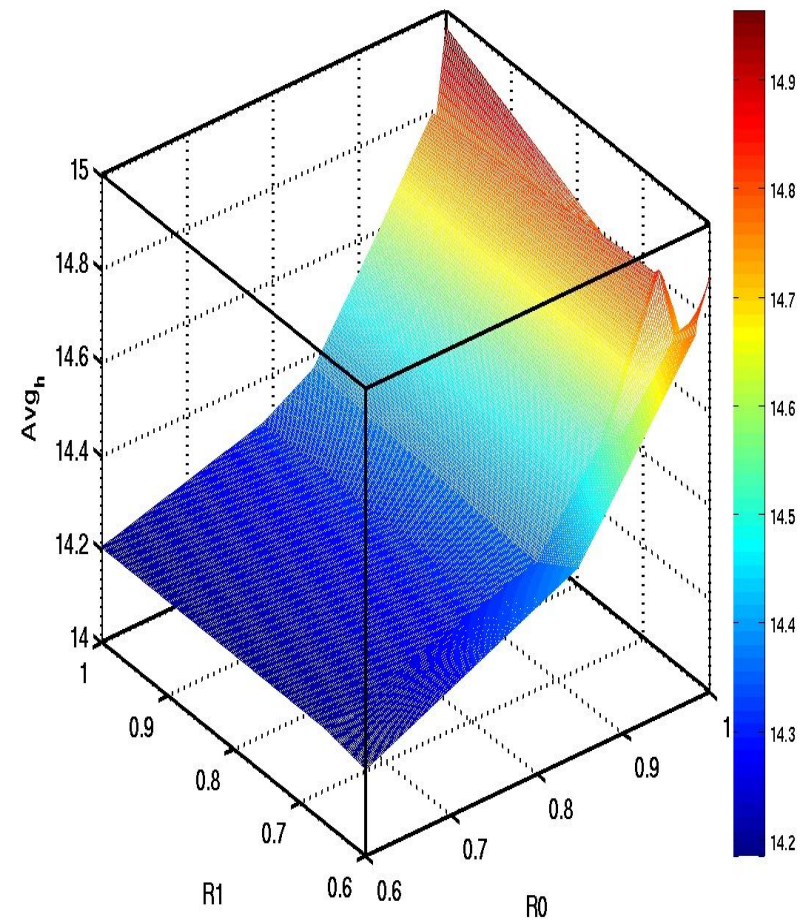
- max prediction uncertainty  $\sim 0.2$  (<2%)
- but it is large relative to the output range (0.7)

# Comparing data and response surface (Avg\_h)

3D 2-Input/1-Output Data Plot

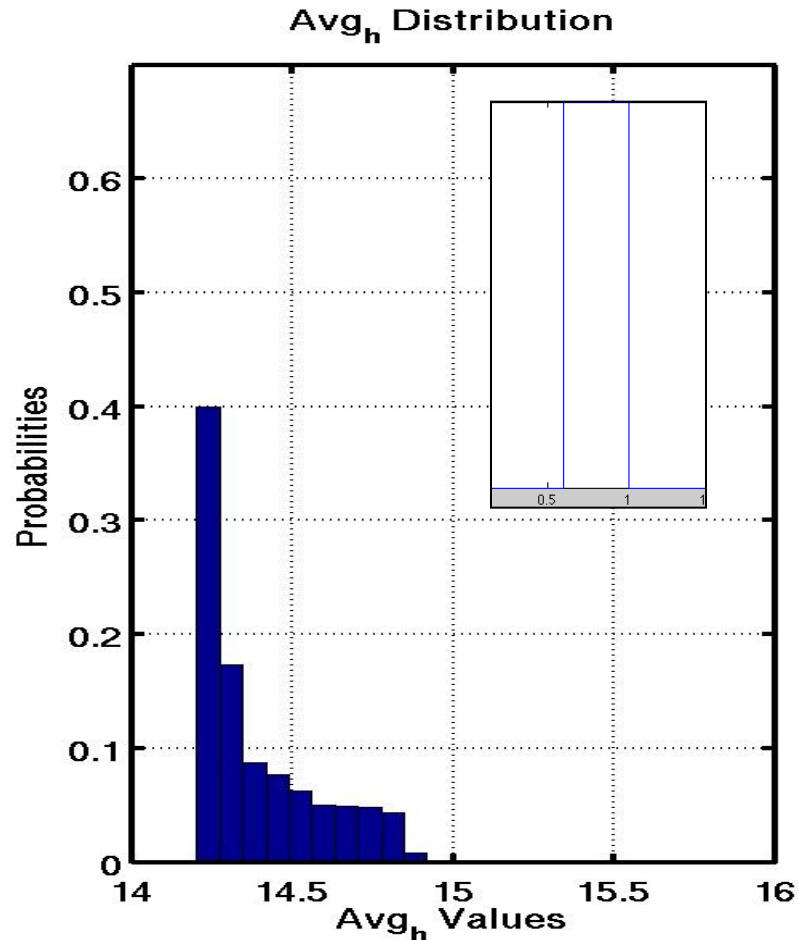


Avg<sub>h</sub> Response Surface

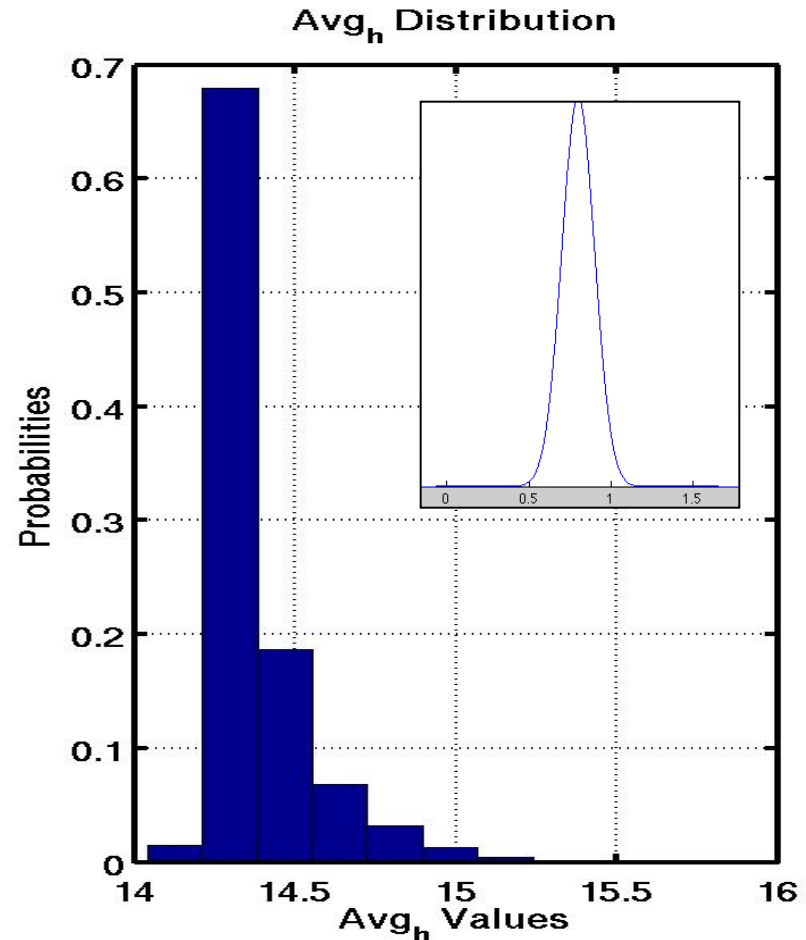


**Q: is Avg<sub>h</sub> smoothly varying with R0 and R1?**

# If R0 and R1 are uncertain, the output is also uncertain



**R0, R1 ~ Uniform(0.6, 1.0)**

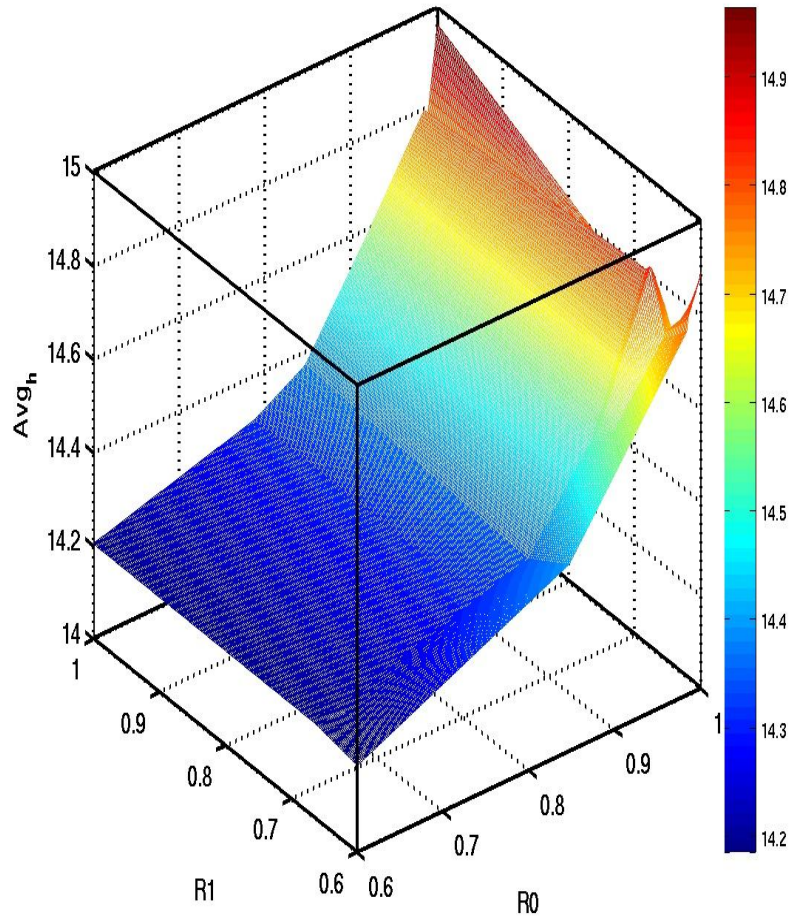


**R0, R1 ~ Normal(0.8, 0.1)  
(give more weight to center)**

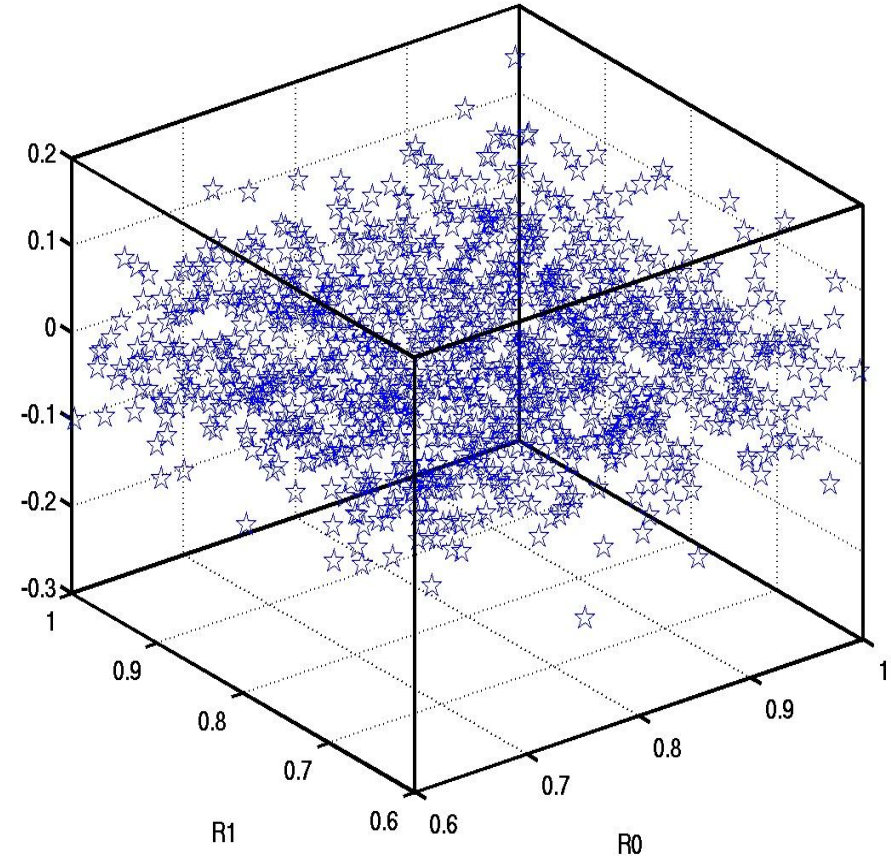


# What happened? Let's examine Avg\_dp more closely.

Avg<sub>h</sub> Response Surface



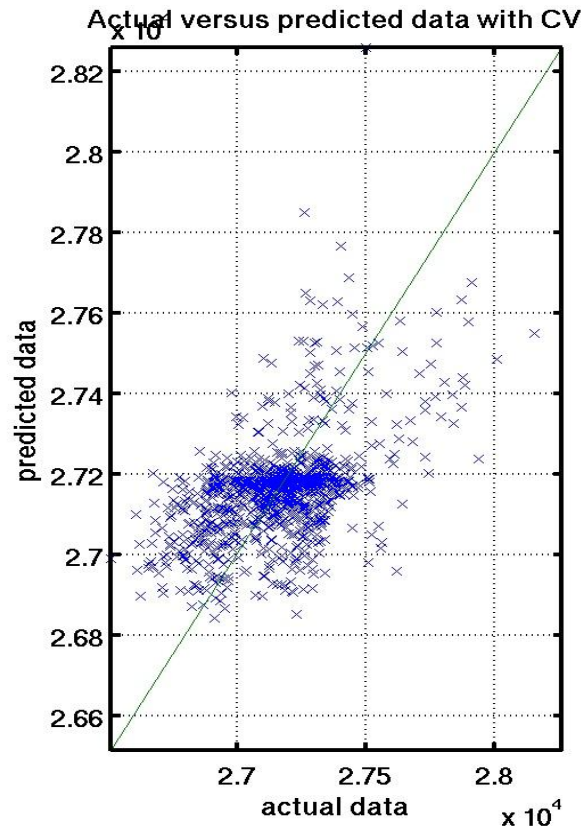
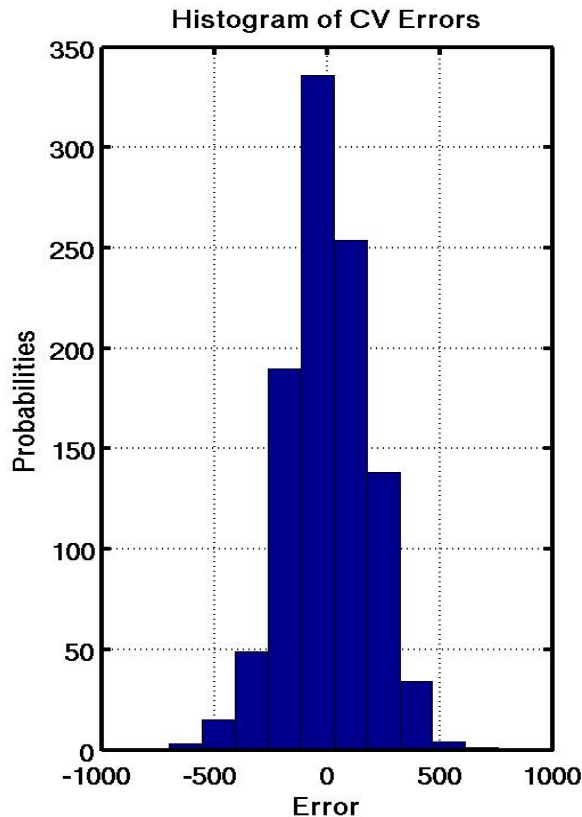
Avg<sub>h</sub> Error Scatter Plot



**Interpolation errors all over – locally smooth?**

# Fluidized Bed Data Analysis (Avg\_dP)

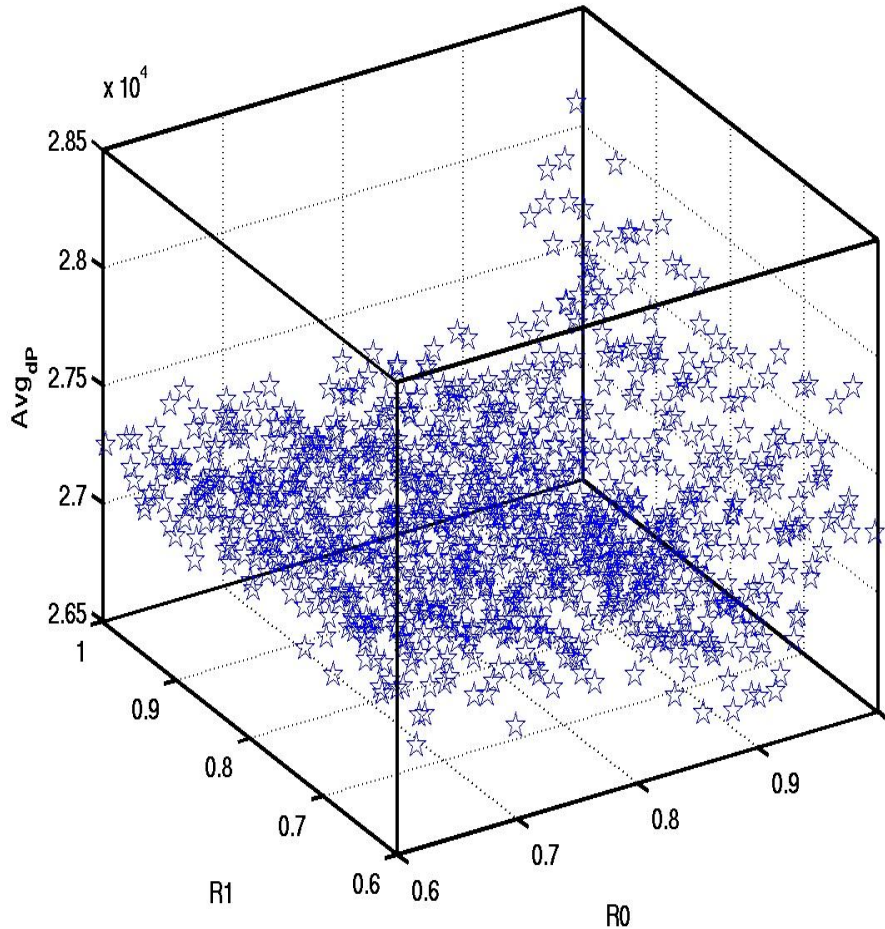
- response surface analysis using cubic splines gives



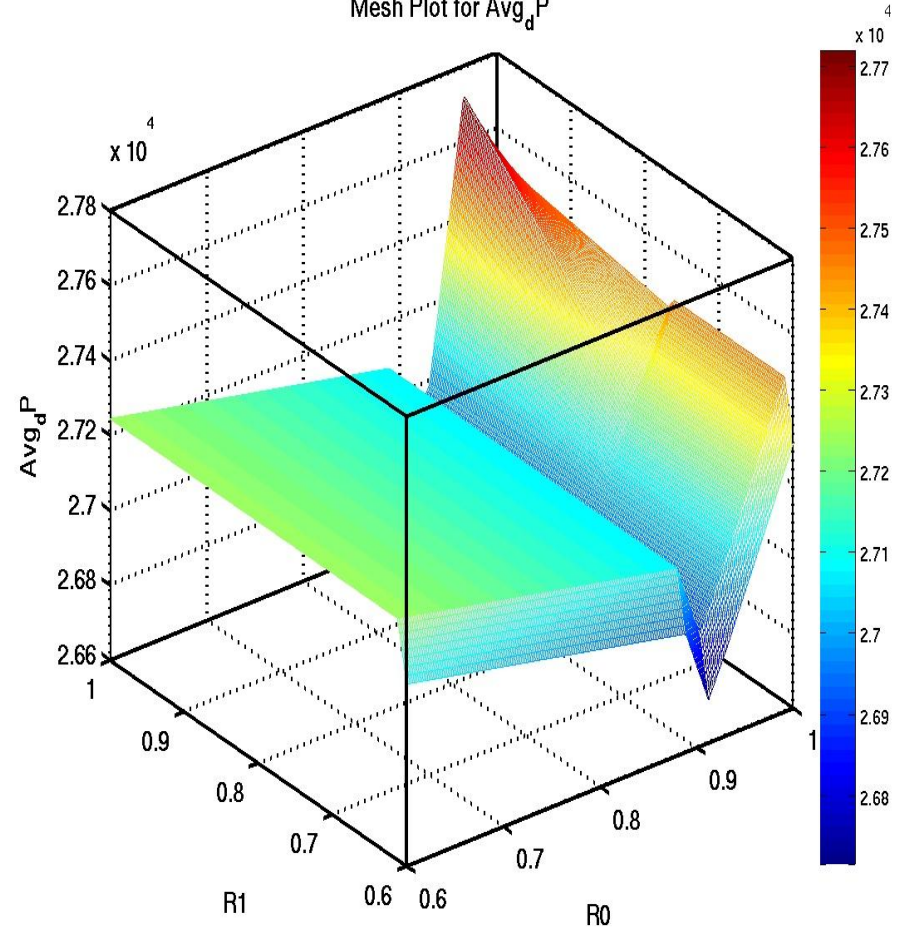
- Errors ~ output range  $\rightarrow$  not acceptable

# What happened? Let's examine Avg\_dp more closely.

3D 2-Input/1-Output Data Plot



Mesh Plot for Avg<sub>d</sub>P



**Abrupt changes: physical?**