# Progress Toward Inclusion of Collision Effects in MP-PIC

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# Background: Original Multi-Phase Particle-in-Cell (MP-PIC) Method

- Andrews and O'Rourke, Int. J. Multiphase Flow 22, 379-402 (1996). (one dimension)
- Snider, J. Comput. Phys. 170, 523-549 (2001). (multi-dimensions)

# Original MP-PIC

- Solve transport equation for particle distribution function $f(x_i,v_i,m,\ldots,t)$

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  ( e.g. $f(x_i,v_i,m,t)$=particle number density in 7-dimensional particle space at time t )
- Similar to Boltzmann Equation except
  - More independent variables
  - No collision terms on right-hand side (original MP-PIC)
  - More complicated acceleration

# Original MP-PIC Particle Acceleration

$$\frac{dv_i}{dt} = A_i^{orig} = D\left(u_i - v_i\right) - \frac{1}{\rho_p}\frac{\partial p}{\partial x_i} + g_i - \frac{1}{\rho_p \theta_p}\frac{\partial \tau\left(\theta_p\right)}{\partial x_i}$$

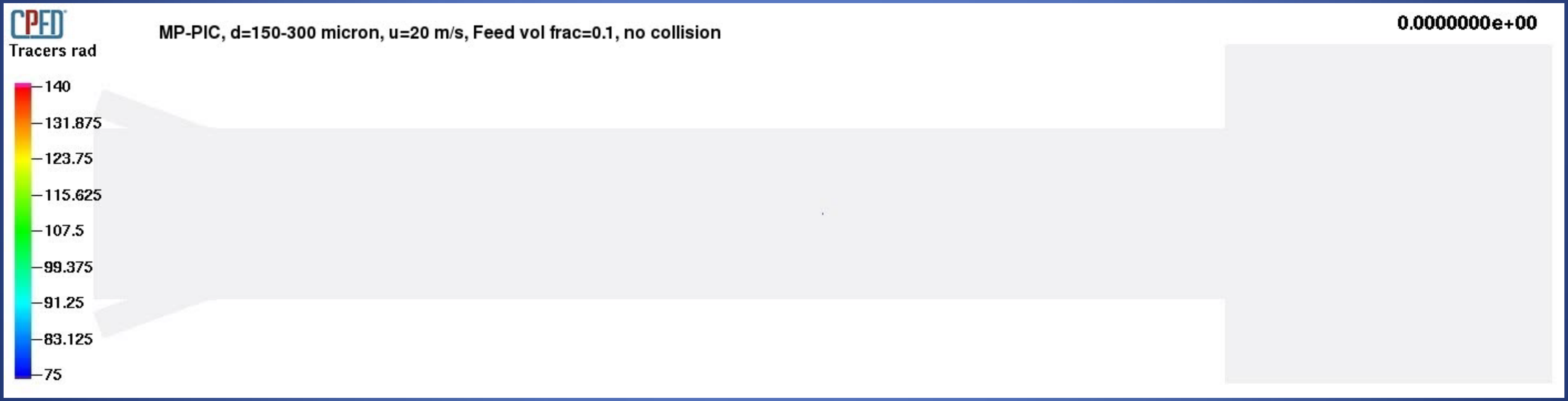
where particle contact stress $\tau(\theta_p)$ given by

$$\tau\left(\theta_p\right) = \frac{P_p \theta_p^{\beta}}{\theta_{cp} - \theta_p}$$

# Original MP-PIC

- Only direct particle-particle interaction is through contact stress $\tau(\theta_p)$
- No collision terms

# Original MP-PIC
# Calculation of Impinging Particle Jets



# Inclusion of Collision Effects

- O'Rourke, et al., Chem. Eng. Sci. 64, 1784-1797 (2009).
- O'Rourke and Snider, Chem. Eng. Sci. 65, 6014-6028 (2010).
- O'Rourke and Snider, submitted to Chem. Eng. Sci. (2011).

# Calculation of Impinging Particle Jets with Collisions

# Inclusion of Collision Effects

- No collision terms on right-hand side of particle distribution function equation
- Add collision acceleration terms

$$\frac{dv_i}{dt} = A_i^{orig} + A_i^{coll}$$

# Collision Acceleration

$$A_i^{coll} = \frac{\overline{v_i} - v_i}{2\tau_G} + \sum_k v'_{k,i}\,\delta(t - t_k) + \frac{\overline{v_i} - v_i}{2\tau_D}$$

- First two terms
  - represent relaxation to isotropic Gaussian distribution on timescale $\tau_G$
  - have no net effect on granular temperature
- Third term
  - represents damping due to inelastic collisions on timescale $\tau_D$
  - gives rise to decay in granular temperature

# Collision Acceleration

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- $t_k$ = time of collision k
- Probability/time = $P(t_k) = 1/\tau_G$
- $v'_{k,i}$ = random velocity induced by collision k
- $v'_{k,i}$ chosen from isotopic Gaussian with variance $\sigma^2(m)$

# Collision Acceleration

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- Analogy with multi-component gas flow
- $\sigma^2(m) = M\sigma^2/m$
- M = local average particle mass
- $\sigma^2$ = local mass-averaged velocity variance of particle velocity distribution

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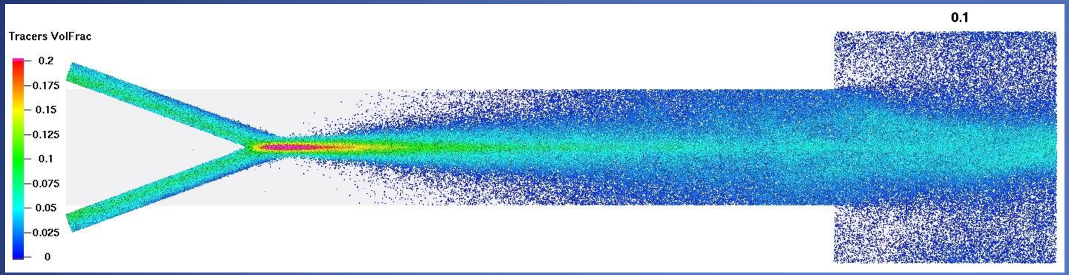
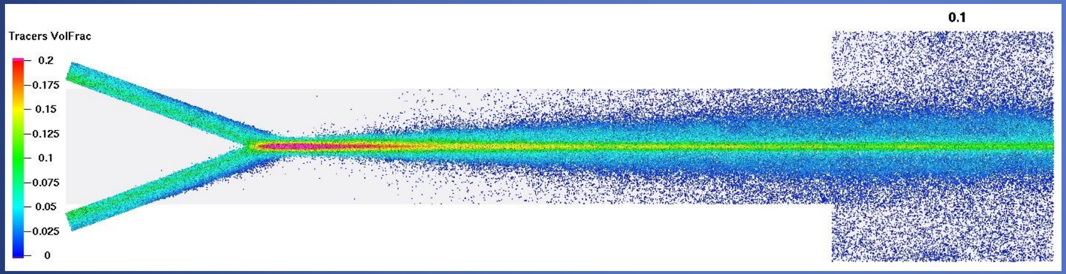
- Computational parcels undergo random walks

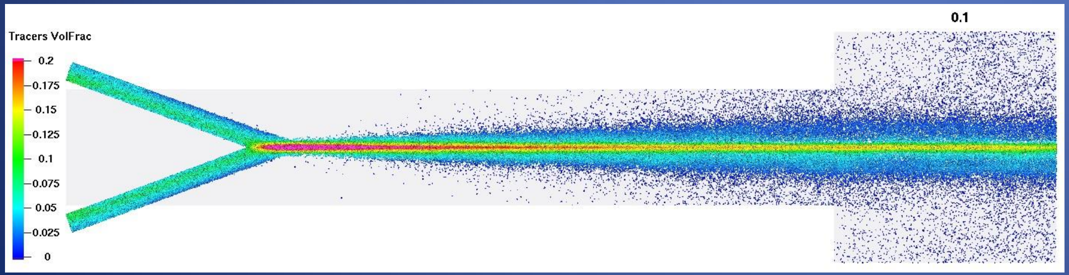
# Effect of Varying Coefficient of Restitution $e_p$




$e_p = 0.999$

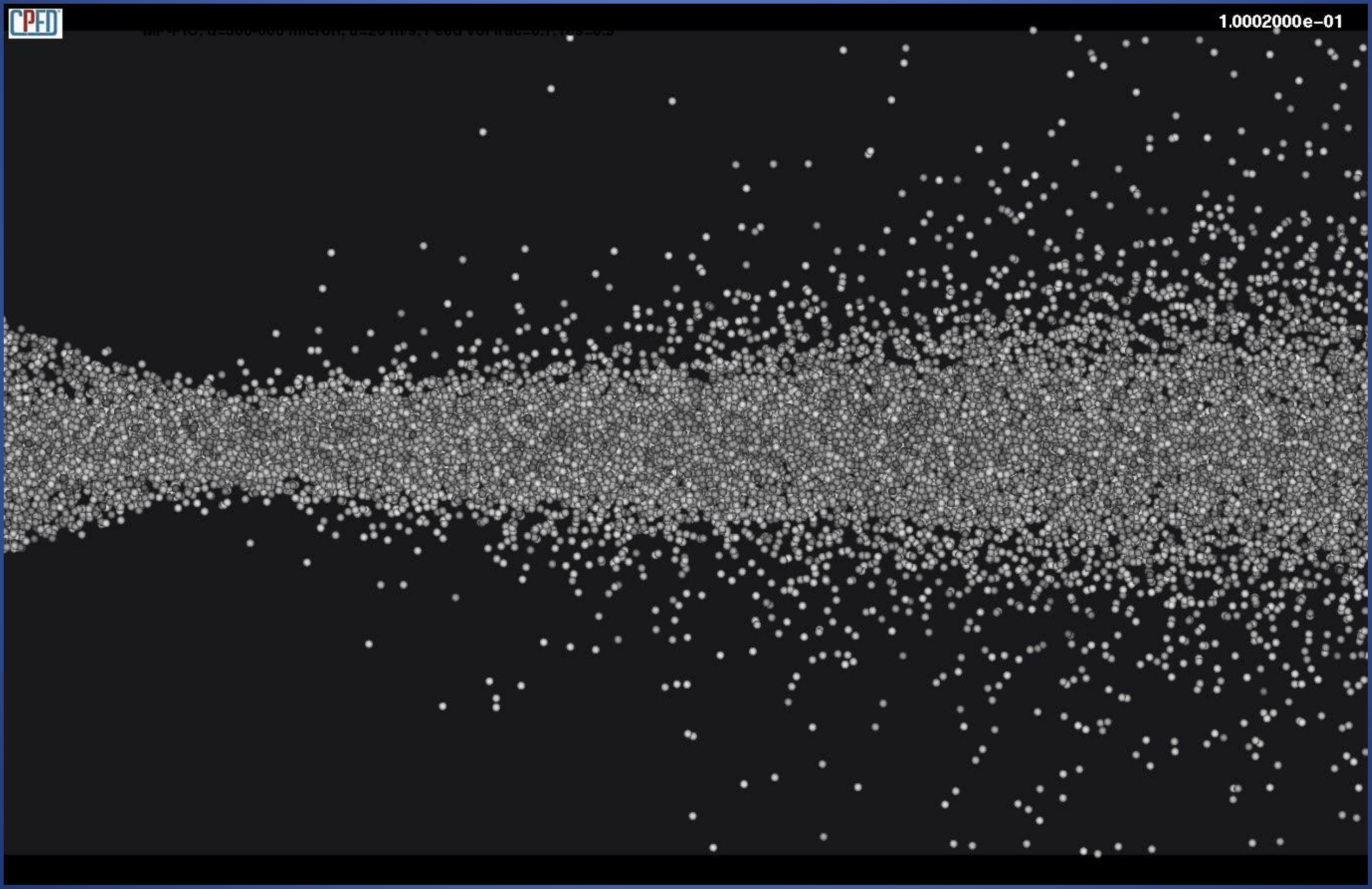



$e_p = 0.9$




$e_p = 0.8$

# Close-up of Impingement Region



# Collision Model
# Future Work

- Collision model V & V
- Assess importance of collisions in practical applications with Barracuda™ code (monitor important parameter $D\tau_G$ )
- Other collision effects