

Uncertainty quantification in chemistry sub-models

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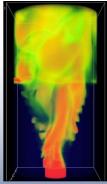
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Carbon Capture Simulation Initiative













Identify promising concepts



Reduce the time for design & troubleshooting



Quantify the technical risk, to enable reaching larger scales, earlier



Stabilize the cost during commercial deployment

National Labs











Academia











FLUOR







Industry



































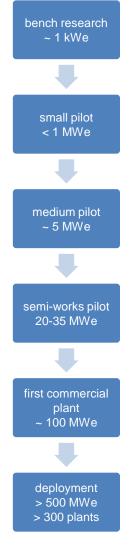






Carbon Capture Challenge

- The traditional pathway from discovery to commercialization of energy technologies can be quite long, i.e., ~ 2-3 decades
- President's plan requires that barriers to the widespread, safe, and cost-effective deployment of CCUS be overcome within 10 years
- To help realize the President's objectives, new approaches are needed for taking carbon capture concepts from lab to power plant, <u>quickly</u>, and at low cost and risk
- CCSI will accelerate the development of carbon capture technology, from discovery through deployment, with the help of science-based simulations

















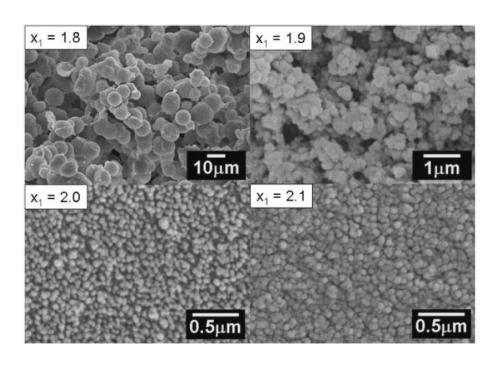
outline

- overview of high-fidelity sorbent modeling
- Bayesian calibration applied to sorbent equilibrium models

the sorbent

- mesoporous silica forms the substrate
- substrate particles agglomerates of micron-sized mesoporous particles
- mesopores impregnated with an active material, such as polyethyleneimine (PEI)





K Kajihara, et al., Bull Chem Soc Jpn, 82 (2009) 1470.







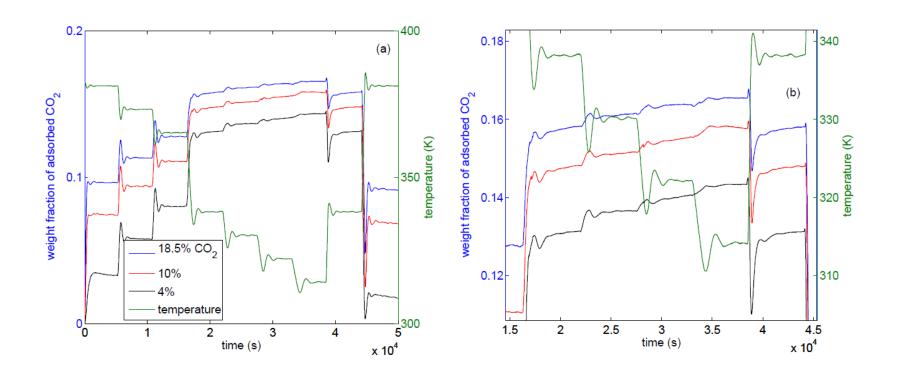








the sorbent: dry TGA behavior



(a)-(b) Sorbent NETL-196C, ~44.1 wt-% PEI, Dry atmosphere. Sorbent synthesis: McMahan Gray, NETL; Sorbent characterization: Daniel Fauth, NETL.















anhydrous model

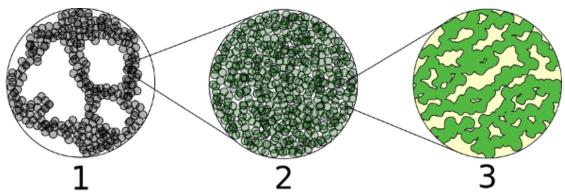
two-step formation of carbamic acid:

$$R_2NH + CO_2(g) \rightleftharpoons R_2NH^+ - CO_2^-$$

 $R_2NH^+ - CO_2^- + R_2NH \rightleftharpoons R_2NCOOH : R_2NH$

three modes of mass transport:

gas phase bulk gas phase Knudsen solid state (zwitterion-mediated hopping)









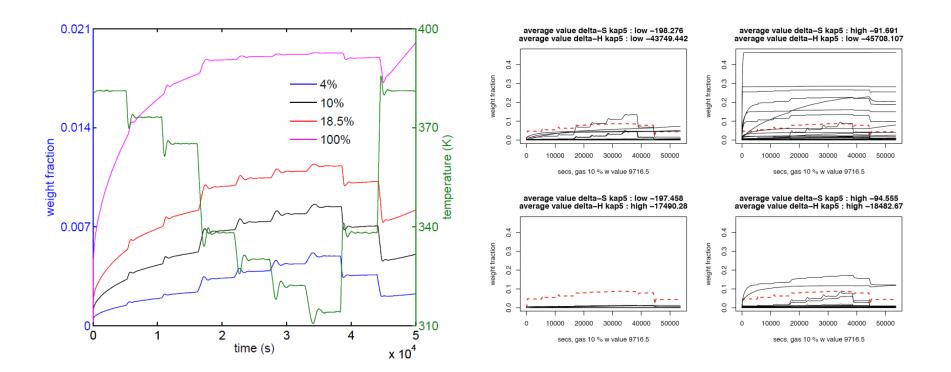








anhydrous model



(left) sample calculated output of the sorbent model showing diffusion effects (right) sensitivity analysis highlighting the importance of zwitterion stability to sorbent working capacity











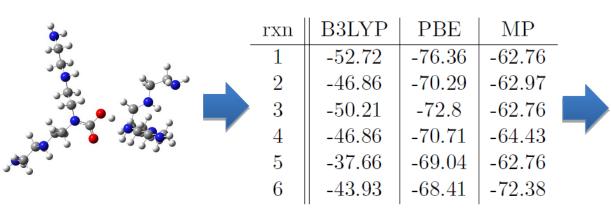


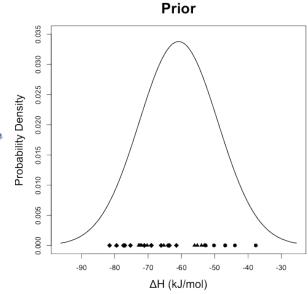


 Bayes' theorem enables the incorporation of prior information in model-based parameter estimates.

$$\widehat{\mathbb{E}}\left[\underbrace{\bigwedge_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\bigwedge_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\prod_{\mathbf{0}} \quad \mathbf{X} \quad \widehat{\mathbb{E}}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\mathbb{E}}_{\mathbf{0}} \right] \quad \mathbf{X} \quad \widehat{\mathbb{E}}\left[\underbrace{\mathbb{E}}_{\mathbf{0}$$

• If model parameters relate to physical quantities, prior information is available through *ab initio* calculations.



















 Bayes' theorem enables the incorporation of prior information in model-based parameter estimates.

$$\widehat{\mathbb{E}} \left[\int_{0}^{\infty} \int_{0.5-1}^{0.5-1} \times \widehat{\mathbb{E}} \left[\int_{0}^{\infty} \int_{0.5-1}^{0.5-1} \times \mathbb{E} \left[\int_{0}^{\infty} \int_{0.5-1}^{0.5-1} \times \pi(\theta|y) \times L(y|\theta) \times \pi(\theta) \right] \right]$$

- The error in the form of the model must also be accounted for.
- A Gaussian process generates a stochastic set of curves adhering to certain general properties.

$$\Sigma(i',j';\boldsymbol{\xi}) = \eta \exp\left[-\frac{(\zeta_{i'} - \zeta_{j'})^2}{\phi^2}\right] \longrightarrow \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{j=1}^{3$$















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$$\widehat{\mathbb{E}} \left[\underbrace{\bigwedge_{0 = 0.5 - 1}^{0.5 - 1}} \right] \times \widehat{\mathbb{E}} \left[\underbrace{\bigwedge_{0 = 0.5 - 1}^{0.5 - 1}} \right] \times \pi(\theta|y) \propto L(y|\theta) \times \pi(\theta)$$

- The error in the form of the model must also be accounted for.
- A Gaussian process generates a stochastic set of curves adhering to certain general properties.

$$\mathbf{Y} = \mathbf{Z}(\boldsymbol{\theta}, \boldsymbol{\zeta}) + \boldsymbol{\delta}(\boldsymbol{\xi}, \boldsymbol{\zeta}) + \boldsymbol{\epsilon}(\boldsymbol{\psi})$$

$$\mathbf{Y} \sim N\left[\mathbf{Z}(\boldsymbol{\theta}, \boldsymbol{\zeta}), \Sigma(\boldsymbol{\xi}) + \psi \mathbf{I}\right] = \mathcal{L}(\mathbf{Y}|\boldsymbol{\theta}, \boldsymbol{\xi}, \psi)$$







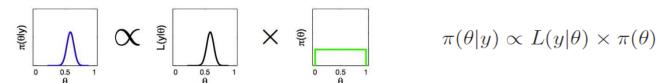




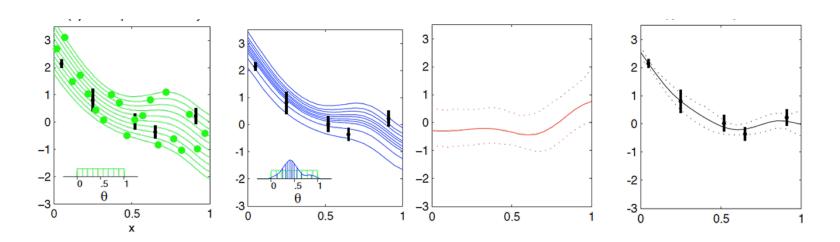




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MC Kennedy and A O'Hagan, J Royal Stat Soc B, 63 (2001) 425.









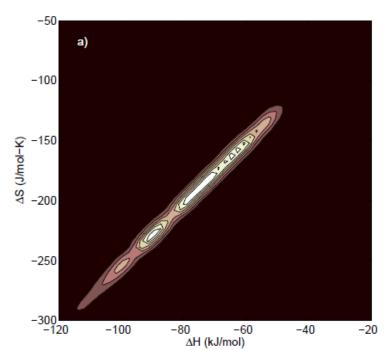




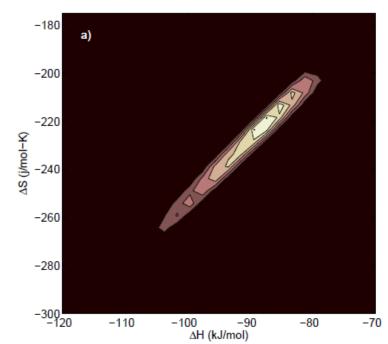


$$2R_2NH + CO_2(g) \rightleftharpoons R_2NCOOH : R_2NH$$

$$\kappa = \frac{x^2}{(1 - 2x)^2 p} = \exp\left(\frac{\Delta S}{R}\right) \exp\left(\frac{-\Delta H}{RT}\right) / P \qquad w = M n_{\rm v} x / \rho$$



posterior distributions (left) without and (right) with informative priors











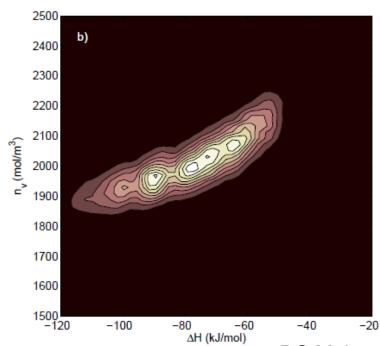




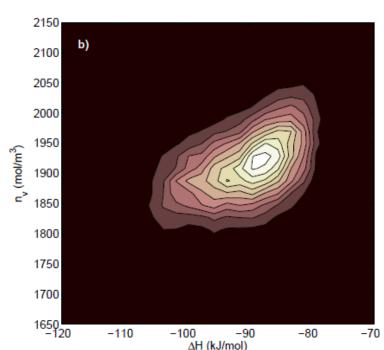


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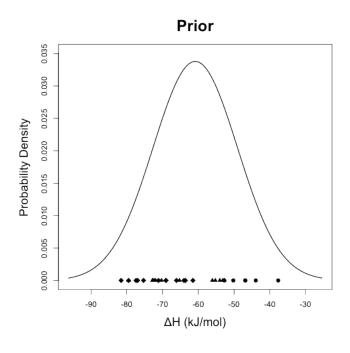




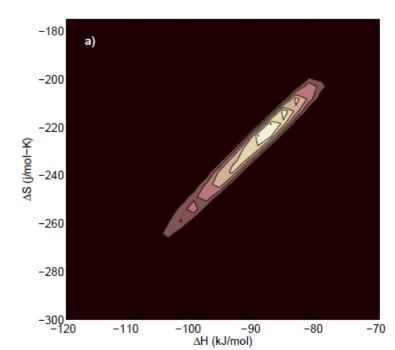


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(left) prior distribution for adsorption enthalpy, and (right) posterior distribution











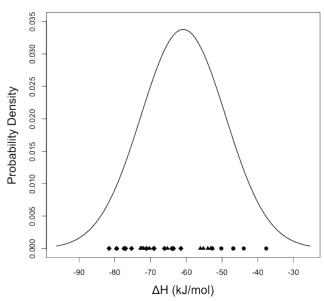




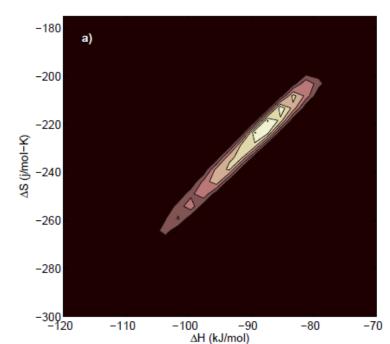


reaction	B3LYP	PBE	PBE0	MP2	MP3
$CO_2+2MMA \rightarrow P-COOH:P$	-52.72	-71.13	-81.59	-52.72	-72.8
$CO_2+MMA+DMA \rightarrow S-COOH:P$	-46.86	-63.60	-76.99	-53.97	-71.96
$CO_2+MMA+DMA\rightarrow P-COOH:S$	-50.21	-66.11	-79.50	-53.14	-72.38
$CO_2+2DMA \rightarrow S-COOH:S$	-46.86	-64.02	-77.40	-56.07	-72.80
$CO_2+DETA+EDA\rightarrow P-COOH:S$	-37.66	-69.04	-69.04	-55.23	-70.29
$CO_2+DETA+EDA \rightarrow S-COOH:P$	-43.93	-61.50	-75.31	-65.27	-79.50





(left) prior distribution for adsorption enthalpy, and (right) posterior distribution











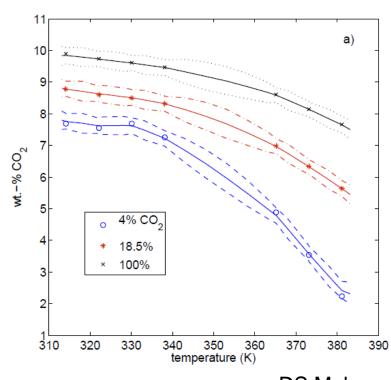




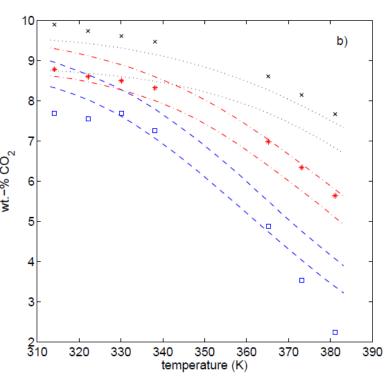


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(left) conditioned model + discrepancy predictions, and (right) model predictions, with 95% confidence bounds











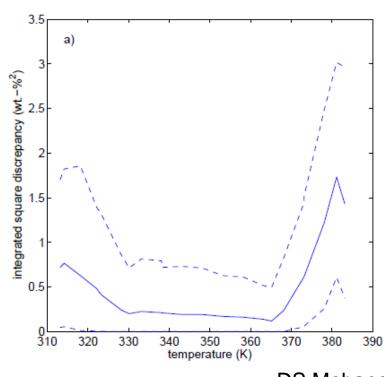




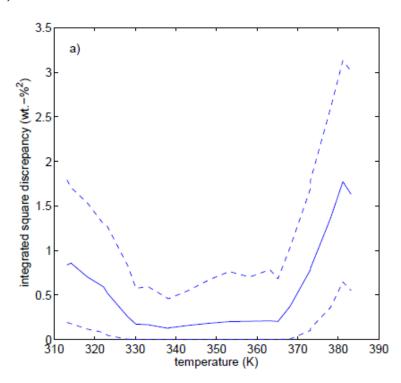


$$2R_2NH + CO_2(g) \rightleftharpoons R_2NCOOH : R_2NH$$

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(left) normalized discrepancy for uniform priors with 95% bounds, and (right) normalized discrepancy for informative priors, 4% CO₂











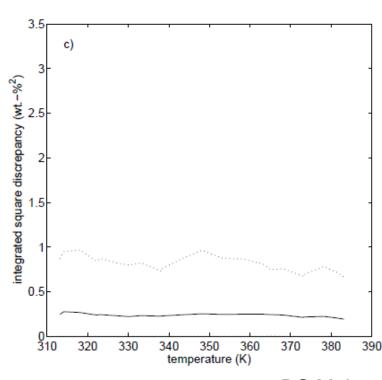




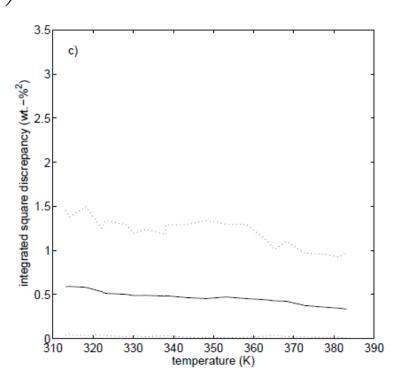


$$2R_2NH + CO_2(g) \rightleftharpoons R_2NCOOH : R_2NH$$

$$\kappa = \frac{x^2}{(1 - 2x)^2 p} = \exp\left(\frac{\Delta S}{R}\right) \exp\left(\frac{-\Delta H}{RT}\right) / P \qquad w = M n_{\rm v} x / \rho$$



(left) normalized discrepancy for uniform priors with 95% bounds, and (right) normalized discrepancy for informative priors, 100% CO₂

















conclusions / future work

- Ab initio calculations can be used in along with a valid model form discrepancy in a Bayesian framework to influence the experimental calibration of engineering-useful models of complex chemical systems.
- A conditioned model form discrepancy enables the direct use of experimental information in scale-up through a model-plusdiscrepancy approach, providing penalties for interpolation and extrapolation that become smaller as models improve.
- Work is underway to demonstrate the effects of model form discrepancy in upscaling of a simple kinetic model for CO₂ capture through to the process scale.















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