Coarse-Grained Energy Balance For Gas-Particle Flows

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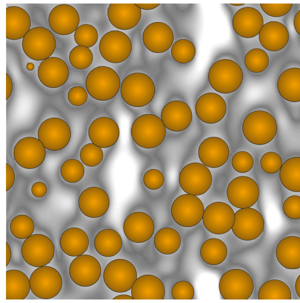
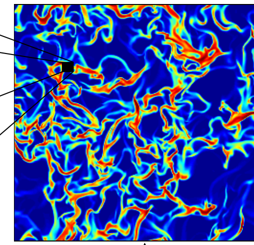
Simulation of Gas-Particle Flows Across Length Scales

MICRO-SCALE
~ 50μ*m-mm*



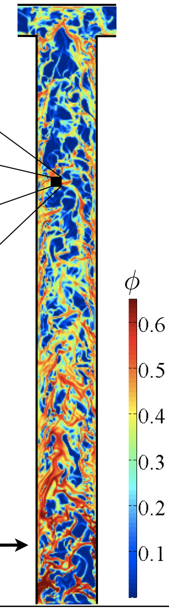
Newton's equations of motion for each particle, Navier-Stokes equations for the fluid flow in the interstices.

MESO-SCALE
~ *mm-cm*



Volume-averaged hydrodynamic models for fluid and particle phases.

MACRO-SCALE
~ *cm-m*



Filtered volume-averaged hydrodynamic models for fluid and particle phases.

Development of Filtered Two-Fluid Models

- Hydrodynamics (*Igci et al, 2011*)
  - Filtered drag *
  - Filtered pressure
  - Filtered viscosity
- Reacting flows (*Holloway & Sundaresan, 2012*)
  - Filtered reaction rate
- Thermal energy & interphase transport (present work)
  - Filtered energy dispersion
  - Filtered interphase heat/mass transfer
  - Filtered scalar dispersion (no interphase transport)
    - Helium Tracer/Solid Particle Tracer

**Li & Kwauk, 1994; Parmentier et. al, 2011*

Two-Fluid Model Equations

Continuity

$$\frac{\partial(\rho_s\phi_s)}{\partial t} + \nabla\cdot(\rho_s\phi_s\boldsymbol{v}_s) = 0$$

$$\frac{\partial(\rho_g\phi_g)}{\partial t} + \nabla\cdot(\rho_g\phi_s\boldsymbol{v}_g) = 0$$

Momentum

Granular kinetic theory

$$\frac{\partial}{\partial t}(\rho_s\phi_s\boldsymbol{v}_s) + \nabla\cdot(\rho_s\phi_s\boldsymbol{v}_s\boldsymbol{v}_s) = -\nabla\cdot\boldsymbol{\sigma}_s - \phi_s\nabla\cdot\boldsymbol{\sigma}_g + \boldsymbol{f} + \rho_s\phi_s\boldsymbol{g}$$

$$\frac{\partial}{\partial t}(\rho_g\phi_g\boldsymbol{v}_g) + \nabla\cdot(\rho_g\phi_g\boldsymbol{v}_g\boldsymbol{v}_g) = -\phi_g\nabla\cdot\boldsymbol{\sigma}_g - \boldsymbol{f} + \rho_g\phi_g\boldsymbol{g}$$

Wen & Yu (1966)

Thermal Energy

$$\rho_s C_{p_s}\left[\frac{\partial}{\partial t}(\phi_s T_s) + \nabla\cdot(\phi_s\boldsymbol{v}_s T_s)\right] = \nabla\cdot(k_s\nabla T_s) + \gamma(T_s - T_g)$$

Gunn (1978)

$$\rho_g C_{p_g}\left[\frac{\partial}{\partial t}(\phi_g T_g) + \nabla\cdot(\phi_g\boldsymbol{v}_g T_g)\right] = \nabla\cdot(k_g\nabla T_g) - \gamma(T_s - T_g)$$

Filtered Equations

Filter $$\overline{\phi_s}(\boldsymbol{x},t) = \int_{V_\infty} G(\boldsymbol{x},\boldsymbol{y})\phi_s(\boldsymbol{x},t)d\boldsymbol{y} \qquad \int_{V_\infty} G(\boldsymbol{x},\boldsymbol{y})(\boldsymbol{x},t)d\boldsymbol{y} = 1$$

Favre Filter $$\widetilde{\alpha_s}(\boldsymbol{x},t) = \frac{\displaystyle\int_{V_\infty} G(\boldsymbol{x},\boldsymbol{y})\alpha_s(\boldsymbol{x},t)\phi_s(\boldsymbol{x},t)d\boldsymbol{y}}{\displaystyle\int_{V_\infty} G(\boldsymbol{x},\boldsymbol{y})\phi_s(\boldsymbol{x},t)d\boldsymbol{y}}$$

Filtered Solids Thermal Energy Equation

$$\rho_s C_{p_s}\left[\frac{\partial}{\partial t}(\overline{\phi_s}\widetilde{T_s}) + \nabla\cdot(\overline{\phi_s}\widetilde{\boldsymbol{v}_s}\widetilde{T_s})\right] = \nabla\cdot(\overline{k_s\nabla T_s}) + \overline{\gamma(T_s - T_g)} + \rho_s C_{p_s}\nabla\cdot\left[\overline{\phi_s}\widetilde{\boldsymbol{v}_s}\widetilde{T_s} - \overline{\phi_s}\widetilde{\boldsymbol{v}_s T_s}\right]$$

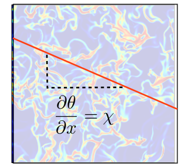
‘dispersive’ flux

Applying Mean Temperature Gradient

Temperature Gradient

$$T_s = T_s^{'} + \theta(x) \qquad \frac{\partial \theta}{\partial x} = \chi = constant$$

$$T_g = T_g^{'} + \theta(x)$$



Thermal Energy

$$\rho_s C_{p_s} \left[\frac{\partial}{\partial t}(\phi_s T_s^{'}) + \nabla \cdot (\phi_s \boldsymbol{v}_s T_s^{'}) \right] = \nabla \cdot (k_s \nabla T_s^{'}) + \gamma (T_s^{'} - T_g^{'}) + \chi \frac{\partial k_s}{\partial x} - \rho_s C_{p_s} \phi_s v_{s_x} \chi$$

$$\rho_g C_{p_g} \left[\frac{\partial}{\partial t}(\phi_g T_g^{'}) + \nabla \cdot (\phi_g \boldsymbol{v}_g T_g^{'}) \right] = \nabla \cdot (k_g \nabla T_g^{'}) - \gamma (T_s^{'} - T_g^{'}) + \chi \frac{\partial k_g}{\partial x} - \rho_g C_{p_g} \phi_g v_{g_x} \chi$$

'New' source terms

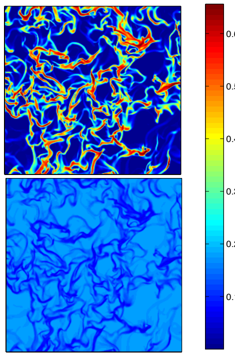
Applying Heat Sources/Sinks

Solids Heat Source

$$\dot{Q}_s\phi_s \qquad \int_D \dot{Q}_s\phi_s dV = \dot{Q}_s\overline{\phi}_s V \qquad \phi_s$$

Gas Heat Sink

$$\dot{Q}_g\phi_g = \dot{Q}_s\frac{\overline{\phi}_s}{\overline{\phi}_g}\phi_g \qquad \int_D \dot{Q}_g\phi_g dV = \dot{Q}_s\overline{\phi}_s V \qquad \frac{\overline{\phi}_s}{\overline{\phi}_g}\phi_g$$



Thermal Energy

$$\rho_s C_{p_s}\left[\frac{\partial}{\partial t}(\phi_s T_s) + \nabla\cdot(\phi_s \boldsymbol{v}_s T_s)\right] = \nabla\cdot(k_s\nabla T_s) + \gamma(T_s - T_g) + \dot{Q}_s\phi_s$$

$$\rho_g C_{p_g}\left[\frac{\partial}{\partial t}(\phi_g T_g) + \nabla\cdot(\phi_g \boldsymbol{v}_g T_g)\right] = \nabla\cdot(k_g\nabla T_g) - \gamma(T_s - T_g) - \dot{Q}_g\phi_g$$

‘New’ source/sink terms

Simulations and Filtering Procedure

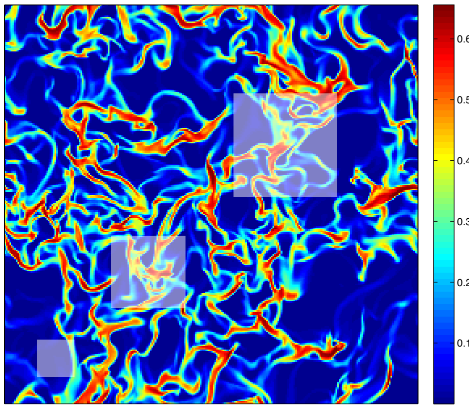
2-D periodic domain with mean vertical pressure drop in gas phase set to balance weight of mixture.

Computational Domain: 32cm x 32cm
Discretization: 256 x 256 cells
Particle diameter: 75μm

Simulations are for FCC particles in air. Results are suitably scaled so that they are applicable to other systems.

Snapshot of solids volume fraction field obtained from highly resolved simulations. ⟶

Shaded squares illustrate regions over which a filtering operation is performed.

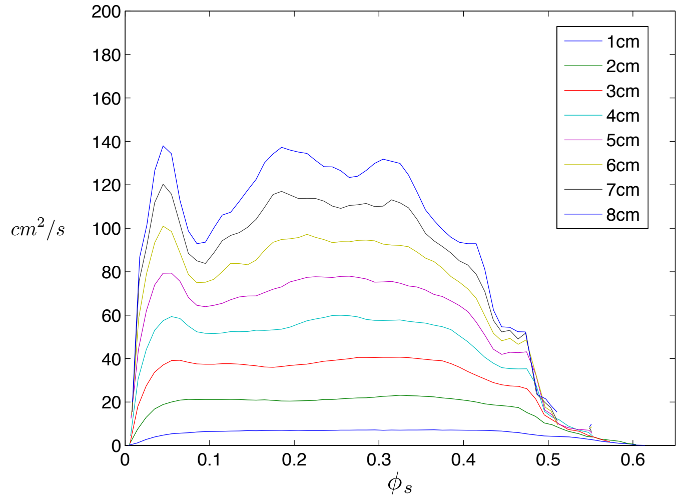


Filtered Solids Thermal Dispersion

$$\alpha_{filt} = \frac{k_{filt}}{\rho_s C_{p_s}} = \frac{\overline{\phi_s}\left(\widetilde{\boldsymbol{v}_{s_x}}\widetilde{T_s} - \widetilde{\boldsymbol{v}_{s_x} T_s}\right)}{\frac{\partial \widetilde{T_s}}{\partial x}} \qquad \frac{\left(\widetilde{\boldsymbol{v}_{s_x}}\widetilde{T_s} - \widetilde{\boldsymbol{v}_{s_x} T_s}\right)}{\frac{\partial \widetilde{T_s}}{\partial x}}$$

Curves correspond to different filter sizes.

FCC particles in air.



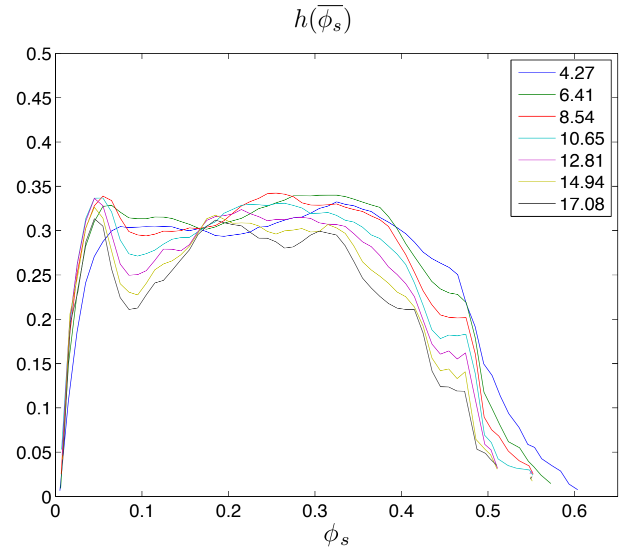
Scaled Filtered Solids Thermal Dispersion

$$\frac{\alpha_{filt}}{(v_t^3/g)} = \overline{\phi_s}\, h(\overline{\phi_s})(Fr_\Delta)^\delta$$

$$\delta = -4/3$$

Dimensionless filter size shown in legend $\dfrac{\Delta}{v_t^2/g}$

$(Fr_\Delta)^{-1}$



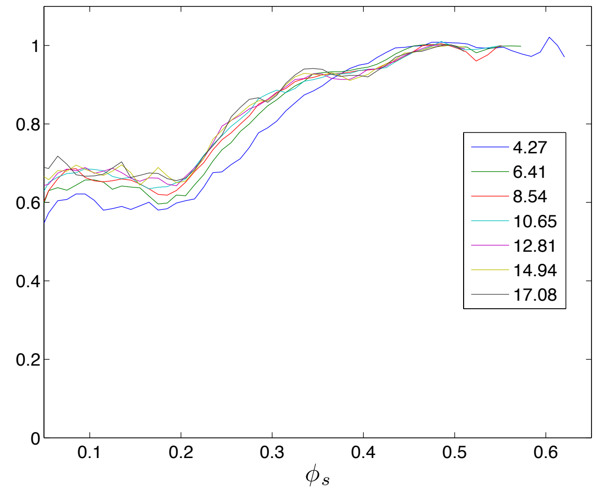
Filtered Gas Thermal Dispersion

Ratio of Gas to Solids Filtered Dispersion

$$\delta = -4/3$$

Dimensionless filter size shown in legend $\dfrac{\Delta}{v_t^2/g}$

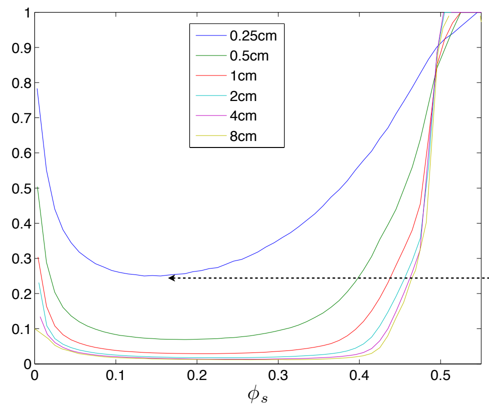
$$\frac{\left[h(\overline{\phi_s})\right]_g}{\left[h(\overline{\phi_s})\right]_s}$$



Filtered Interphase Heat Transfer Coefficient

$$\frac{\gamma_{filt}}{\gamma(\overline{\phi_s}, \widetilde{\boldsymbol{v}}_{slip})}$$

$$\gamma_{filt} = \frac{\overline{\gamma(T_s - T_g)}}{(\widetilde{T_s} - \widetilde{T_g})}$$



$$1 - \left[\frac{\gamma_{filt}}{\gamma(\overline{\phi_s}, \widetilde{\boldsymbol{v}}_{slip})}\right]_{min} \sim \left(1 + \frac{1}{\left[(Fr_{\Delta})^{-1} - (Fr_{\Delta res})^{-1}\right]^{+}}\right)^{-0.1}$$

Curves correspond to different filter sizes.

Dong et. al (2008); Kashyap & Gidaspow (2010,2011)

Summary

- Filtered dispersion coefficient

$$\frac{\alpha_{filt}}{(v_t^3/g)} = \overline{\phi_s}\, h(\overline{\phi_s})(Fr_\Delta)^{-4/3}$$

- Filtered interphase mass/heat transfer coefficient

$$1 - \frac{\gamma_{filt}}{\gamma(\overline{\phi_s}, \widetilde{\boldsymbol{v}}_{slip})} \sim f(\overline{\phi_s}) \left(1 + \frac{1}{\left[(Fr_\Delta)^{-1} - (Fr_{\Delta res})^{-1}\right]^+}\right)^{-0.1}$$

Next Steps

- Explore alternate scaling

$$\frac{\alpha_{filt}}{\Delta^2|\widetilde{S}|} = G(\overline{\phi_s}) \qquad |\widetilde{S}| \sim \Delta^{2/3}$$

- Relate momentum dispersion to energy/scalar dispersion and interphase mass/heat transfer
- Impact of going from 2-D to 3-D
  - Results for hydrodynamics suggest the effect is small (*Igci et. al, 2011*)