

# A MASSIVELY PARALLEL EULER-LAGRANGE STRATEGY FOR SIMULATING FLUIDIZED BED REACTORS

2012 MULTIPHASE CONFERENCE  
MAY 22, 2012

JESSE CAPECELATRO, OLIVIER DESJARDINS  
*SIBLEY SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING  
CORNELL UNIVERSITY*

FUNDED IN PART BY:  
*DOE OFFICE OF BIOMASS PROGRAM  
NATIONAL RENEWABLE ENERGY LABORATORY*



Cornell University  
Computational Thermo-Fluids  
Laboratory

# Motivation

- Particle-laden flows are an important type of multiphase flow
- Common in many natural and industrial processes
- Fluidized bed reactors are ideal for gasification /pyrolysis
  - Easily scalable
  - Efficient mixing
  - Uniform temperature distribution
- Range of phenomenon exist in particle flows
  - Bubbling
  - Clustering



Spiegel (2009)

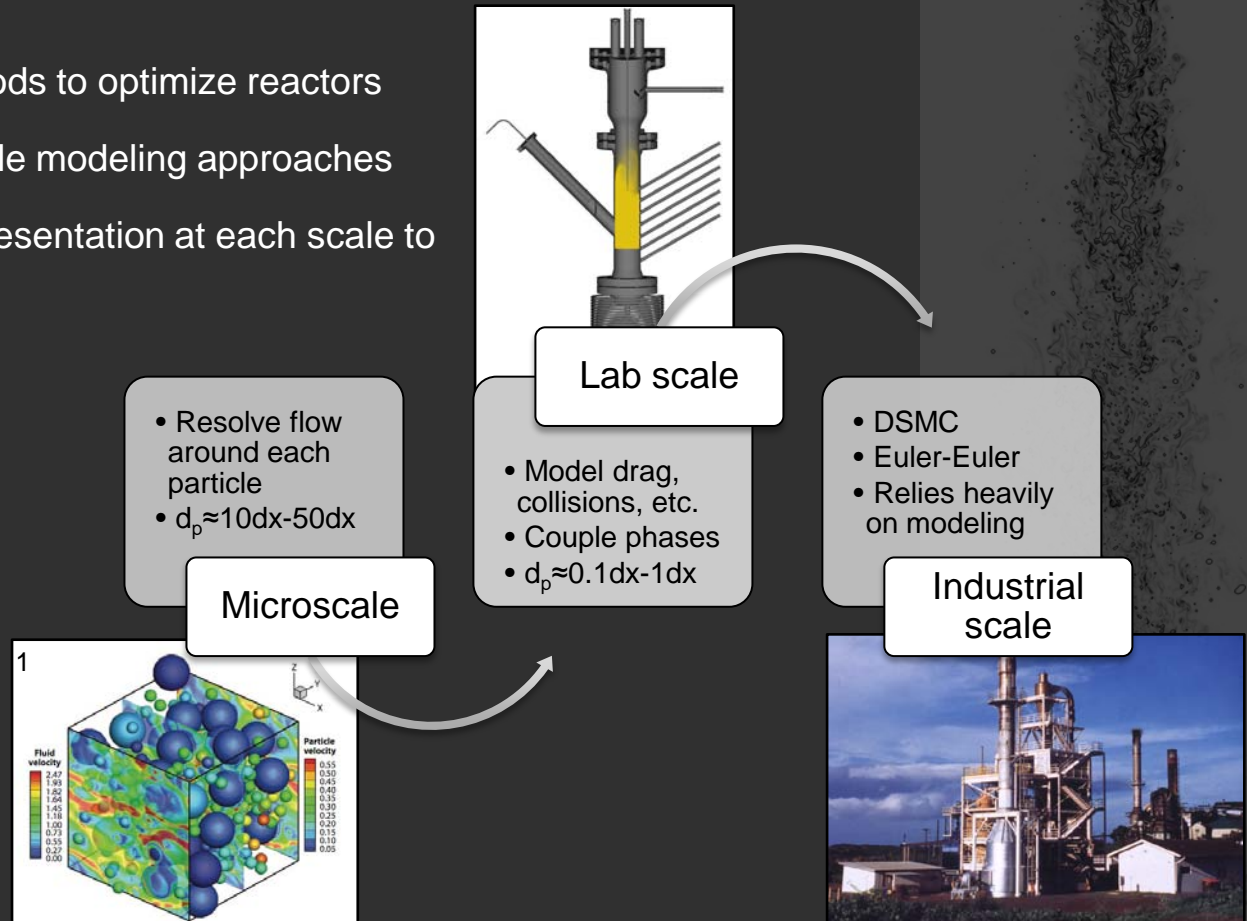


Horio & Kuroki (1994)



# Objective

- Develop simulation strategy investigate complex multiphase flow dynamics
- Use first-principle based methods to optimize reactors
- Provide closures for larger scale modeling approaches
- Useful to have successful representation at each scale to elucidate the physics



# Mathematical Formulation

## POINTWISE DESCRIPTION

- Gas phase: Variable-density / low-Mach Navier-Stokes equations

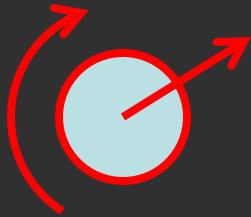
$$\frac{\partial \rho_f}{\partial t} + \nabla \cdot (\rho_f \mathbf{u}_f) = 0$$

$$\frac{\partial}{\partial t} (\rho_f \mathbf{u}_f) + \nabla \cdot (\rho_f \mathbf{u}_f \otimes \mathbf{u}_f) = \nabla \cdot \boldsymbol{\tau} + \rho_f \mathbf{g}$$

$$\boldsymbol{\tau} = -p \mathbf{I} + \mu \left[ (\nabla \mathbf{u}_f + \nabla \mathbf{u}_f^T) - \frac{2}{3} (\nabla \cdot \mathbf{u}_f) \mathbf{I} \right]$$

- Particles: Newton's second law of motion

$$\frac{d\mathbf{x}_p}{dt} = \mathbf{u}_p$$



$$m_p \frac{d\mathbf{u}_p}{dt} = \int_{S_p} \boldsymbol{\tau} \cdot \mathbf{n} \, d\mathbf{y} + F^{\text{col}} + m\mathbf{g}$$

$$I_p \frac{d\boldsymbol{\omega}_p}{dt} = \int_{S_p} \frac{d_p}{2} \mathbf{n} \times (\boldsymbol{\tau} \cdot \mathbf{n}) \, d\mathbf{y} + \sum_{j=1}^{n_p} \mathbf{f}_{t,j \rightarrow p}^{\text{col}}$$

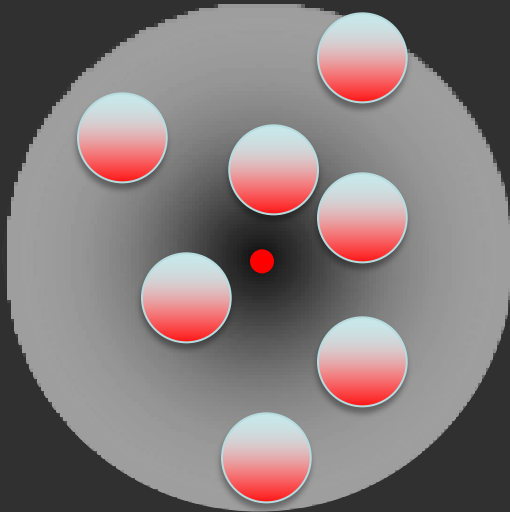
- Boundary conditions: no-slip and no-penetration at surface of particle
- Collision force: contact mechanics



# Mathematical Formulation

## VOLUME-FILTERED DESCRIPTION

- Following the work of Anderson & Jackson (1967)
- Objective: formulate equations for particle-laden flows that allow  $\Delta x \gg d_p$
- Introduce **local volume filter** based on convolution product with kernel  $g(r)$ 
  - Characteristic width of kernel needs to be  $\gg d_p$
  - Flow features on the scale of the particles are filtered out, enabling  $\Delta x \gg d_p$ 
    - Leads to **local mean voidage**:



$$\varepsilon_f(\mathbf{x}, t) = \int_{V_{f\infty}(t)} g(|\mathbf{x} - \mathbf{y}|) d\mathbf{y}$$

- Allows to define **filtered variable**  $\bar{a}$  from point variable  $a$  :

$$\varepsilon_f \bar{a}(\mathbf{x}, t) = \int_{V_{f\infty}(t)} a(\mathbf{y}, t) g(|\mathbf{x} - \mathbf{y}|) d\mathbf{y}$$

- No commutation between filtering and differentiation due to particle surface contributions



# Mathematical Formulation

## VOLUME-FILTERED DESCRIPTION

- For variable density flows  $\tilde{a} = \frac{\varepsilon_f \overline{\rho_f a}}{\varepsilon_f \overline{\rho_f}}$
- In general, decompose point variable  $a$  as  $a = \bar{a} + a'$  or  $a = \tilde{a} + a''$

$$\frac{\partial \varepsilon_f \overline{\rho_f}}{\partial t} + \nabla \cdot (\varepsilon_f \overline{\rho_f} \widetilde{\mathbf{u}_f}) = S_\rho$$

$$\frac{\partial}{\partial t} (\varepsilon_f \overline{\rho_f} \widetilde{\mathbf{u}_f}) + \nabla \cdot (\varepsilon_f \overline{\rho_f} \widetilde{\mathbf{u}_f} \otimes \widetilde{\mathbf{u}_f}) = \nabla \cdot \bar{\boldsymbol{\tau}} - \nabla \cdot \mathbf{R}_u + \varepsilon_f \overline{\rho_f} \mathbf{g} + S_{\rho u} - \mathbf{F}^{\text{inter}}$$

$$S_\rho = \sum_{i=1}^{n_p} \int_{S_p} \rho_f \mathbf{n} \cdot \frac{d\mathbf{r}_i}{dt} g(|\mathbf{x} - \mathbf{y}|) d\mathbf{y}$$

$$S_{\rho u} = \sum_{i=1}^{n_p} \int_{S_p} \rho_f \mathbf{n} \cdot \frac{d\mathbf{r}_i}{dt} \otimes \mathbf{u}_f g(|\mathbf{x} - \mathbf{y}|) d\mathbf{y}$$

$$\bar{\boldsymbol{\tau}} = -p\mathcal{I} + \mu \left[ (\nabla \bar{\mathbf{u}_f} + \nabla \bar{\mathbf{u}_f}^T) - \frac{2}{3} (\nabla \cdot \bar{\mathbf{u}_f}) \mathcal{I} \right] + \mathbf{R}_\mu$$

$$\mathbf{F}^{\text{inter}} = \sum_{i=1}^{n_p} g(|\mathbf{x} - \mathbf{x}_p|) \int_{S_p} \boldsymbol{\tau} \cdot \mathbf{n} d\mathbf{y}$$

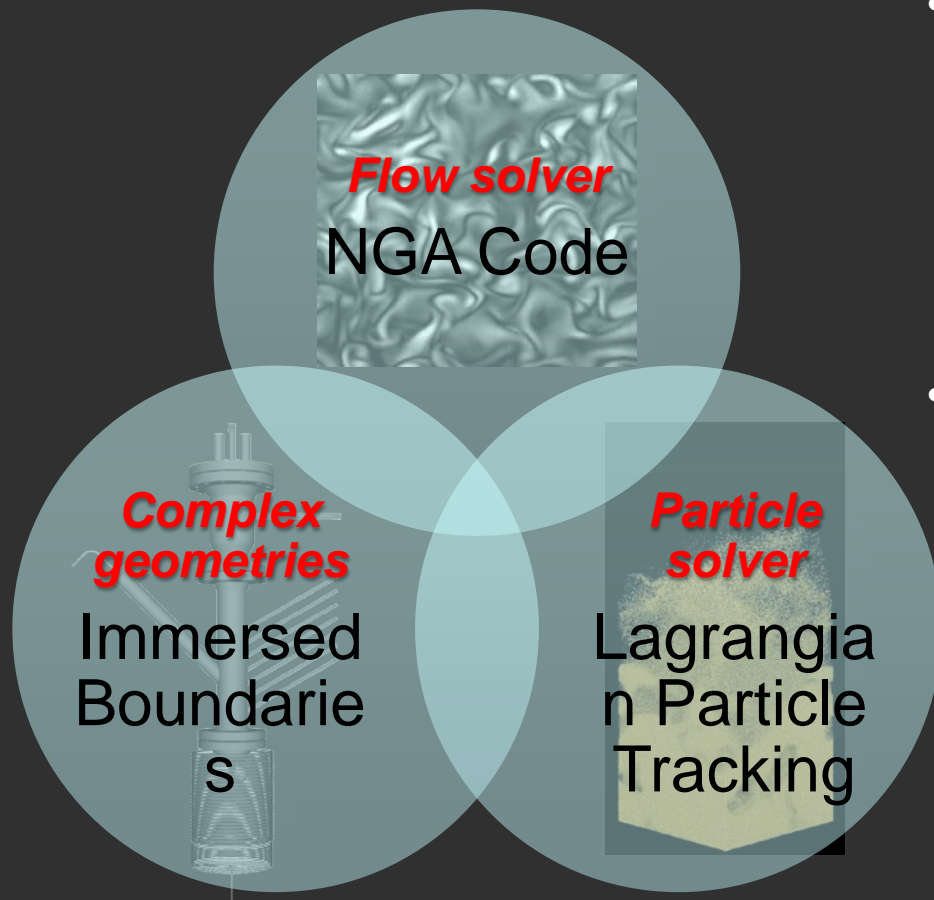
$$\int_{S_p} \boldsymbol{\tau} \cdot \mathbf{n} d\mathbf{y} \approx V_p \nabla \cdot \bar{\boldsymbol{\tau}} + \mathbf{f}_p^{\text{drag}}$$

$$\mathbf{R}_u = \varepsilon_f \overline{\rho_f} \widetilde{\mathbf{u}_f'' \otimes \mathbf{u}_f''}$$





# Computational Approach



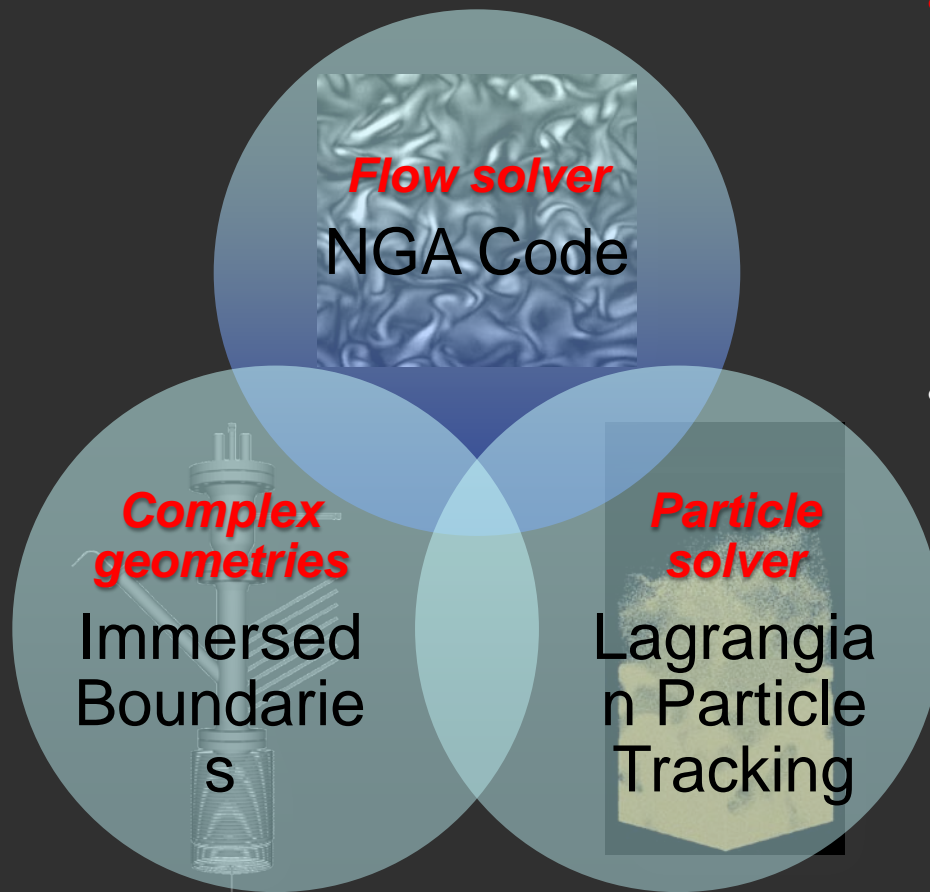
- NGA<sup>2</sup>
  - Arbitrarily high-order DNS/LES code
  - Massively parallel
  - Conservation of mass, momentum, and kinetic energy
- Immersed Boundary<sup>3</sup>
  - Based on cut-cell approach
  - Discrete conservation of mass and momentum
  - Fully implicit implementation to handle small cut-cells

2. O. Desjardins, G. Blanquart, G. Balarac, H. Pitsch, High order conservative finite difference scheme for variable density low Mach number turbulent flows, *Journal of Computational Physics* 227 (2008) 7125– 7159.

3. P. Peipert, O. Desjardins, Direct numerical simulation of dense particle-laden flows using a conservative immersed-boundary technique, *Center of Turbulence Research, Summer program 2010*.



# Computational Approach



- **NGA<sup>2</sup>**
  - Arbitrarily high-order DNS/LES code
  - Massively parallel
  - Conservation of mass, momentum, and kinetic energy
- **Immersed Boundary<sup>3</sup>**
  - Based on cut-cell approach
  - Discrete conservation of mass and momentum
  - Fully implicit implementation to handle small cut-cells

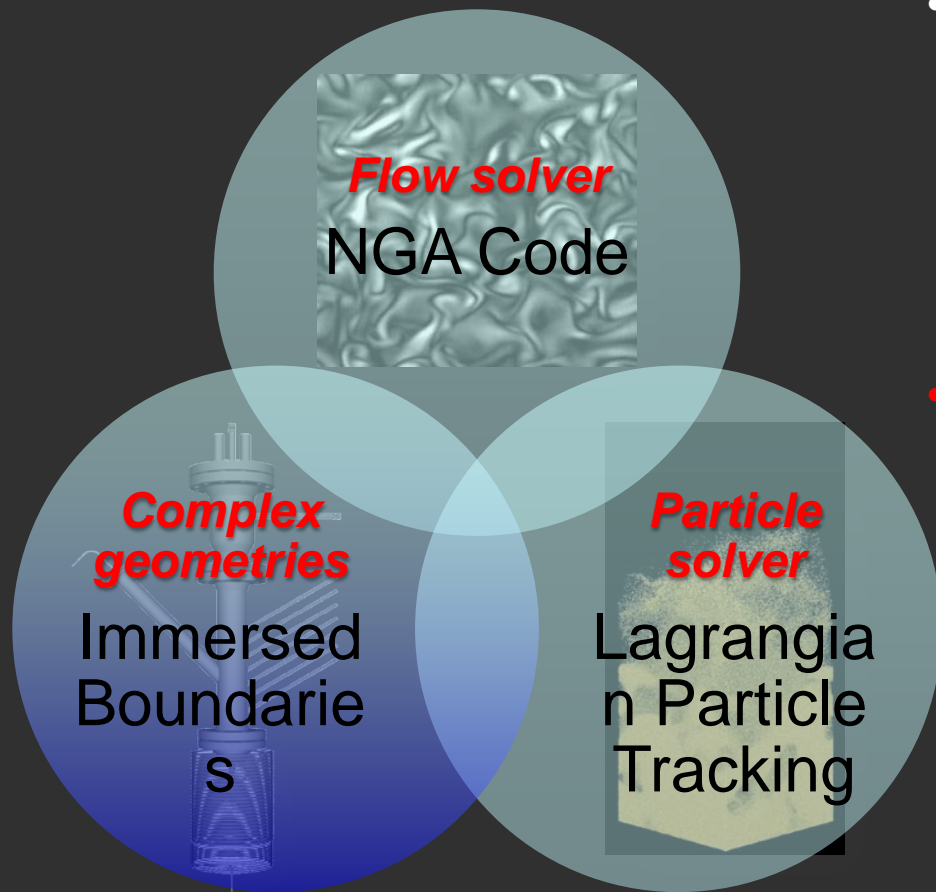
2. O. Desjardins, G. Blanquart, G. Balarac, H. Pitsch, High order conservative finite difference scheme for variable density low Mach number turbulent flows, *Journal of Computational Physics* 227 (2008) 7125– 7159.

3. P. Peipert, O. Desjardins, Direct numerical simulation of dense particle-laden flows using a conservative immersed-boundary technique, *Center of Turbulence Research, Summer program 2010*.





# Computational Approach



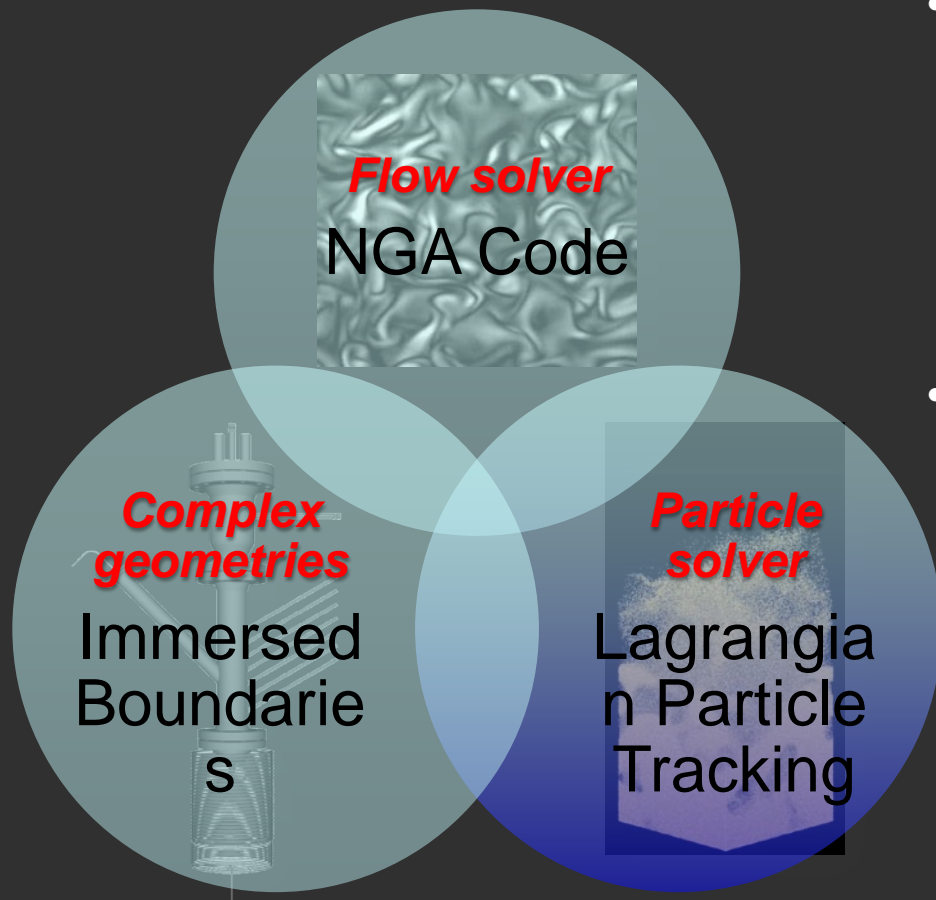
- NGA<sup>2</sup>
  - Arbitrarily high-order DNS/LES code
  - Massively parallel
  - Conservation of mass, momentum, and kinetic energy
- Immersed Boundary<sup>3</sup>
  - Based on cut-cell approach
  - Discrete conservation of mass and momentum
  - Fully implicit implementation to handle small cut-cells

2. O. Desjardins, G. Blanquart, G. Balarac, H. Pitsch, High order conservative finite difference scheme for variable density low Mach number turbulent flows, *Journal of Computational Physics* 227 (2008) 7125– 7159.

3. P. Peipert, O. Desjardins, Direct numerical simulation of dense particle-laden flows using a conservative immersed-boundary technique, *Center of Turbulence Research, Summer program 2010*.



# Computational Approach



- NGA<sup>2</sup>
  - Arbitrarily high-order DNS/LES code
  - Massively parallel
  - Conservation of mass, momentum, and kinetic energy
- Immersed Boundary<sup>3</sup>
  - Based on cut-cell approach
  - Discrete conservation of mass and momentum
  - Fully implicit implementation to handle small cut-cells

2. O. Desjardins, G. Blanquart, G. Balarac, H. Pitsch, High order conservative finite difference scheme for variable density low Mach number turbulent flows, *Journal of Computational Physics* 227 (2008) 7125– 7159.

3. P. Peipert, O. Desjardins, Direct numerical simulation of dense particle-laden flows using a conservative immersed-boundary technique, *Center of Turbulence Research, Summer program 2010*.



# NGA Computational Platform

- NGA solves the volume-filtered equations with the following assumptions & models

$$\frac{\partial \varepsilon_f \bar{\rho}_f}{\partial t} + \nabla \cdot (\varepsilon_f \bar{\rho}_f \widetilde{\mathbf{u}}_f) = S_\rho$$

$$\frac{\partial}{\partial t} (\varepsilon_f \bar{\rho}_f \widetilde{\mathbf{u}}_f) + \nabla \cdot (\varepsilon_f \bar{\rho}_f \widetilde{\mathbf{u}}_f \otimes \widetilde{\mathbf{u}}_f) = \nabla \cdot \bar{\boldsymbol{\tau}} - \nabla \cdot \mathbf{R}_u + \varepsilon_f \bar{\rho}_f \mathbf{g} + S_{\rho u} - \mathbf{F}^{\text{inter}}$$



# NGA Computational Platform

- NGA solves the volume-filtered equations with the following assumptions & models

$$\frac{\partial \varepsilon_f \bar{\rho}_f}{\partial t} + \nabla \cdot (\varepsilon_f \bar{\rho}_f \widetilde{\mathbf{u}}_f) = \cancel{S_p}$$

$$\frac{\partial}{\partial t} (\varepsilon_f \bar{\rho}_f \widetilde{\mathbf{u}}_f) + \nabla \cdot (\varepsilon_f \bar{\rho}_f \widetilde{\mathbf{u}}_f \otimes \widetilde{\mathbf{u}}_f) = \nabla \cdot \bar{\boldsymbol{\tau}} - \nabla \cdot \mathbf{R}_u + \varepsilon_f \bar{\rho}_f \mathbf{g} + \cancel{S_{pu}} - \mathbf{F}^{\text{inter}}$$



# NGA Computational Platform

- NGA solves the volume-filtered equations with the following assumptions & models

$$\frac{\partial \varepsilon_f \bar{\rho}_f}{\partial t} + \nabla \cdot (\varepsilon_f \bar{\rho}_f \widetilde{\mathbf{u}}_f) = \cancel{S_p}$$

$$\frac{\partial}{\partial t} (\varepsilon_f \bar{\rho}_f \widetilde{\mathbf{u}}_f) + \nabla \cdot (\varepsilon_f \bar{\rho}_f \widetilde{\mathbf{u}}_f \otimes \widetilde{\mathbf{u}}_f) = \nabla \cdot \bar{\boldsymbol{\tau}} - \nabla \cdot \mathbf{R}_u + \varepsilon_f \bar{\rho}_f \mathbf{g} + \cancel{S_{\rho u}} - \mathbf{F}^{\text{inter}}$$



$$\mathbf{R}_u = -\nu_T \left( \nabla \widetilde{\mathbf{u}}_f + \nabla \widetilde{\mathbf{u}}_f^T \right)$$



# NGA Computational Platform

- NGA solves the volume-filtered equations with the following assumptions & models

$$\frac{\partial \varepsilon_f \bar{\rho}_f}{\partial t} + \nabla \cdot (\varepsilon_f \bar{\rho}_f \widetilde{\mathbf{u}}_f) = \cancel{S_p}$$

$$\frac{\partial}{\partial t} (\varepsilon_f \bar{\rho}_f \widetilde{\mathbf{u}}_f) + \nabla \cdot (\varepsilon_f \bar{\rho}_f \widetilde{\mathbf{u}}_f \otimes \widetilde{\mathbf{u}}_f) = \nabla \cdot \bar{\boldsymbol{\tau}} - \nabla \cdot \mathbf{R}_u + \varepsilon_f \bar{\rho}_f \mathbf{g} + \cancel{S_{pu}} - \mathbf{F}^{\text{inter}}$$

$$\mathbf{R}_u = -\nu_T \left( \nabla \widetilde{\mathbf{u}}_f + \nabla \widetilde{\mathbf{u}}_f^T \right)$$

*Drag model of Tenneti & Subramaniam (2011)*

$$\mathbf{f}_i^{\text{drag}} = \frac{1}{\tau_p} (\widetilde{\mathbf{u}}_f - \mathbf{u}_p) F(\varepsilon_f, \text{Re}_p)$$

$$\tau_p = \frac{\rho_p d_p^2}{18\mu\varepsilon_f} \quad \text{Re}_p = \frac{\varepsilon_f \rho_f |\widetilde{\mathbf{u}}_f - \mathbf{u}_p| d_p}{\mu}$$

$$F(\varepsilon_f, \text{Re}_p) = \frac{1 + 0.15 \text{Re}_p^{0.687}}{\varepsilon_f^2} + \frac{5.81(1 - \varepsilon_f)}{\varepsilon_f^2} + \frac{0.48(1 - \varepsilon_f)^{1/3}}{\varepsilon_f^3} + \varepsilon_f(1 - \varepsilon_f)^3 \text{Re}_p \left( 0.95 + \frac{0.61(1 - \varepsilon_f)^3}{\varepsilon_f^2} \right)$$





# NGA Computational Platform

- NGA solves the volume-filtered equations with the following assumptions & models

$$\frac{\partial \varepsilon_f \bar{\rho}_f}{\partial t} + \nabla \cdot (\varepsilon_f \bar{\rho}_f \widetilde{\mathbf{u}}_f) = \cancel{S_p}$$

$$\frac{\partial}{\partial t} (\varepsilon_f \bar{\rho}_f \widetilde{\mathbf{u}}_f) + \nabla \cdot (\varepsilon_f \bar{\rho}_f \widetilde{\mathbf{u}}_f \otimes \widetilde{\mathbf{u}}_f) = \nabla \cdot \bar{\boldsymbol{\tau}} - \nabla \cdot \mathbf{R}_u + \varepsilon_f \bar{\rho}_f \mathbf{g} + \cancel{S_{pu}} - \mathbf{F}^{\text{inter}}$$

$$\mathbf{R}_u = -\nu_T \left( \nabla \widetilde{\mathbf{u}}_f + \nabla \widetilde{\mathbf{u}}_f^T \right)$$

$$\bar{\boldsymbol{\tau}} = -p \mathbf{I} + \mu^* \left[ \left( \nabla \bar{\mathbf{u}}_f + \nabla \bar{\mathbf{u}}_f^T \right) - \frac{2}{3} (\nabla \cdot \bar{\mathbf{u}}_f) \mathbf{I} \right] + \cancel{\mathbf{R}_p} \quad \text{Drag model of Tenneti \& Subramaniam (2011)}$$

*Effective viscosity model of Gibilaro et al.*

$$\mu^* = \mu \varepsilon_f^{-2.8}$$

$$\mathbf{f}_i^{\text{drag}} = \frac{1}{\tau_p} (\widetilde{\mathbf{u}}_f - \mathbf{u}_p) F(\varepsilon_f, \text{Re}_p)$$

$$\tau_p = \frac{\rho_p d_p^2}{18 \mu \varepsilon_f} \quad \text{Re}_p = \frac{\varepsilon_f \rho_f |\widetilde{\mathbf{u}}_f - \mathbf{u}_p| d_p}{\mu}$$

$$F(\varepsilon_f, \text{Re}_p) = \frac{1 + 0.15 \text{Re}_p^{0.687}}{\varepsilon_f^2} + \frac{5.81 (1 - \varepsilon_f)}{\varepsilon_f^2} + \frac{0.48 (1 - \varepsilon_f)^{1/3}}{\varepsilon_f^3} + \varepsilon_f (1 - \varepsilon_f)^3 \text{Re}_p \left( 0.95 + \frac{0.61 (1 - \varepsilon_f)^3}{\varepsilon_f^2} \right)$$



# Lagrangian Particle Tracking

- A set of 9 coupled ODEs are solved for each individual particle
- Time advancement based on
  - 2<sup>nd</sup> order Runge-Kutta for particle ODEs
  - 2<sup>nd</sup> order coupling between gas and particles phase

$$\frac{d\mathbf{x}_p}{dt} = \mathbf{u}_p$$

$$m_p \frac{d\mathbf{u}_p}{dt} = \int_{S_p} \boldsymbol{\tau} \cdot \mathbf{n} \, d\mathbf{y} + F^{\text{col}} + m\mathbf{g}$$

$$I_p \frac{d\boldsymbol{\omega}_p}{dt} = \int_{S_p} \frac{d_p}{2} \mathbf{n} \times (\boldsymbol{\tau} \cdot \mathbf{n}) \, d\mathbf{y} + \sum_{j=1}^{n_p} \mathbf{f}_{t,j \rightarrow p}^{\text{col}}$$

- Collisions based on soft sphere approach<sup>6</sup> modified for parallel efficiency
- Exchange between phases
  - Gas phase data is interpolated to the particle location using trilinear interpolation
  - Particle data is filtered onto the Eulerian mesh with an implicit/conservative smoothing operation



# Lagrangian Particle Tracking

- A set of 9 coupled ODEs are solved for each individual particle
- Time advancement based on
  - 2<sup>nd</sup> order Runge-Kutta for particle ODEs
  - 2<sup>nd</sup> order coupling between gas and particles phase

$$\frac{d\mathbf{x}_p}{dt} = \mathbf{u}_p$$

$$m_p \frac{d\mathbf{u}_p}{dt} = \int_{S_p} \boldsymbol{\tau} \cdot \mathbf{n} \, d\mathbf{y} + F^{\text{col}} + m\mathbf{g}$$

$$I_p \frac{d\boldsymbol{\omega}_p}{dt} = \int_{S_p} \frac{d_p}{2} \cancel{\mathbf{n}} (\boldsymbol{\tau} \cdot \mathbf{n}) \, d\mathbf{y} + \sum_{j=1}^{n_p} \mathbf{f}_{t,j \rightarrow p}^{\text{col}}$$

- Collisions based on soft sphere approach<sup>6</sup> modified for parallel efficiency
- Exchange between phases
  - Gas phase data is interpolated to the particle location using trilinear interpolation
  - Particle data is filtered onto the Eulerian mesh with an implicit/conservative smoothing operation

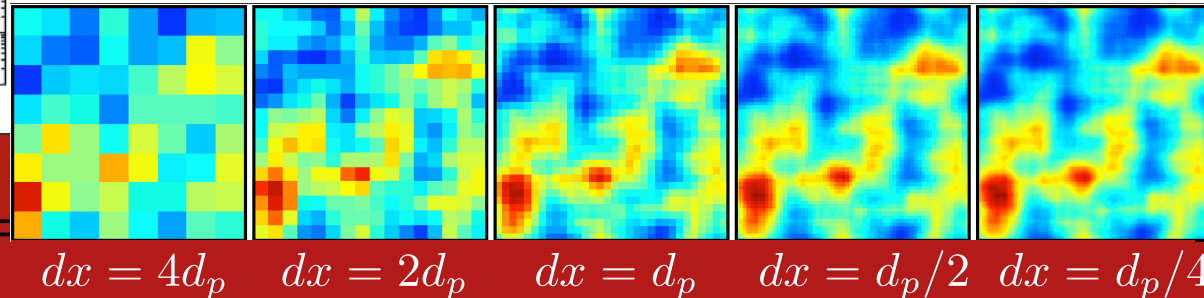
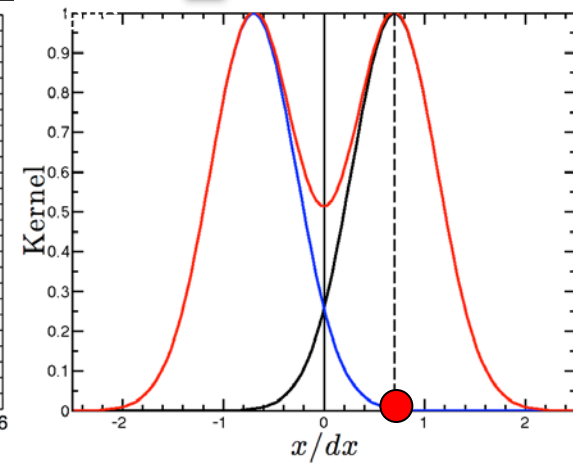
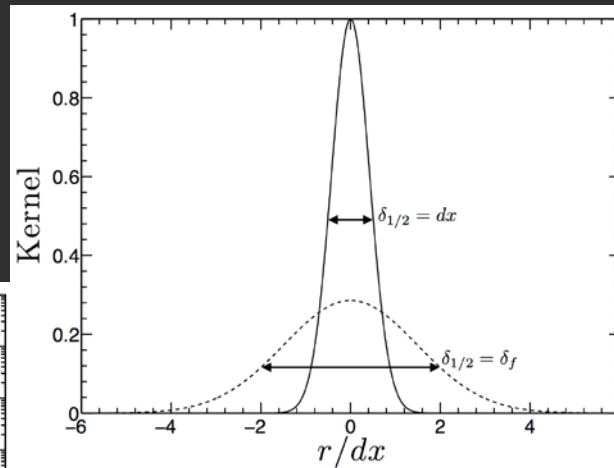
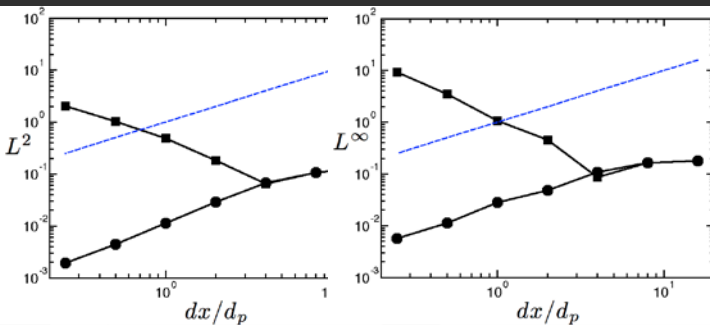


# Particle Data to Eulerian Mesh

- Need to transfer  $\varepsilon_f$  and  $\mathbf{F}^{\text{inter}}$  to underlying mesh consistent with mathematical formulation
- Filtering needs to be based on particle size not mesh
- Filter based on the convolution of mollification and Laplacian smoothing
  - Mollification: extrapolate particle data to neighboring cells
  - Diffusion: smooth data with specified width
- Filter width  $\delta_f$  is independent of the mesh  $\delta_x$  Gaussian diffusion

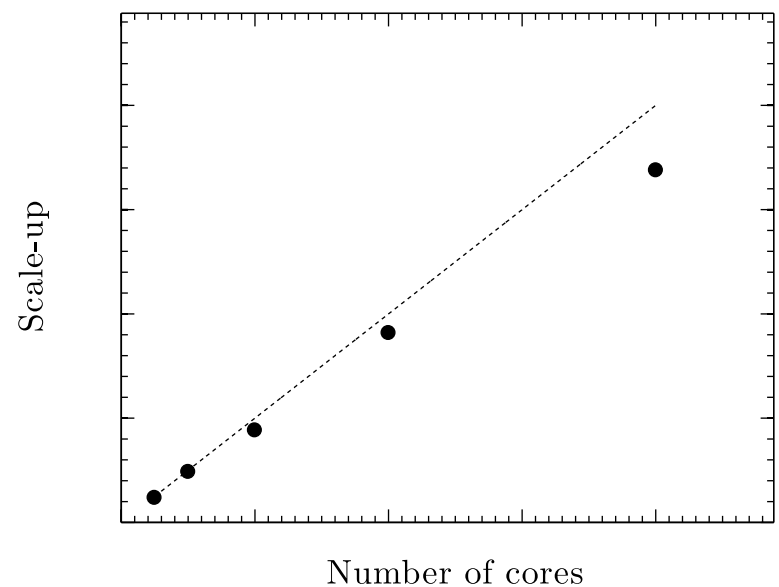
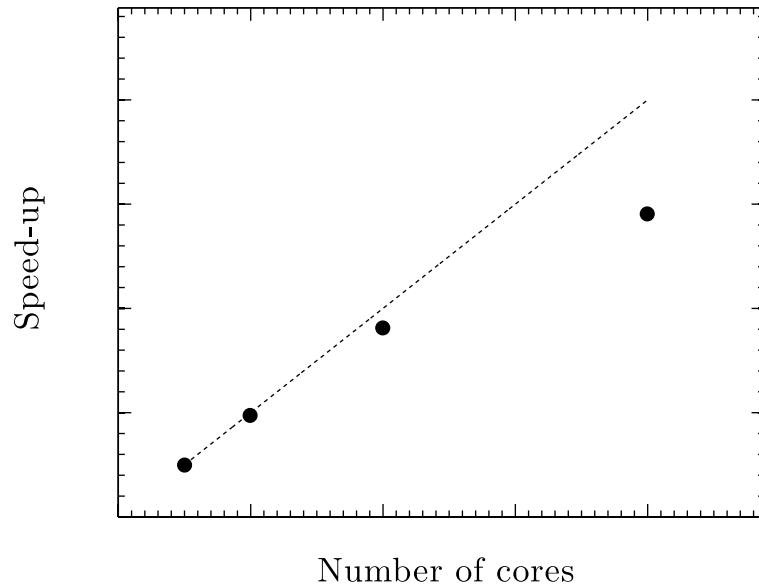
Introduce image particle to apply Neumann condition near

L-norm error



# Parallel Performance

- Parallelization: MPI (domain decomposition)
- Scaling performed on Red Mesa (Sandia National Labs)
- 134 million cells, 383 million particles



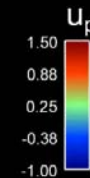
# Initial validation

Spout fluidization (*Link et al., 2005*)

- Case A:  $u_{bg} = 1.5$  m/s,  $u_{sp} = 30$  m/s
- Case B:  $u_{bg} = 3.0$  m/s,  $u_{sp} = 20$  m/s

Bed dimensions [m]	0.75 x 0.15 x 0.015
Spout width [m]	0.01
Particle diameter [m]	0.0025
Particle density [kg/m <sup>3</sup> ]	2526
Number of particles [-]	245,000
Grid [-]	300 x 60 x 6

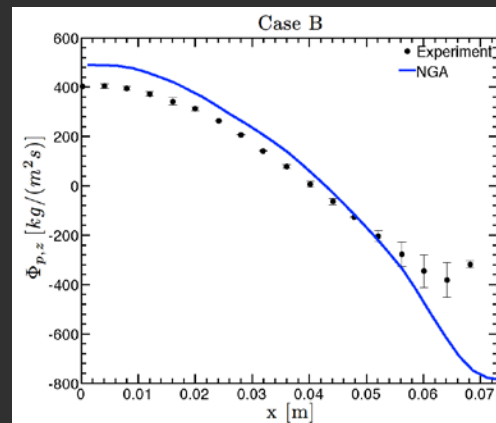
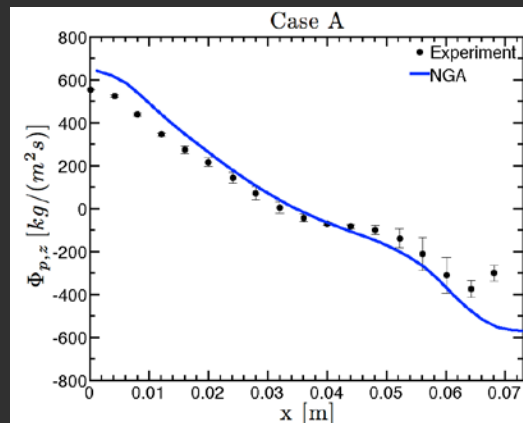
Time = 0.00000



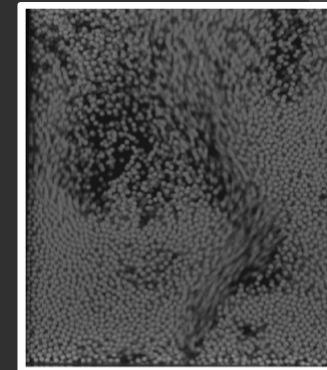
Spout-fluidization

Jet in fluidized bed

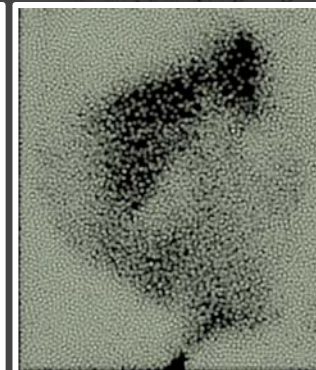
Particle mass flux



Case B (exp)



Case B (sim)



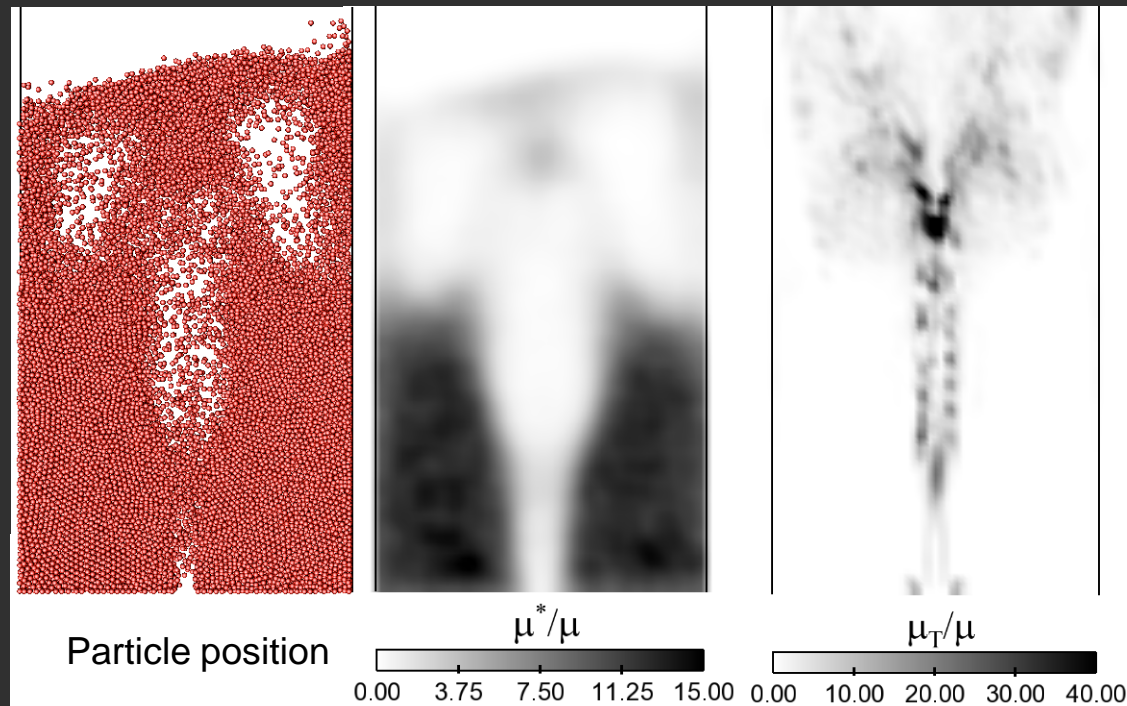
Cornell University  
Computational Thermo-Fluids  
Laboratory



# Initial validation

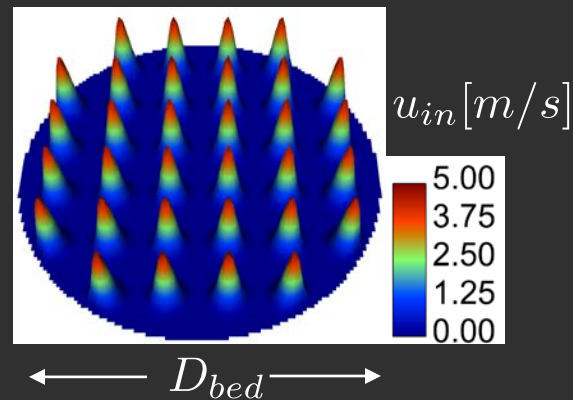
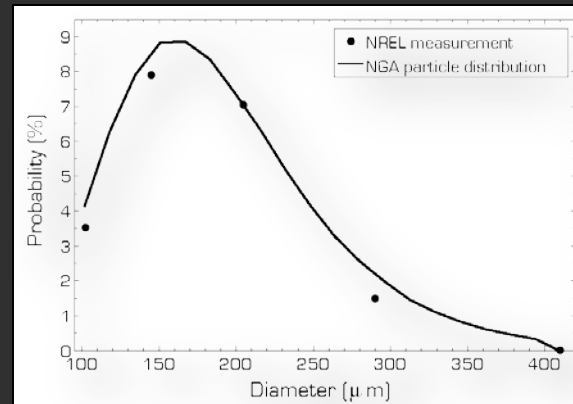
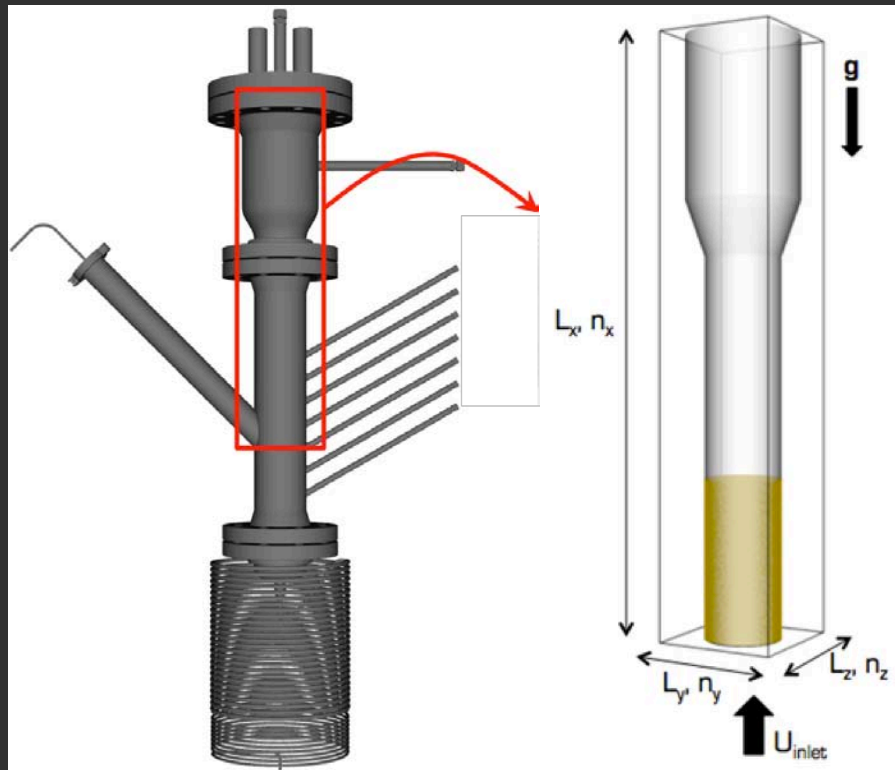
## Spout fluidization, case A

- Dynamic Smagorinsky eddy viscosity model<sup>8</sup> to close  $\tau_{ij}$
- Based on Lagrangian averaging<sup>9</sup>
- SGS model does not account for turbulence modulation by particles
- Spatial segregation between models



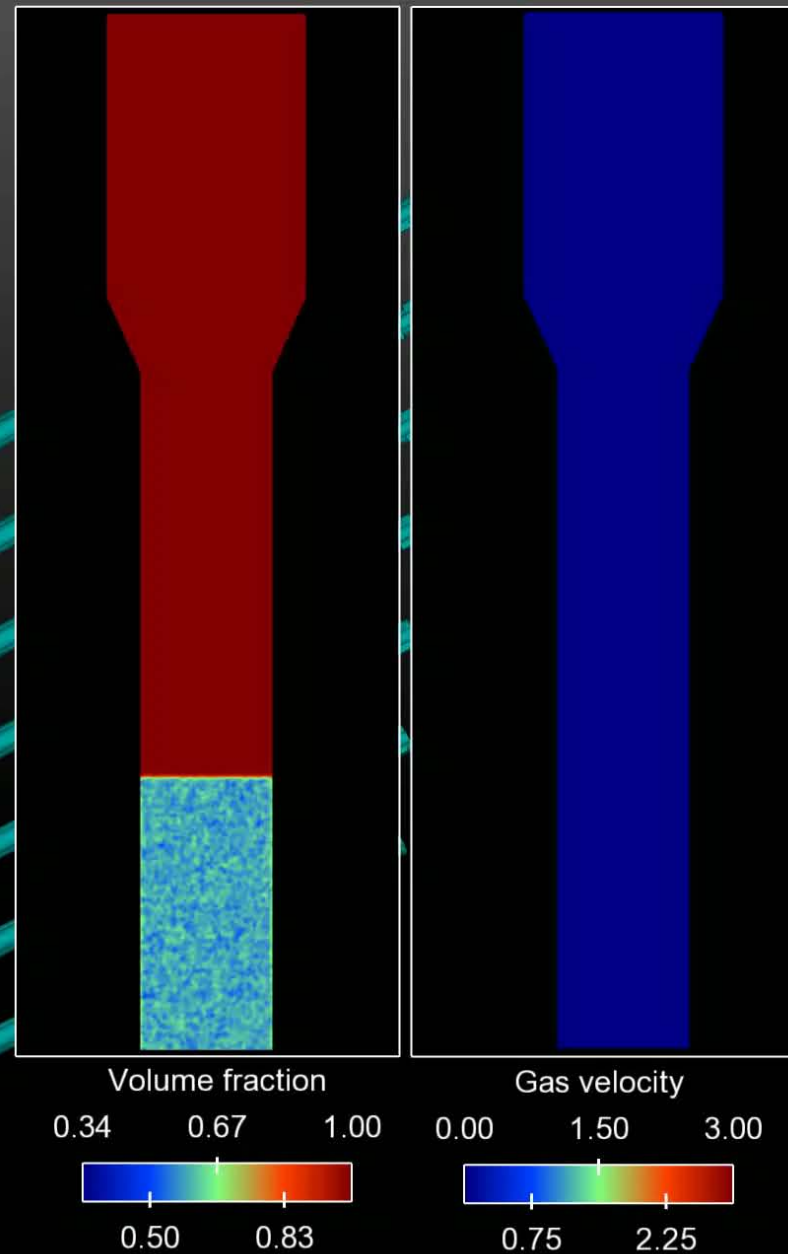
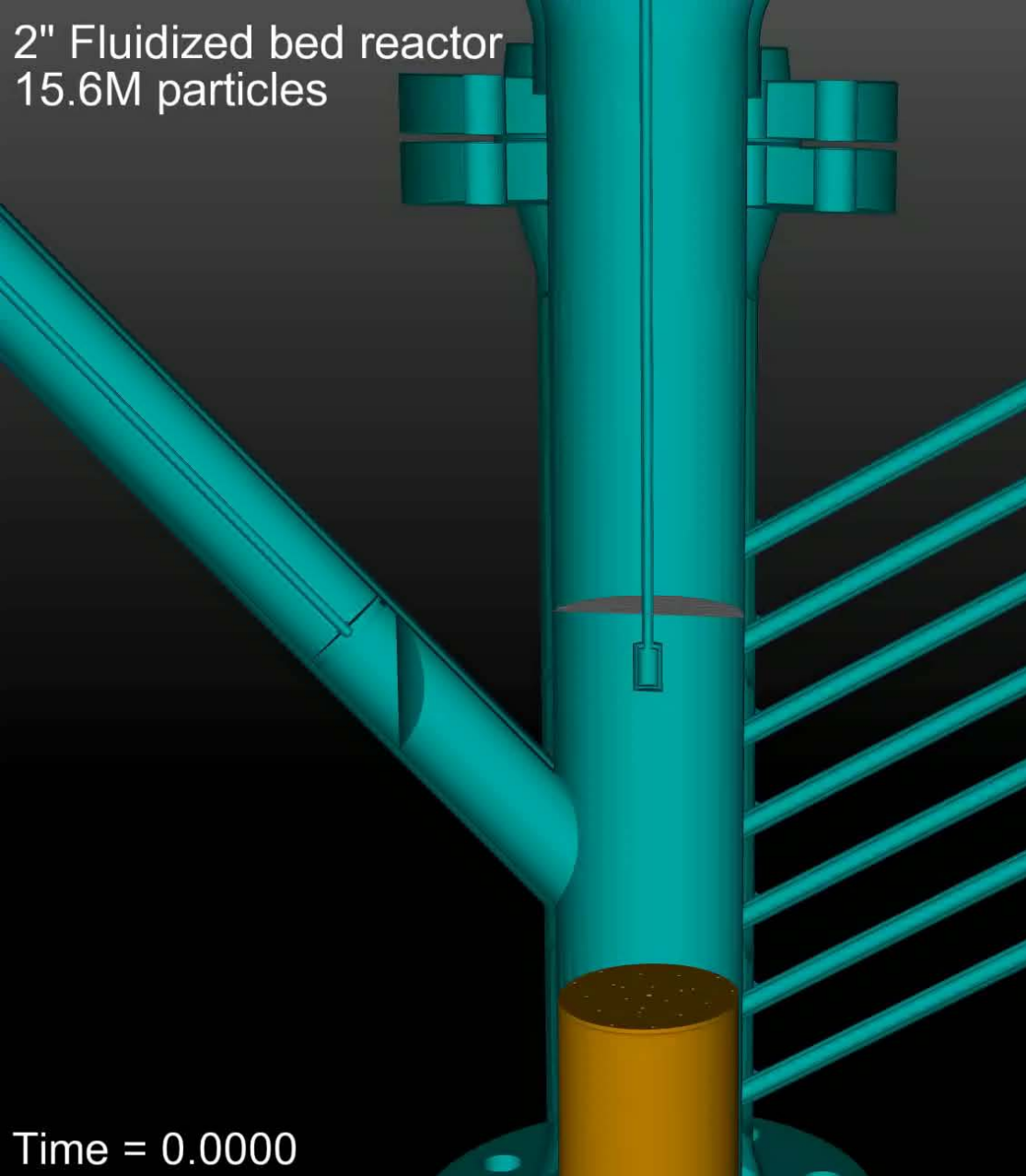
# Half-scale simulation of NREL's 4-in fluidized bed reactor

- National Renewable Energy Laboratory operates a 4-inch fluidized bed reactor for biomass gasification



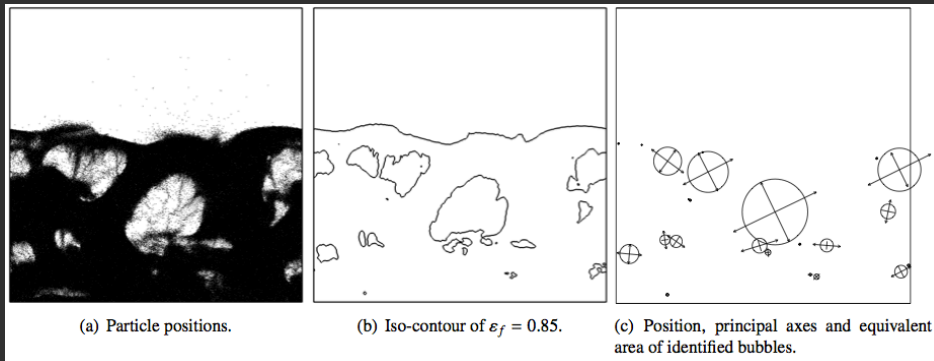
- $u_{in} = 6 \rightarrow 12U_{mf}$
- 15.6 M particles
- 20 M grid cells
- 576 cores on Marvin (Cornell cluster)





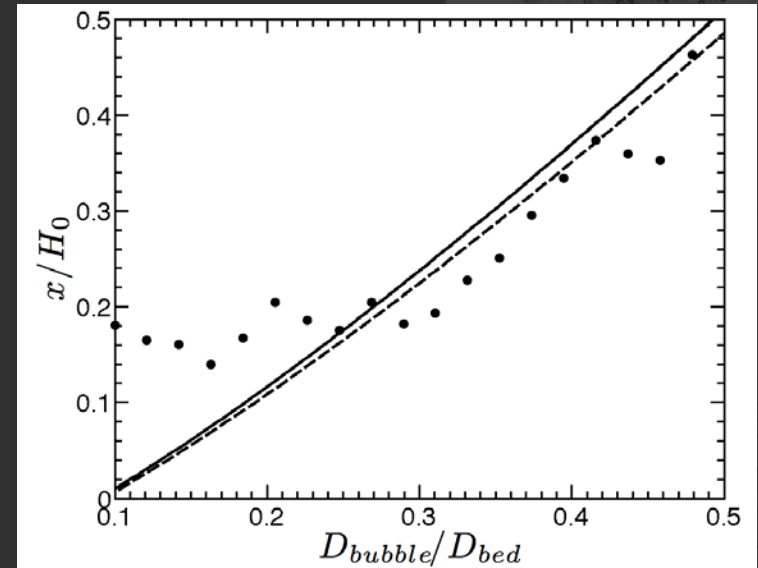
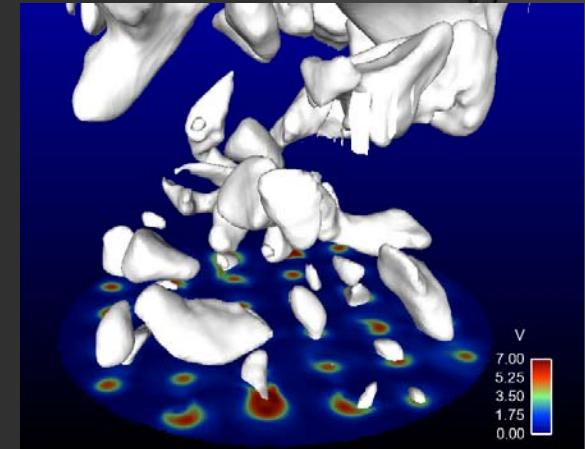
# Bubble characterization

- **Structure identification** algorithm<sup>10</sup>
  - Band-growth algorithm to identify bubbles



- Mean bubble diameter:  $0.18 \cdot D_{bed}$
- On average 8 bubbles in bed at once
- Compared results with Darton<sup>11</sup> correlation
  - $U_{in} = 6 U_{mf}$  (solid line)
  - $U_{in} = 12 U_{mf}$  (Dashed line)

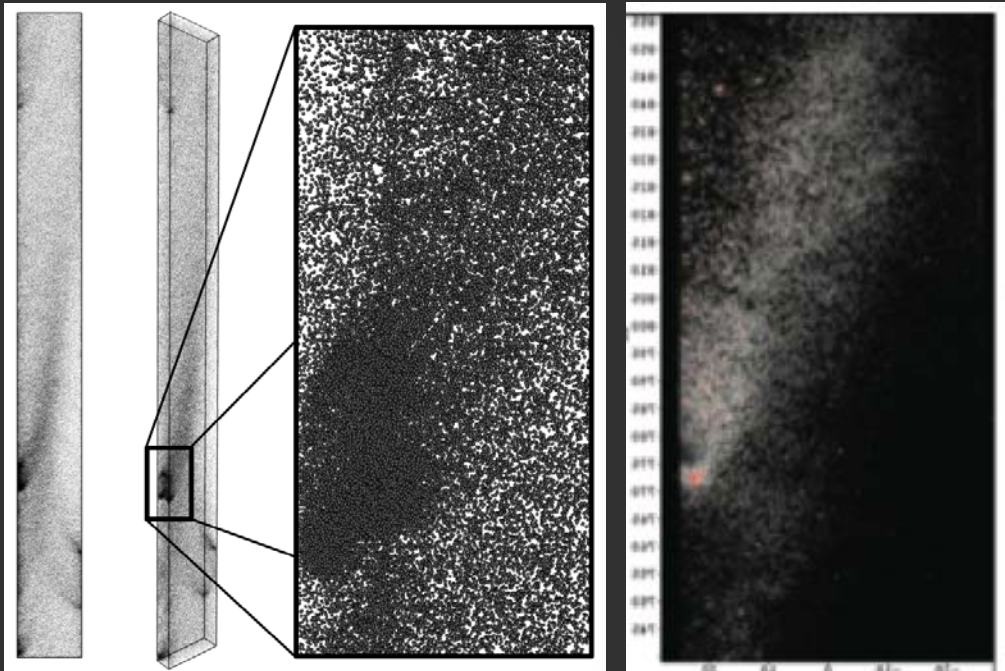
$$D_b = 0.54 (U_{in} - U_{mf})^{0.4} \left( h + 4\sqrt{A_0} \right)^{0.8} g^{-0.2}$$





# Simulation of a Turbulent Riser

- Periodic in stream-wise directions
- 266,760 particles initially uniform distribution
- 800 x 82 x 26 mesh for 0.5 m domain
- Formation of clusters along walls is observed – excellent qualitative agreement with experiments<sup>12</sup>



# Simulation of a Turbulent Riser

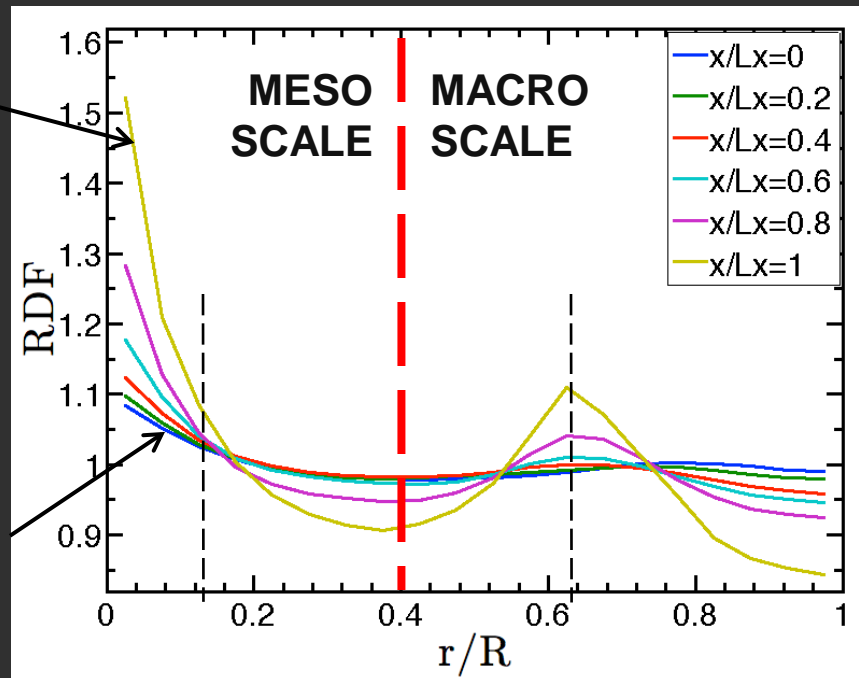
- Radial Distribution Function (RDF) to characterize clustering

$$RDF(x, r) = \frac{N_p L_z}{2drN(N-1)}$$

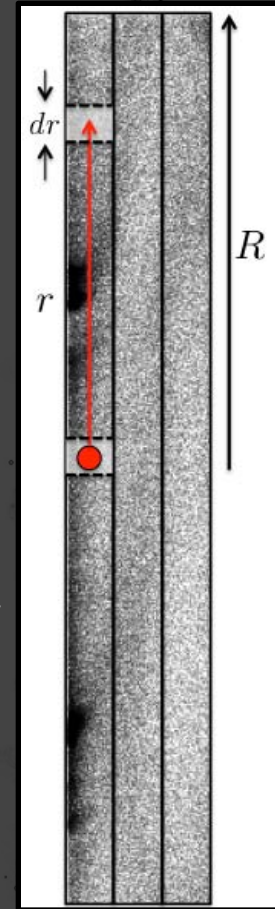
RDF=1: Uniform distribution

RDF>1: Clustering

- N: Number of particles in column
- $N_p$ : Number of particle pairs
- Total possible number of pairs:  $N(N-1)$



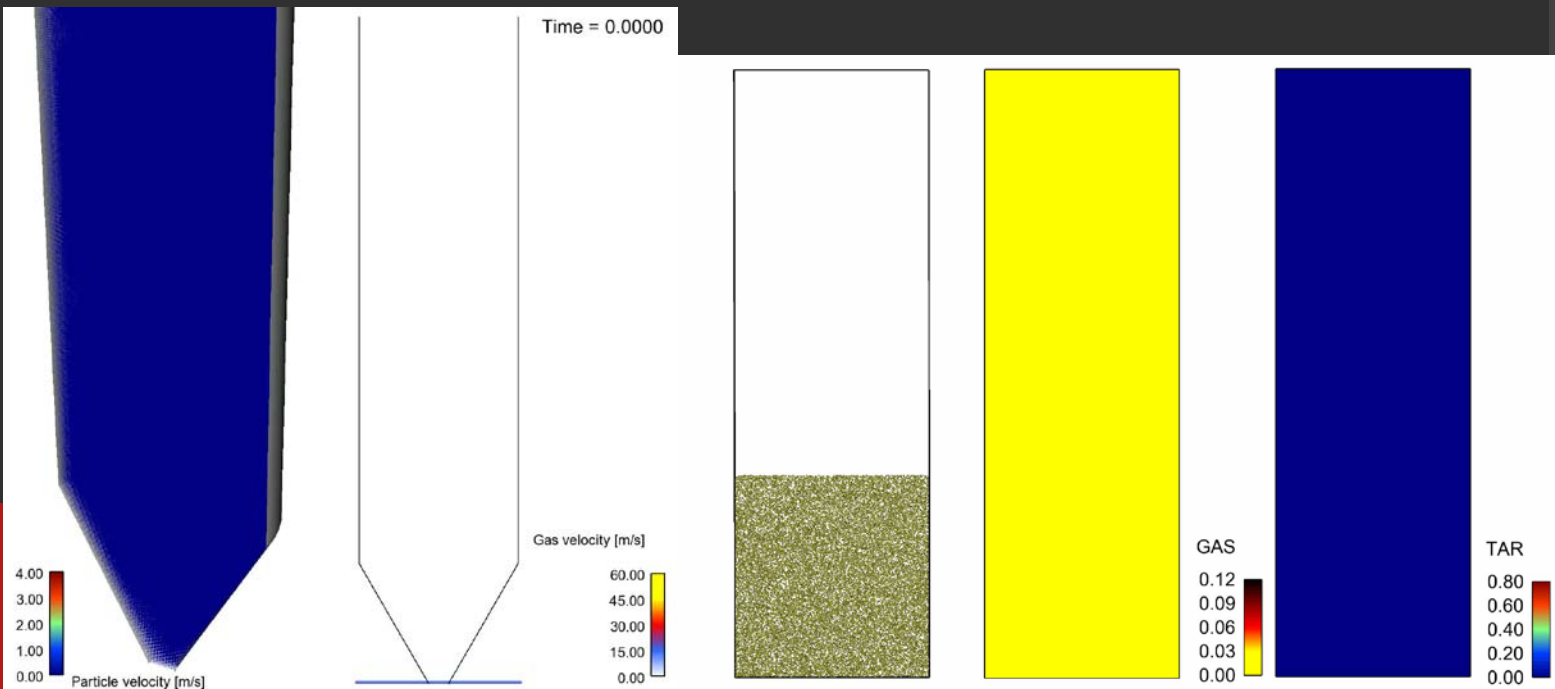
- Meso-scale
  - Maximum clustering at the walls
  - Characteristic cluster size  $\sim 90 d_p$
- Macro-scale
  - Second peak in RDF
  - Characteristic length scale  $\sim L_z / 3$  (or  $460 d_p$ )





# Conclusions

- Presented a simulation strategy for turbulent particle-laden flows in complex geometries
- Simulations of fluidized bed reactors show good agreement with experimental data
- Looking forward:
  - Further validation
  - Chemistry is currently being incorporated (Pepiot research group)
  - Investigate clustering in turbulent risers
  - Use resolved particle simulations for exploring the closure of the filtered equations
  - Transition to industrial scale approaches (QMOM, parcels...)



Time = 0.00000

# Questions?

