Heat transfer enhancement in suspensions of agitated solids

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Theory

Particle agitation and conductivity

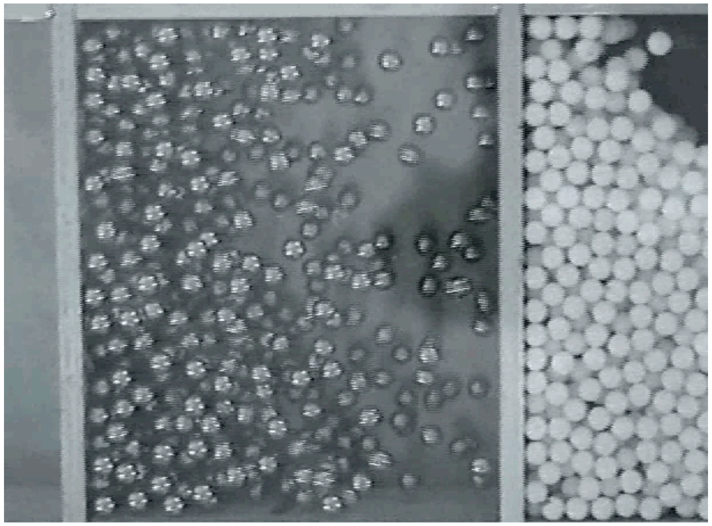
"granular temperature"

$$\Theta = \frac{1}{3}\,\overline{u'_i\,u'_i}$$

fluctuation velocity u'_i

fluctuation kinetic energy per grain mass

Jenkins, Savage, Lun, others



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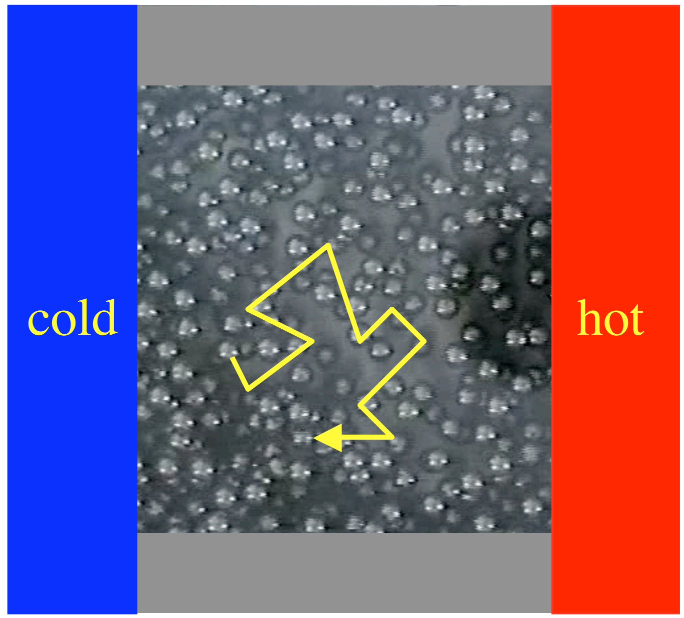
$$\Theta = \frac{1}{3}\,\overline{u'_i\,u'_i}$$



Predictions of the kinetic theory

Self-diffusivity $D_s = \dfrac{d\sqrt{\Theta}}{9\sqrt{\pi}\nu g_{12}} \Rightarrow K_s = \rho_s \nu c_s D_s = \dfrac{\rho_s c_s d\sqrt{\Theta}}{9\sqrt{\pi}\, g_{12}}$

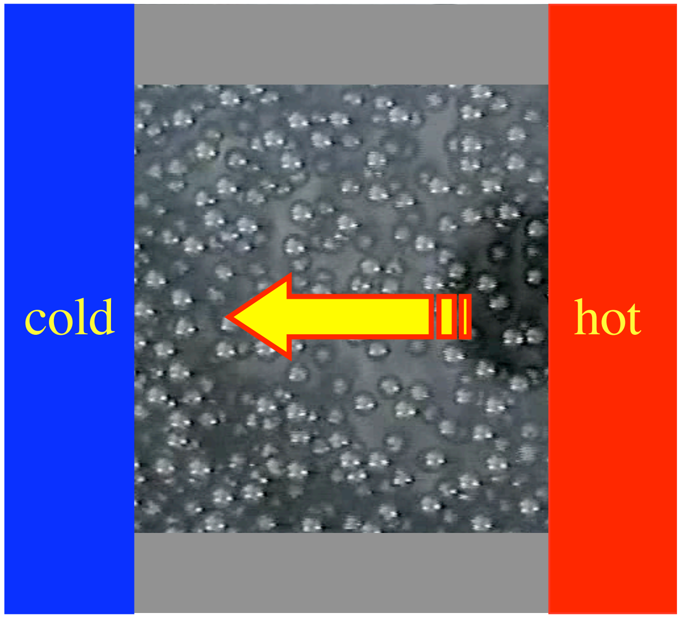
diameter d, solid volume fraction ν, density ρ_s, specific heat c_s, pair correlation g_{12}

Mixture (static) conductivity

Maxwell model (1873)

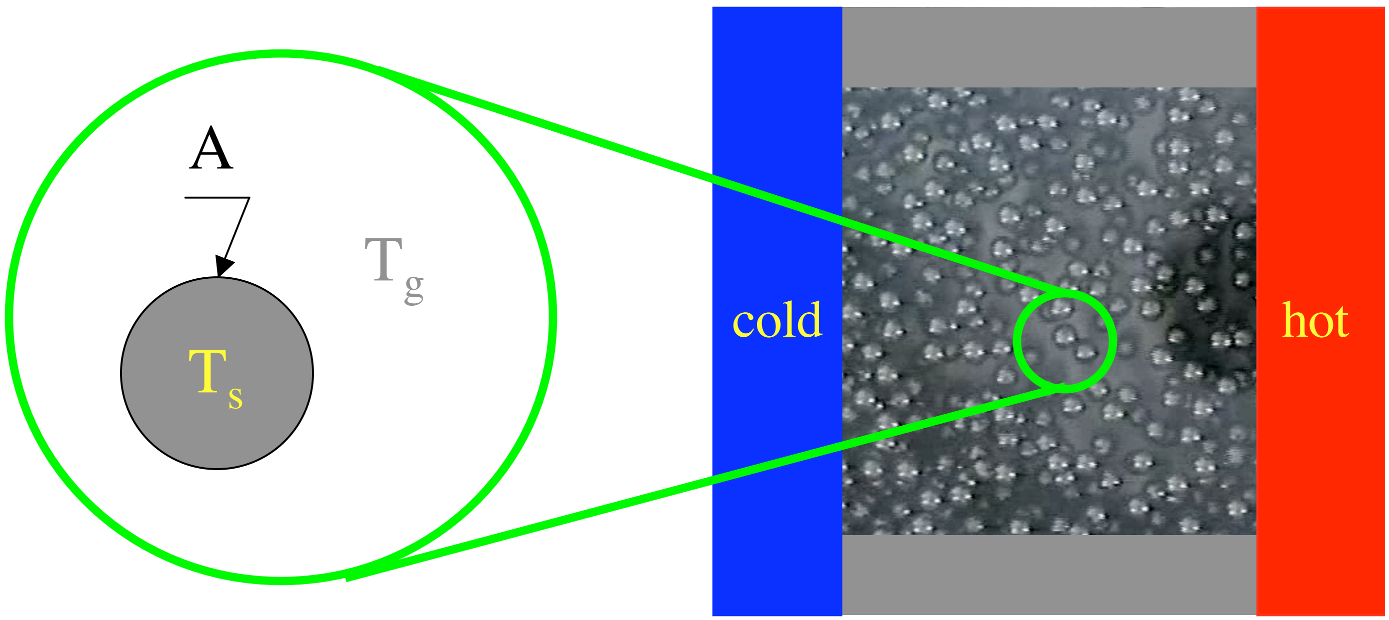
$$\frac{K_g}{k_g} = \frac{(2+\xi_s) + 2\nu(\xi_s - 1)}{(2+\xi_s) - \nu(\xi_s - 1)}$$

$$\xi_s \equiv \frac{k_s}{k_g}$$



solid volume fraction ν, solid conductivity k_s, gas conductivity k_g

Heat exchange

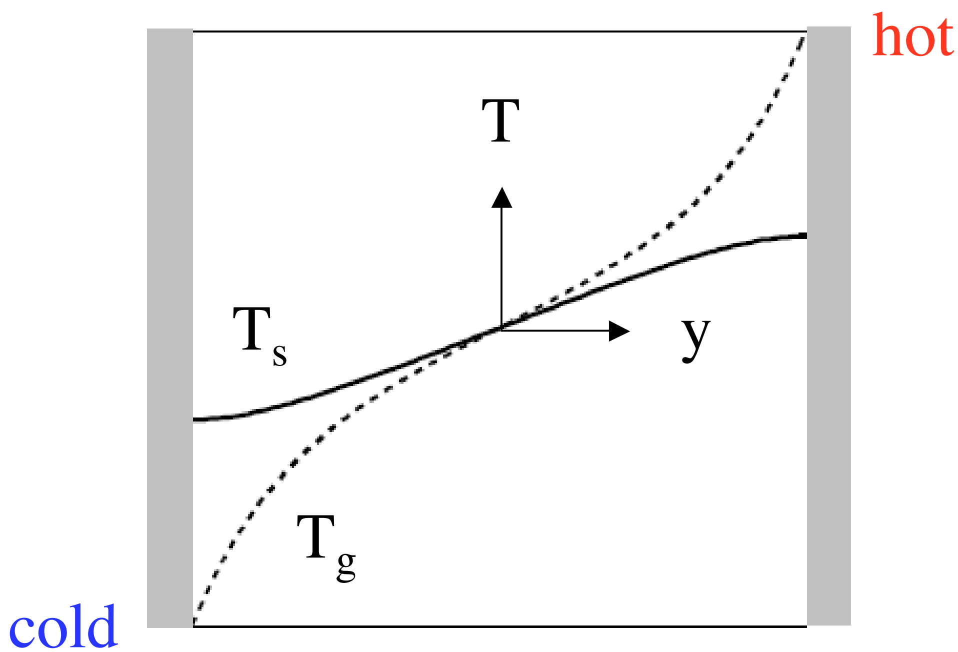


$$H = nAh(T_s - T_g) = 12\nu \frac{k_g}{d^2} \mathrm{Nu}(T_s - T_g) \qquad \mathrm{Nu} = \frac{h(d/2)}{k_g} \approx 1$$

solid volume fraction ν, number density n, diameter d and surface area A, gas conductivity k_g

Heat transfer in agitated suspensions

$$\partial/\partial t = 0,\ \partial/\partial x = 0,\ \nabla v = 0,\ \mathrm{v} = 0$$



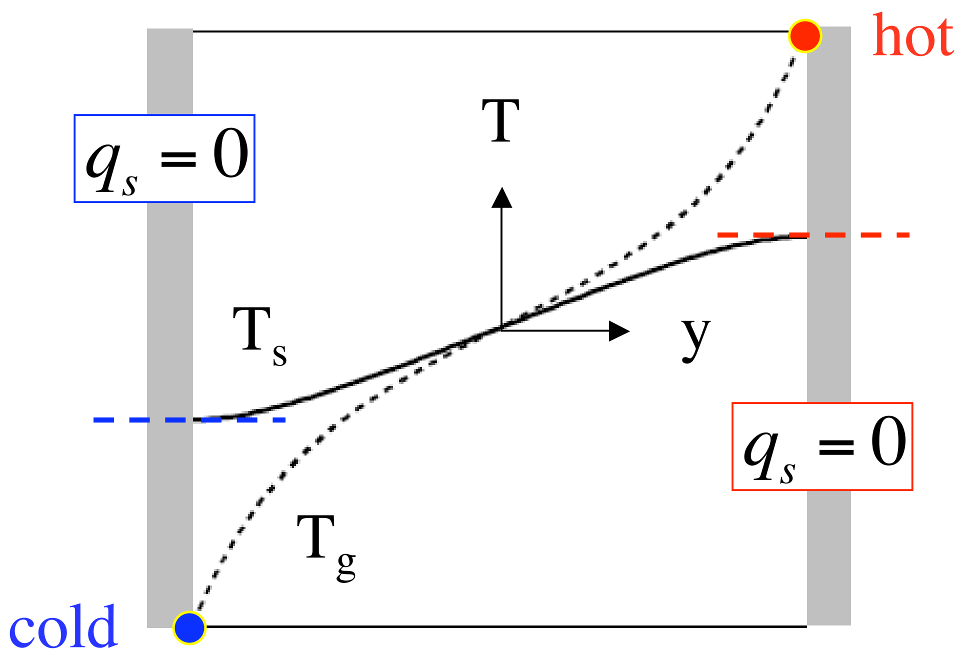
$$0 = -\frac{d}{dy}\left(-K_g \frac{dT_g}{dy}\right) + H$$

$$0 = -\frac{d}{dy}\left(-K_s \frac{dT_s}{dy}\right) - H$$

phase conductivities K_g and K_s; volumetric exchange rate H

Heat transfer in agitated suspensions

Boundary conditions



Governing equations

$$0 = -\frac{d}{dy}\left(-K_g \frac{dT_g}{dy}\right) + H$$

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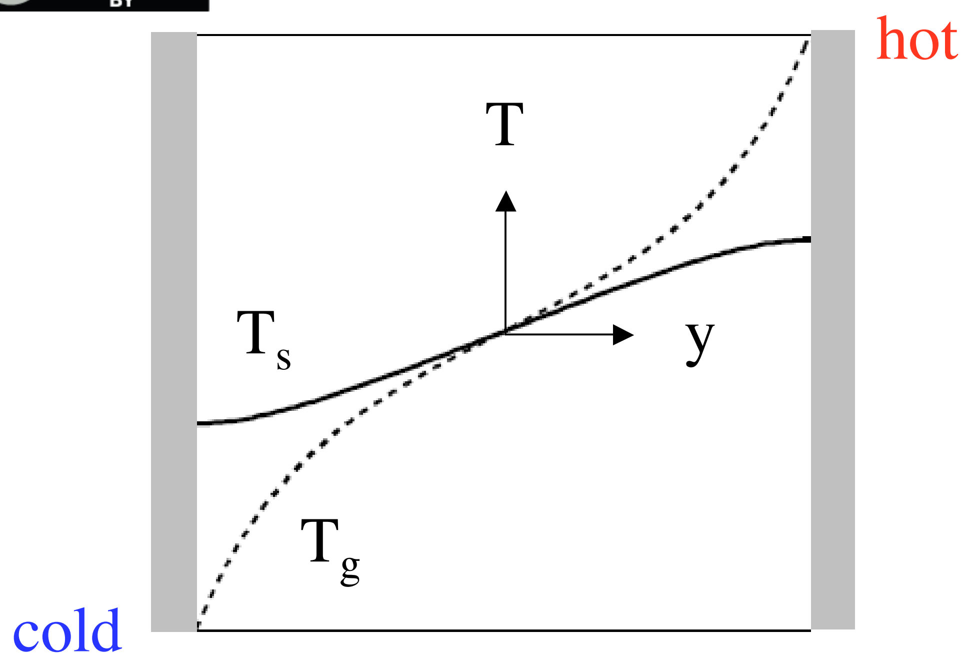
Heat transfer in agitated suspensions

Boundary conditions



Governing equations

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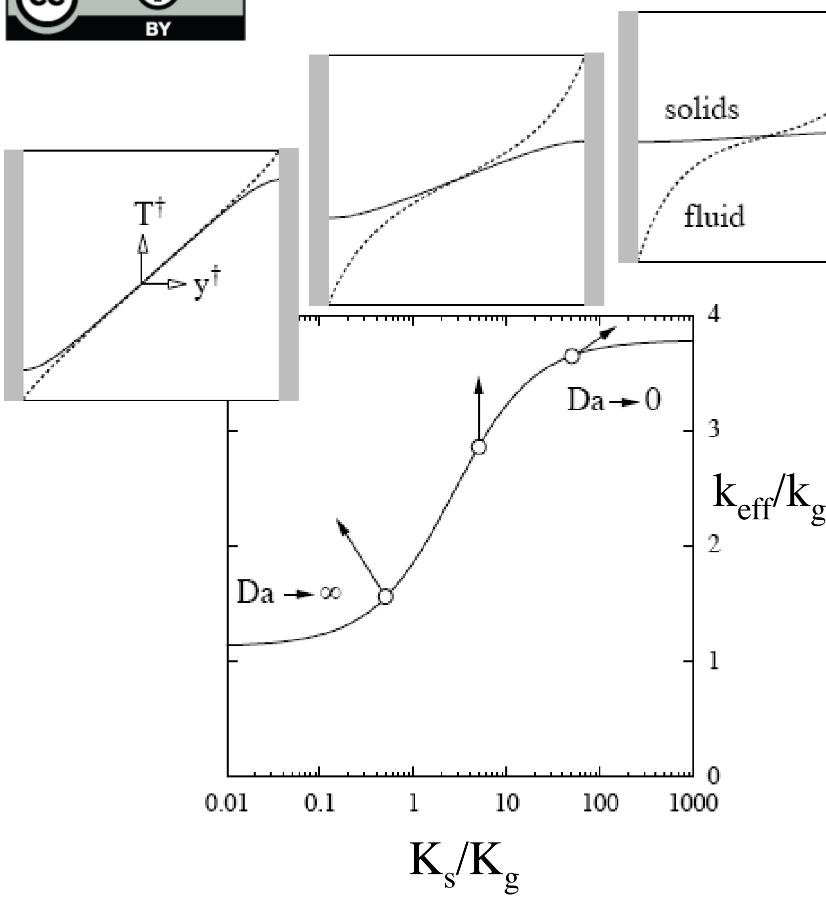
$$T_g(y) \;\&\; T_s(y) \;\Rightarrow\; q = -K_g \frac{\partial T_g}{\partial y} \;\Rightarrow\; \frac{q(\nu)}{q(\nu = 0)} = \frac{k_{eff}}{k_g}$$

Diffusion and Exchange limits



Conductivity enhancement

$$\frac{k_{\text{eff}}}{k_g} = \left(\frac{K_g}{k_g}\right)\left(\frac{1 + K_s / K_g}{1 + 1/\text{Da}}\right)$$

$$\boxed{\text{Da} = \frac{(K_g / K_s)(L^\dagger / 2)}{\tanh(L^\dagger / 2)}}$$

$$L^\dagger = \frac{L}{d}\sqrt{12\nu\text{Nu}\left(\frac{k_g}{K_g} + \frac{k_g}{K_s}\right)}$$

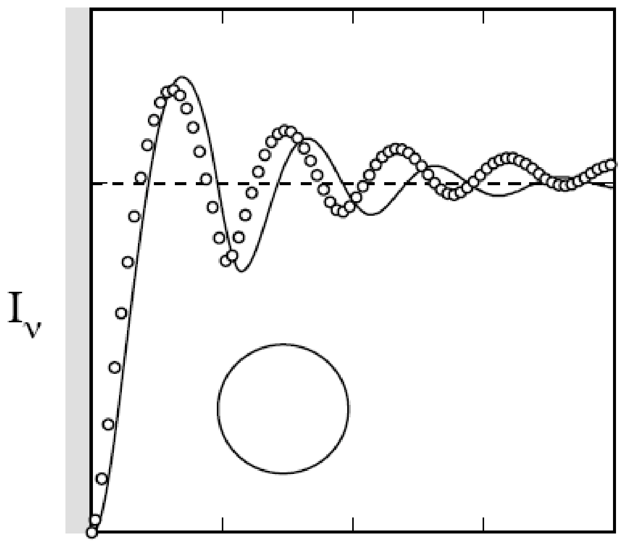
phase conductivities K_g and K_s; fluid conductivity k_g; particle diameter d

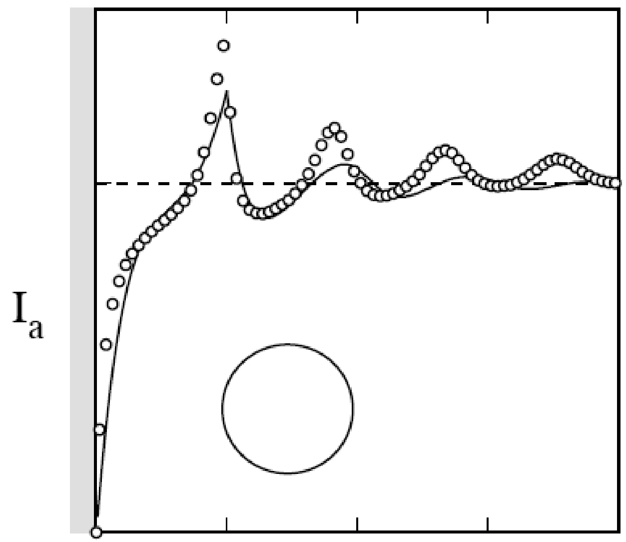
Complications

Role of ordering and oscillations

$$\frac{K_g}{k_g} = \frac{(2+\xi_s) + 2\nu(\xi_s - 1)}{(2+\xi_s) - \nu(\xi_s - 1)}$$

$$H = (nA)\, h(T_s - T_g)$$





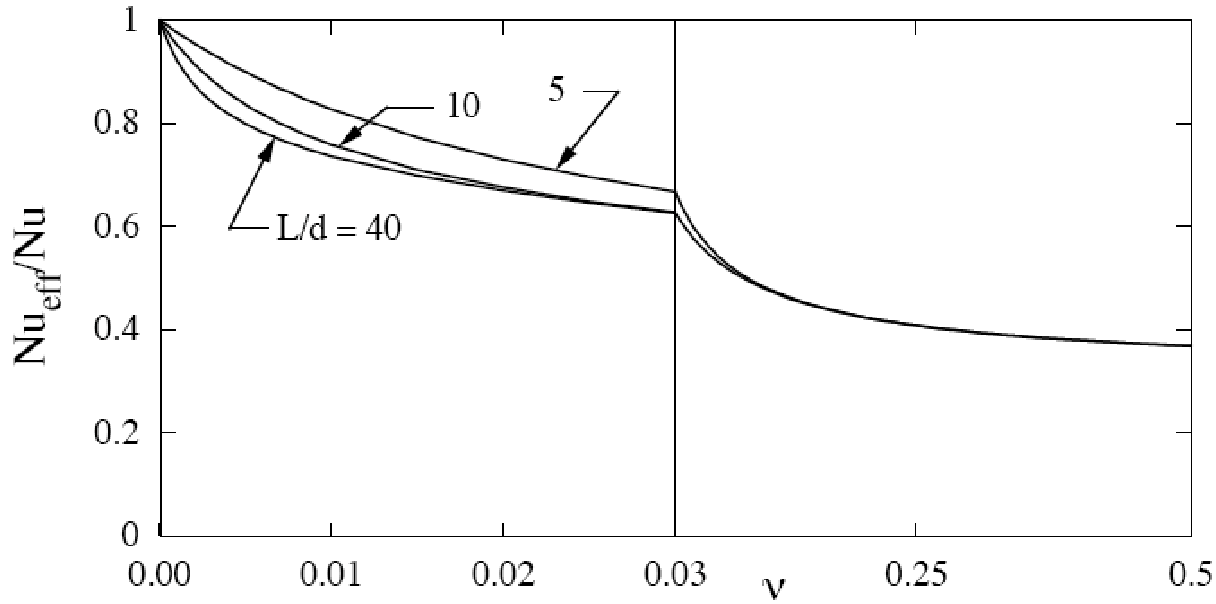
$$\tilde{\nu} = \nu I_\nu$$

$$\widetilde{nA} = \left(\frac{6\nu}{d}\right) I_a$$

Henderson, Abraham & Barker, Molec. Phys. (1976)

Role of ordering and oscillations

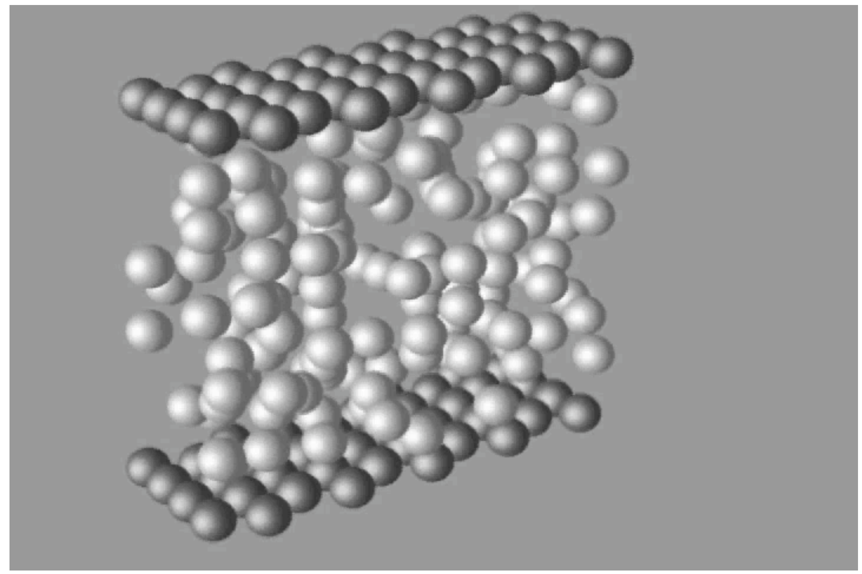
$$\tilde{H} = 12\nu \frac{k_g}{d^2} \mathrm{Nu}_{\mathrm{eff}} (T_s - T_g)$$



Turbulent enhancements in the exchange limit

Verberg and Koch, Phys. Fluids 2006

$$\boxed{\overline{u'_i u'_j} = -\omega\, d\sqrt{\Theta}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}$$



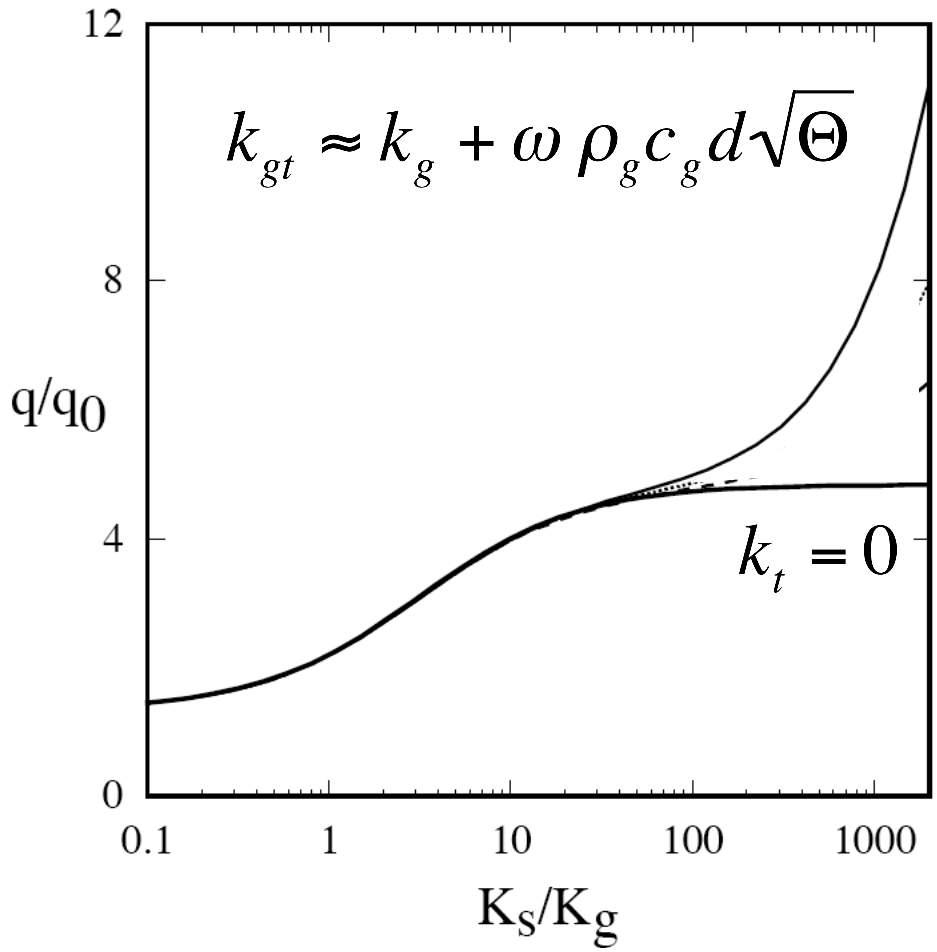
$$\omega \approx 0.04 \text{ at } \nu = 0.3$$

movie: Chris Pelkie

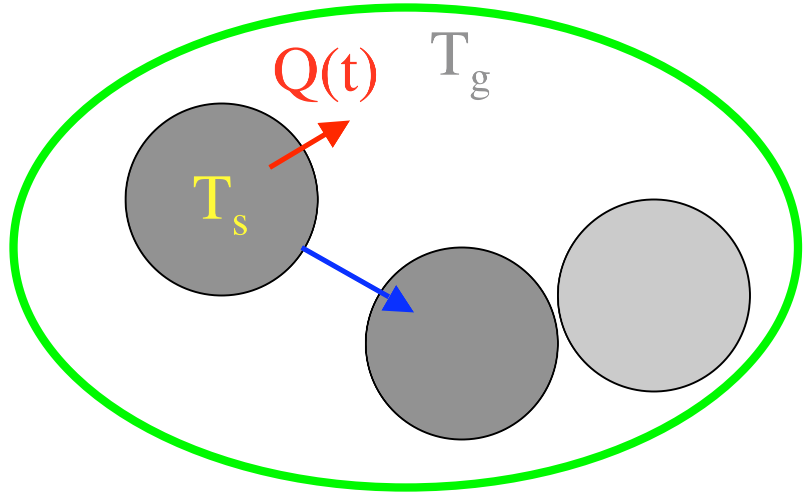
$$\boxed{k_{gt} = k_g + k_t}$$

$$\boxed{\frac{k_t}{\rho_g c_g} \approx \omega\, d\sqrt{\Theta}}$$

Turbulent enhancements in the exchange limit



Role of $\mathrm{Bi} = \dfrac{h(d/2)}{k_s}$



Source term $\quad \tilde{H} = n I_a \dfrac{dQ}{dt}$

$$\frac{1}{Q_0}\frac{dQ}{d\mathrm{Fo}} = \sum_{i=1}^{\infty} \frac{6\ \exp(-\xi_i^2 \mathrm{Fo})}{1 - \frac{1}{\mathrm{Bi}} + (\frac{\xi_i}{\mathrm{Bi}})^2}$$

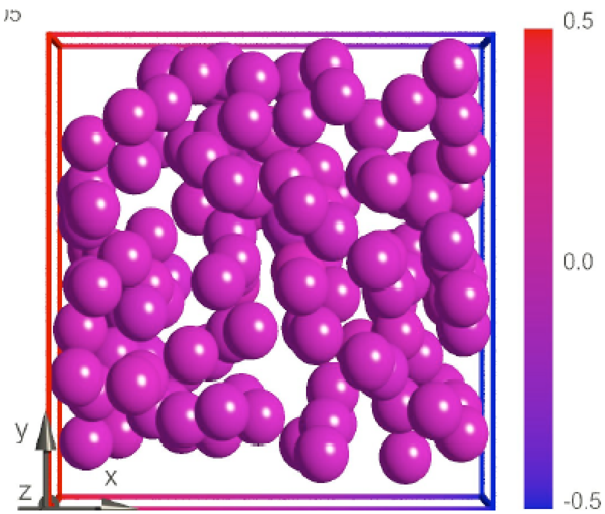
$$\xi_1^2 = 3\mathrm{Bi}\ [1 - \frac{\mathrm{Bi}}{10} + o(\mathrm{Bi})^2]^2$$

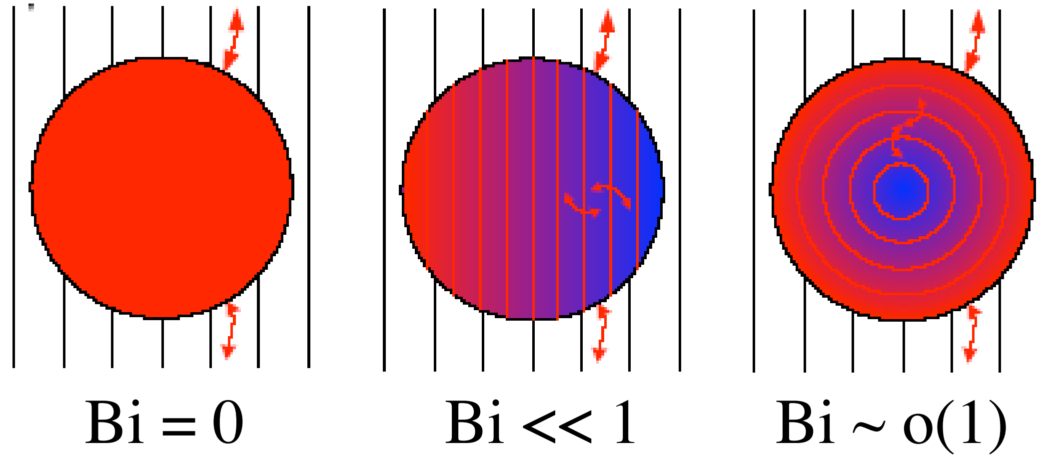
$$\mathrm{Fo} = \frac{4 k_s\ \mathrm{t}}{\rho_s c_s d^2}$$

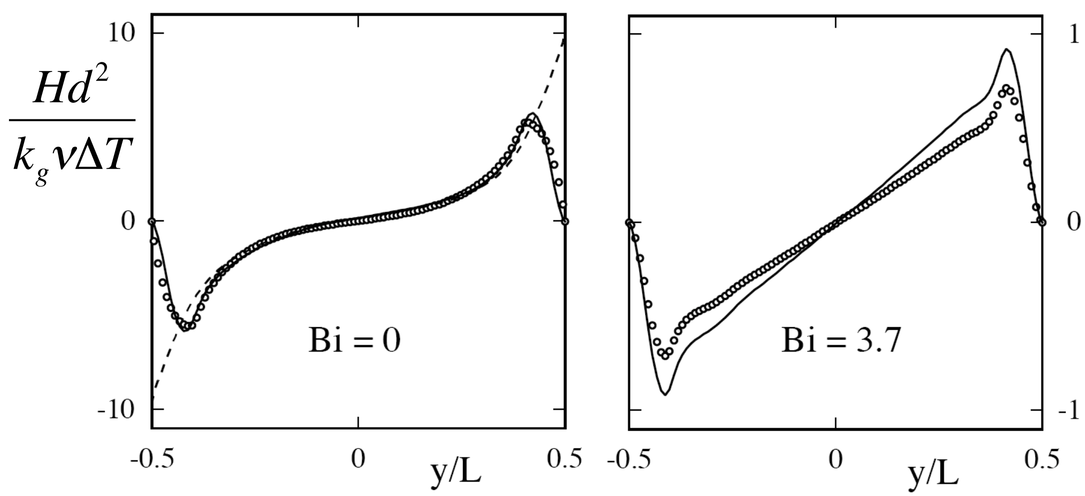
$$t^{-1} \sim f_c = \frac{24}{\sqrt{\pi}} \nu g_{12} \frac{\sqrt{\Theta}}{d}$$ collision frequency

$$\tilde{H} = 12\nu \frac{k_g}{d^2} \mathrm{Nu}(T_g - T_s) I_a \exp(-\xi_1^2 \mathrm{Fo})\left[1 - \frac{\mathrm{Bi}}{5} + o(\mathrm{Bi}^2)\right]$$

Thermal simulations

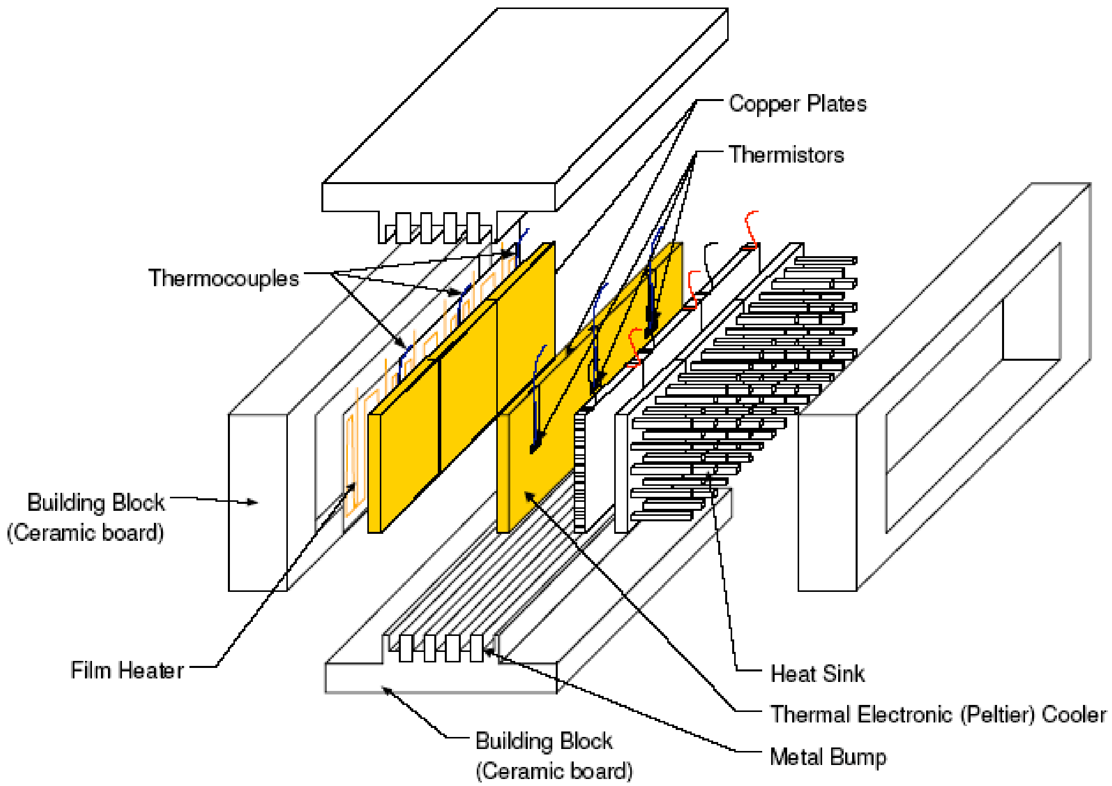


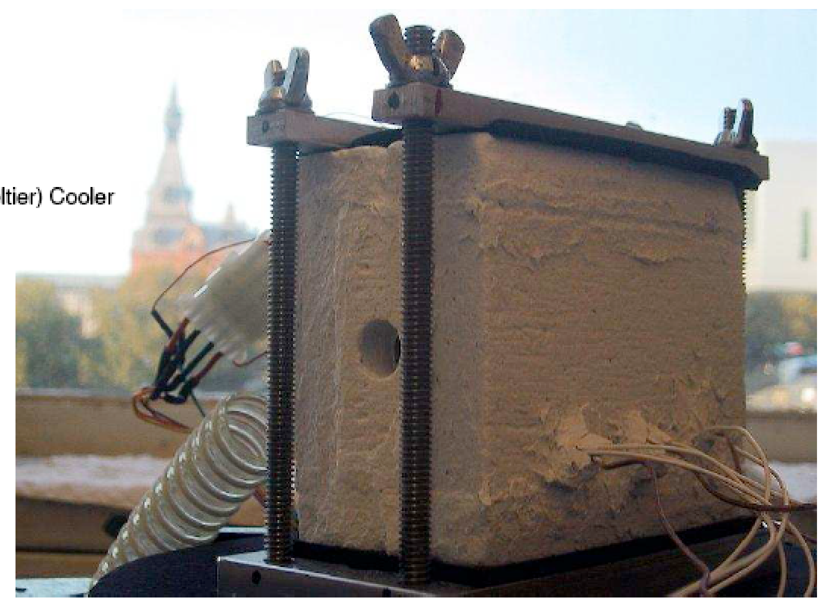




Experiments

Experiments in the exchange limit

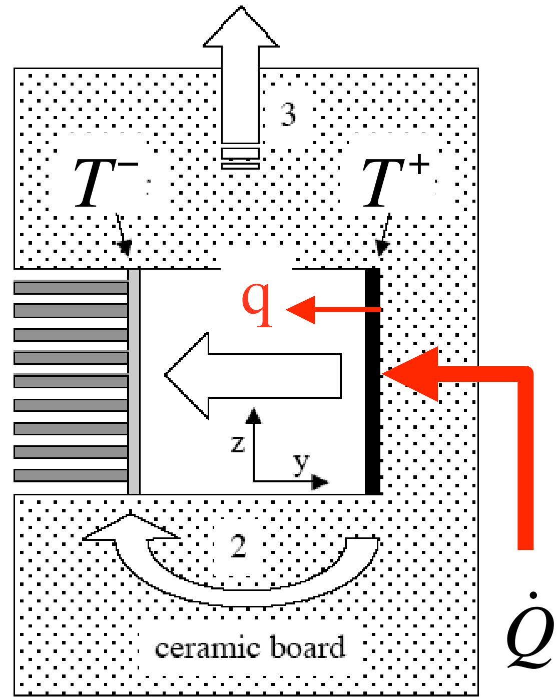


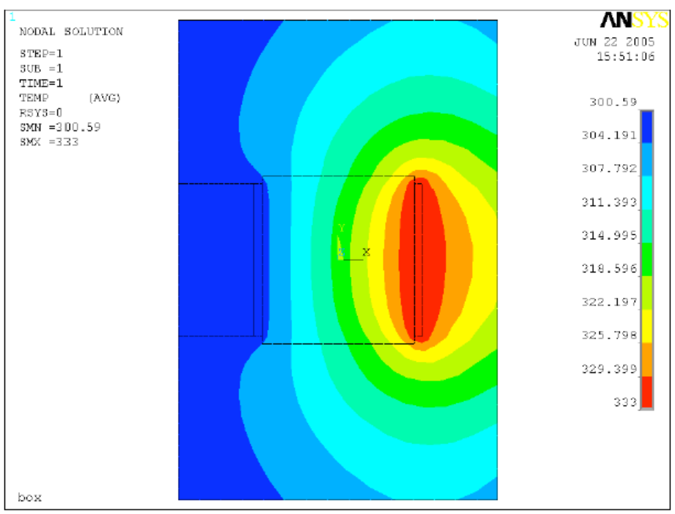


$$\boxed{\text{Measure } k_{eff} = \frac{qL}{T^+ - T^-}}$$

$$\boxed{\dot{Q} = Aq + \frac{T^+ - T^-}{R_b} + h_b S_b \left(\frac{T^+ + T^-}{2} - T_\infty \right)}$$

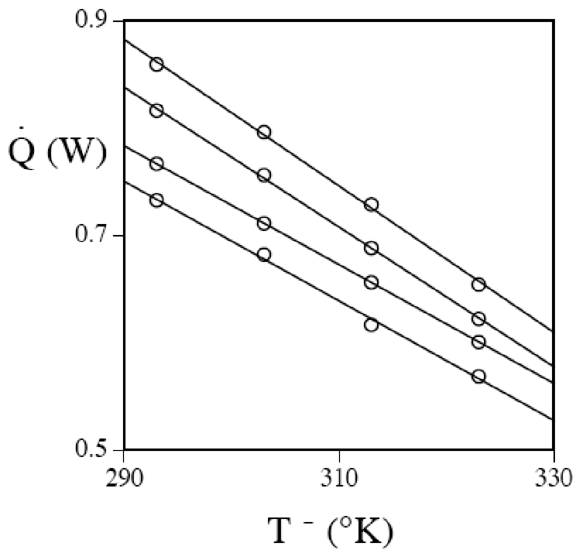
$$\dot{Q} = f(T^+) - \Omega T^-$$

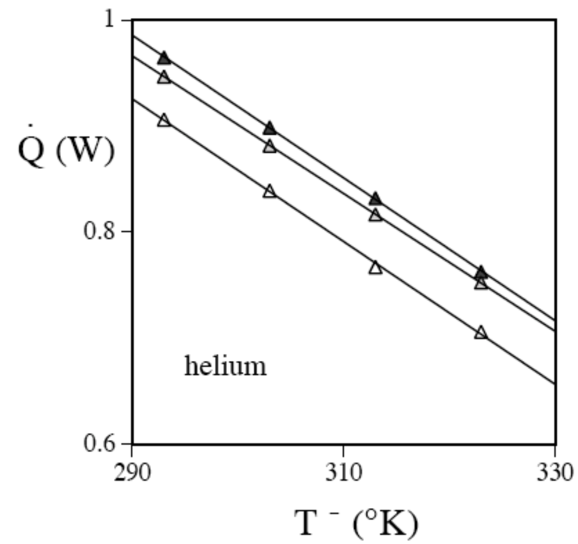




$$\boxed{Measure\ \ k_{eff} = \frac{qL}{T^+ - T^-}}$$

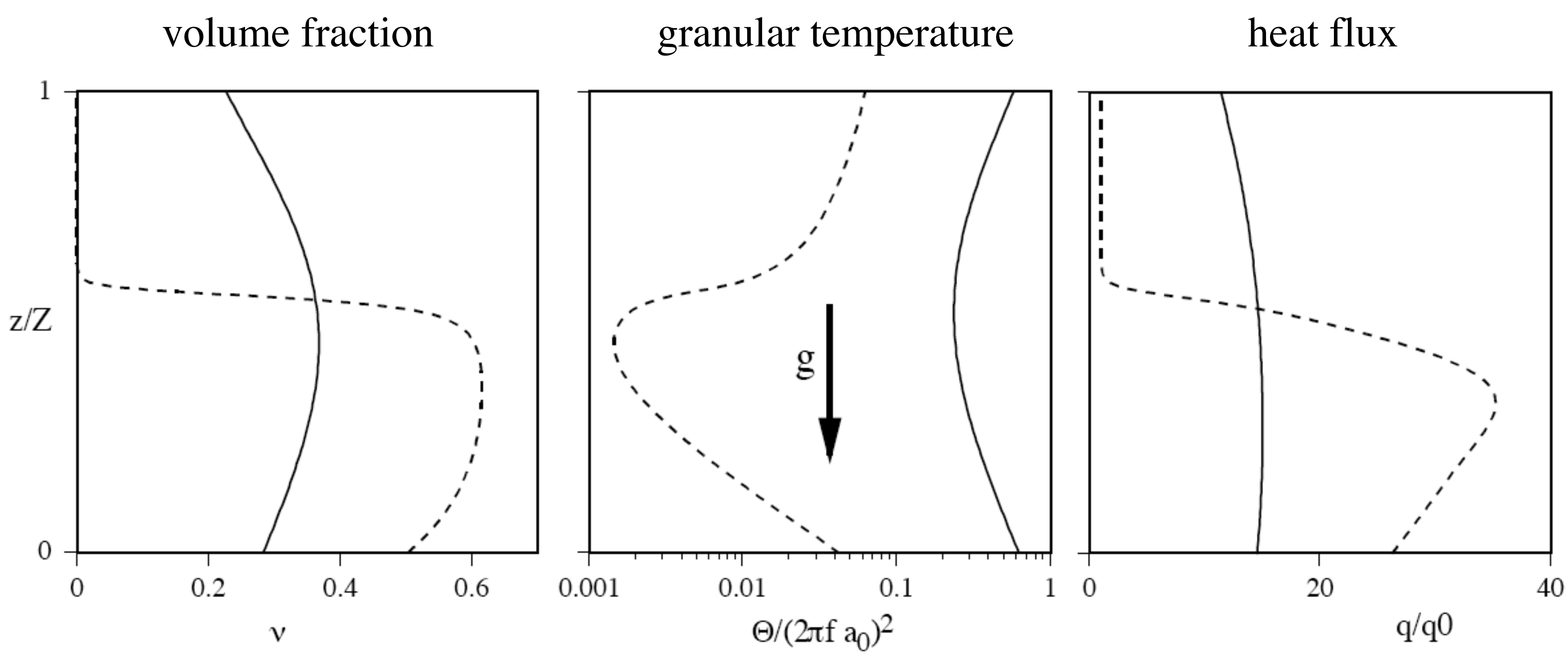
$$\Omega = k_{\text{eff}}\frac{A}{L} - \frac{h_b S_b}{2} + \frac{1}{R_b} \qquad \Omega_{\text{He}} = k_{\text{He}}\frac{A}{L} - \frac{h_b S_b}{2} + \frac{1}{R_b}$$





$$\frac{q}{q_0} = \frac{k_{\text{eff}}}{k_g} = \frac{k_{\text{He}}}{k_{\text{air}}} + (\Omega - \Omega_{\text{He}})\frac{L}{k_{\text{air}}A}$$

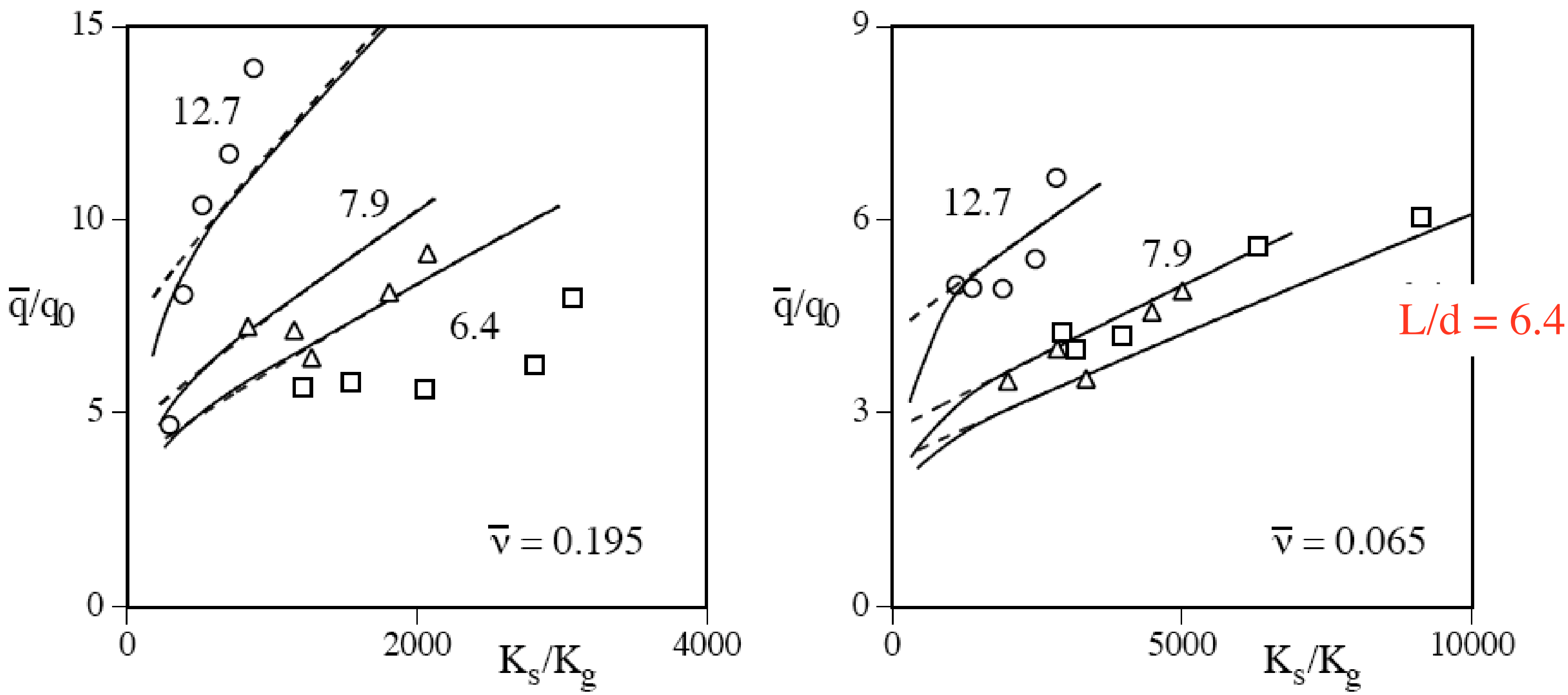
Model the vibrated box



$$\frac{K_s}{K_g} = \left(\frac{\rho_s c_s d\sqrt{gd}}{k_g}\right)\frac{\Theta/\sqrt{gd}}{9\sqrt{\pi}\,g_{12} f_M(\nu;\xi_s)}$$

$$\overline{q} = \int_0^Z q(z)\,dz$$

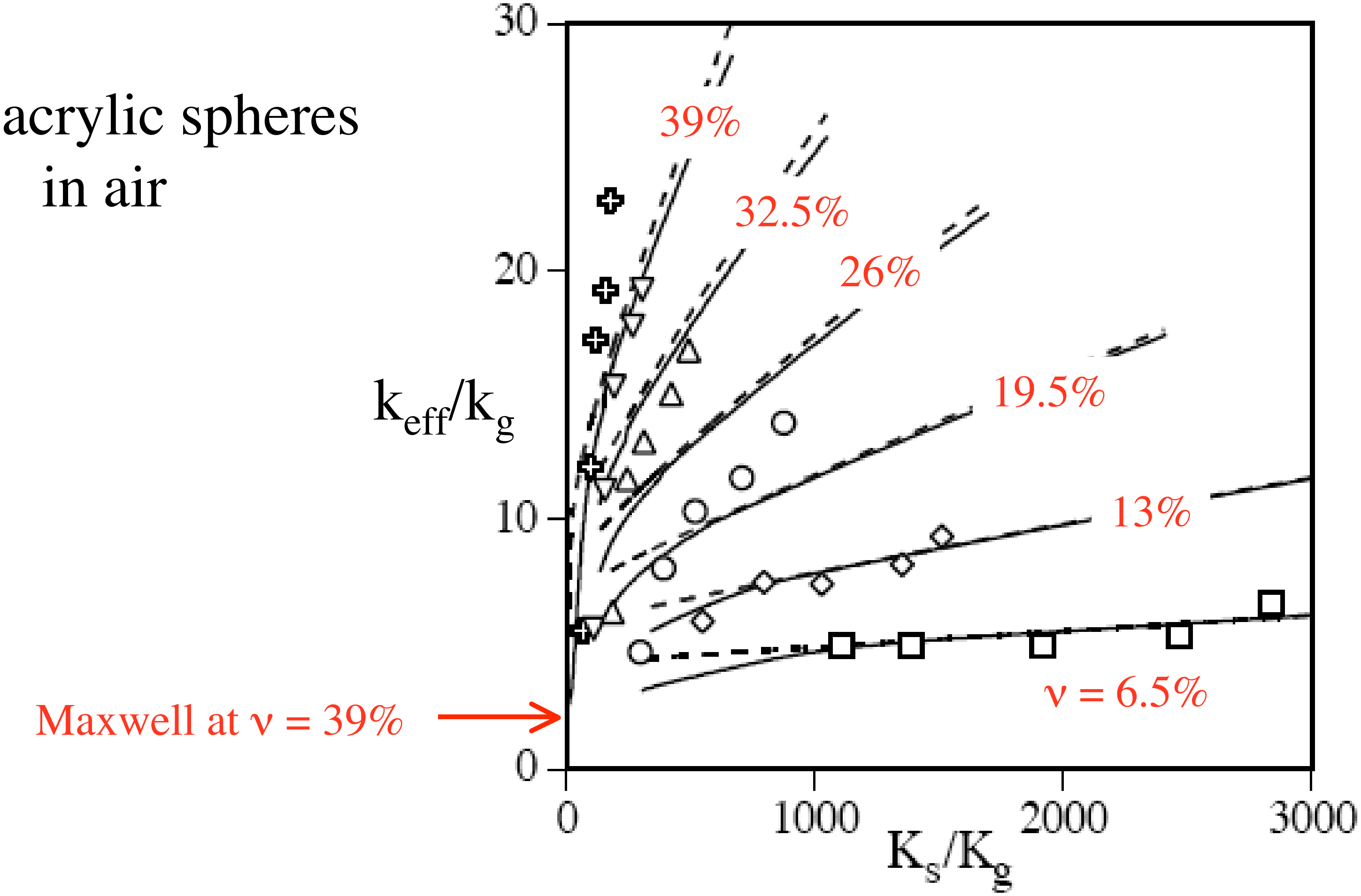
Role of size in the exchange limit



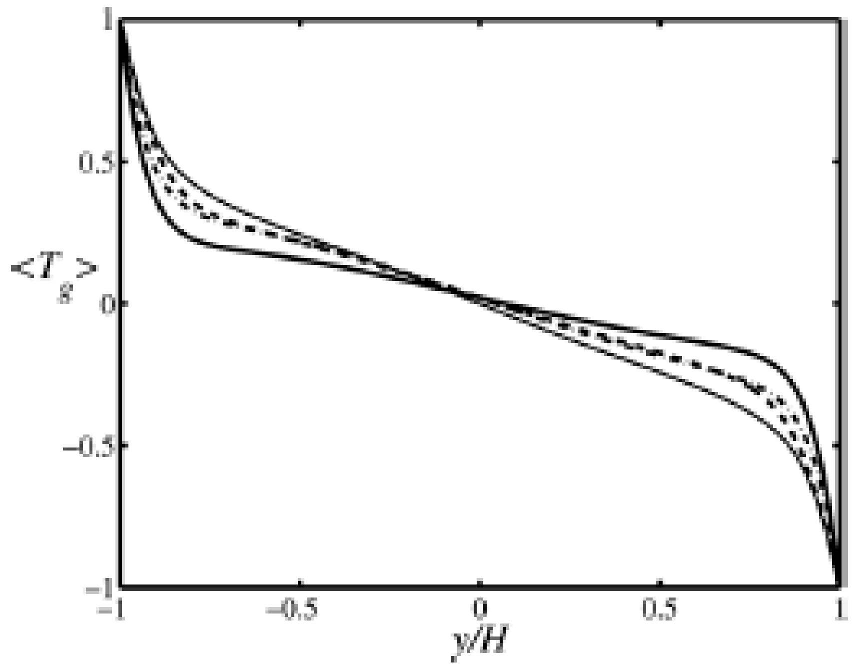
4mm, 3.2mm, and 2 mm acrylic spheres in air

Role of volume fraction in the exchange limit

2 mm acrylic spheres
in air



DNS of turbulent heat transfer between plates



Kuerten, et al "Turbulence modification and heat transfer enhancement
by inertial particles in turbulent channel flow" Phys. Fluids 23, 123301 (2011)

Conclusions

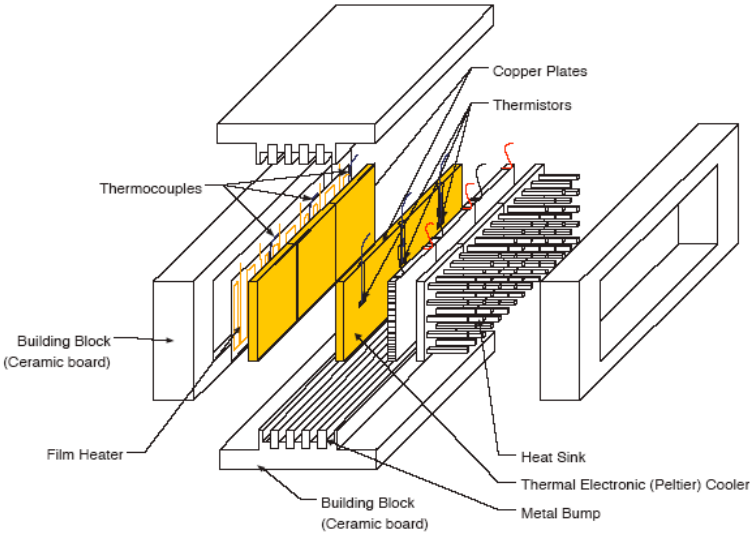
- Agitated suspensions have “diffusion” and “exchange” heat transfer limits.
- Granular ordering affect heat transfer.
- Granular agitation further enhances heat transfer in the exchange limit by augmenting turbulence.

Heat transfer enhancement in dense suspensions of agitated solids.
Part I: Theory. Part II: Experiments in the exchange limit

Xinglong Chen, Michel Louge

Int. J. of Heat & Mass Transfer 51 (2008) 5108–5118; 5119–5129.

Thank you



http://grainflowresearch.mae.cornell.edu/index.html