A Comparison of QUICKER with Conventional Meta-Modeling Methods for Input Uncertainty Propagation

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Motivation and Objectives

- **Motivation for QUICKER**
  - Most engineering systems have some degree of uncertainty in their input parameters
  - Quantifying this uncertainty is computationally expensive

- **Objective of QUICKER**
  - Develop a non-intrusive method for propagating input uncertainty that is less computationally expensive than conventional methods
The inherent uncertainty of input parameters for a nonlinear computational model results in variability in the measured outputs.

Uncertain input distributions

Nonlinear computational model, \( f(x) \)

Output distributions

Due to the complexity of most nonlinear computational models, traditional sampling methods (e.g. factorial, Latin Hypercube, etc.) are too expensive.
Meta-model methods allow for rapid sampling from the response surface instead of the full computational simulation.

Only three sample points are necessary to define the location ($\mu$), shape ($\sigma$), and shift ($\theta$) of the output distribution.

The output from QUICKER is always 1-D, and thus scales easily.

Meta-models scale poorly with increased dimensionality.
1. Select input data points

2. Determine location of output distribution
   \[ f(x_2) = \mu \]
   \( f(x) \) represents the nonlinear computational model

3. Determine shape of output distribution
   \[
   \frac{(\mu - f(x_1)) + (\mu - f(x_3))}{2 \times N} = \sigma
   \]

4. Determine shift of output distribution
   \[ f(0) = \theta \]

5a. \( \sigma < 0 \)
   Lognormal\([\mu - \theta, |\sigma|]\)

5b. \( \sigma > 0 \)
   Lognormal\([f_{\text{max}} - \mu, |\sigma|]\)

6. Determine secondary distributions
   \[
   \frac{\mu_L - f(x_1) + \mu_L - f(x_2)}{2 \times N} = \sigma_L
   \]
   \[
   \frac{\mu_H - f(x_2) + \mu_L - f(x_3)}{2 \times N} = \sigma_H
   \]

7. Weighted average of output distributions
Test Scenario:

\[ f(x_1, x_2, x_3 \ldots x_n) = x_1 H(x_1 - 4) + x_2 \]
\[ \mu_x = 3.5; \sigma_x = 0.75 \]

The output from QUICKER remains constant, but the meta-model’s output is affected poorly by increased dimensionality.
Computational Scenarios

Circulating fluidized bed

3 uncertain input parameters

- Gas outlet
- Mixed outlet
- Periodic outlet
- Periodic inlet
- Radial symmetry

Turbulent fluidized bed

11 uncertain input parameters

- Gas inlet
- Mixed outlet
- Particle inlet
- Periodic outlet
- Periodic inlet

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Circulating Fluidized Bed Results

3 Uncertain Input Parameters

Gaussian Process Meta-model, Radial Basis Function Meta-model, and QUICKER have comparable RMS errors
11 Uncertain Input Parameters

Only QUICKER has a small RMS error
Future Work

- QUICKER for correlated input distributions
  - For when multiple uncertain inputs have some mutual dependence
    (e.g. proximate and ultimate analysis of coal)

- QUICKER for epistemic uncertainty
  - For when the distribution of a given input distribution is not fully known or defined
Meta-models scale poorly because they require the dimensionality of the response surface to match the number of inputs.

QUICKER scales well because the output distribution is always 1-D.

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Multiple input parameters

1. Select input data points

2. Determine location of output distribution
   \[ f(x_2, y_2, z_2) = \mu \]

3. Determine shape of output distribution with Orthogonal Arrays
   \[ \mu - f \left( \begin{array}{c} x_1, y_1, z_1 \\ x_1, y_3, z_3 \\ x_3, y_1, z_3 \\ x_3, y_3, z_1 \end{array} \right) \frac{4 \times N}{= \sigma} \]

   Cholesky decomposition can be used to correlate inputs

4. Determine shift of output distribution
   \[ f(0,0,0) = \theta \]