A Comparison of QUICKER with Conventional Meta-Modeling Methods for Input Uncertainty Propagation

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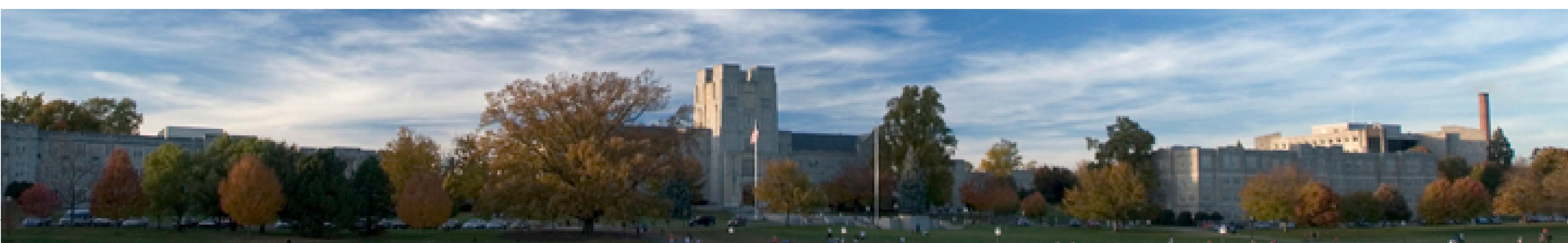


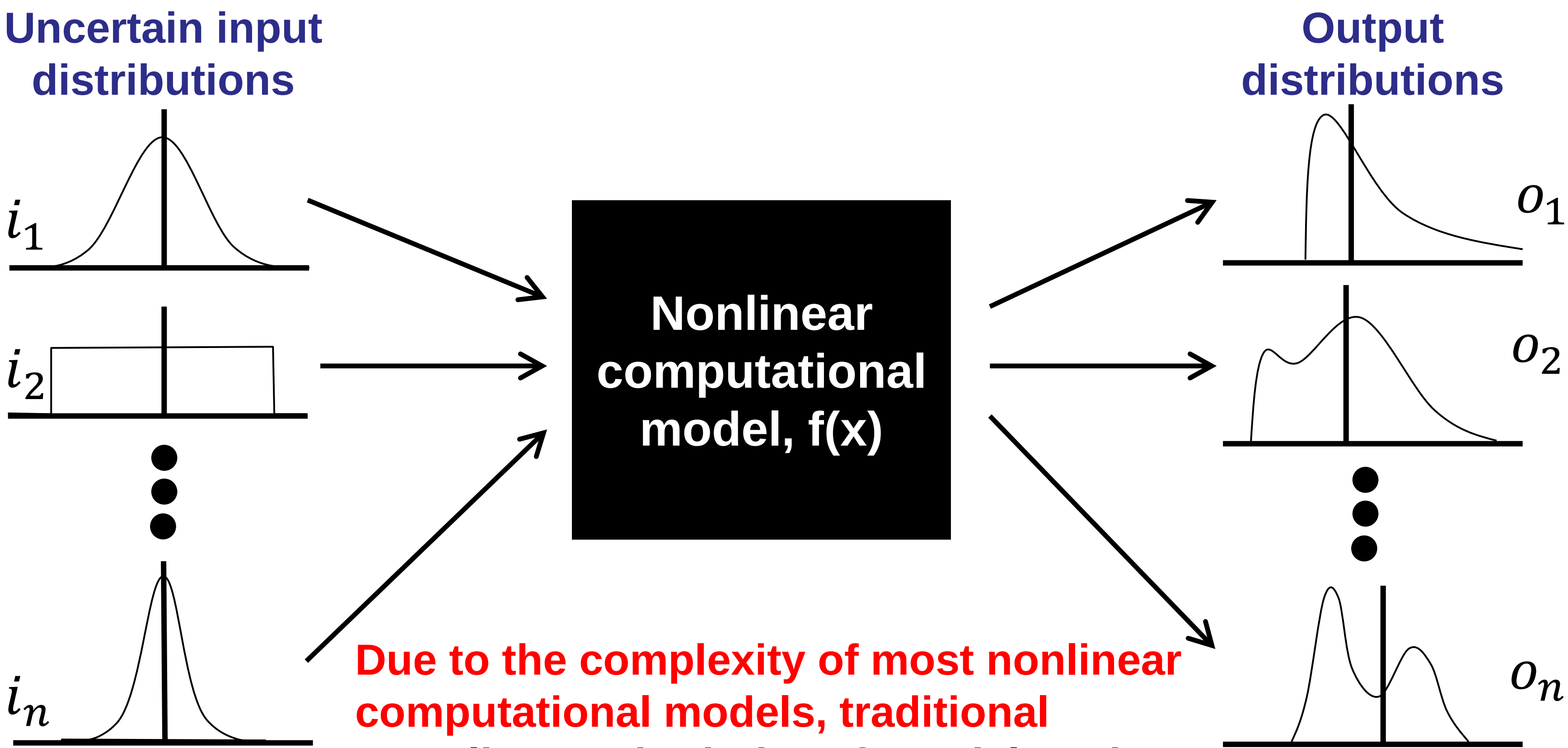
Motivation and Objectives

- **Motivation for QUICKER**
  - **Most engineering systems have some degree of uncertainty in their input parameters**
  - **Quantifying this uncertainty is computationally expensive**

- **Objective of QUICKER**
  - **Develop a non-intrusive method for propagating input uncertainty that is less computationally expensive than conventional methods**

Input Uncertainty Propagation

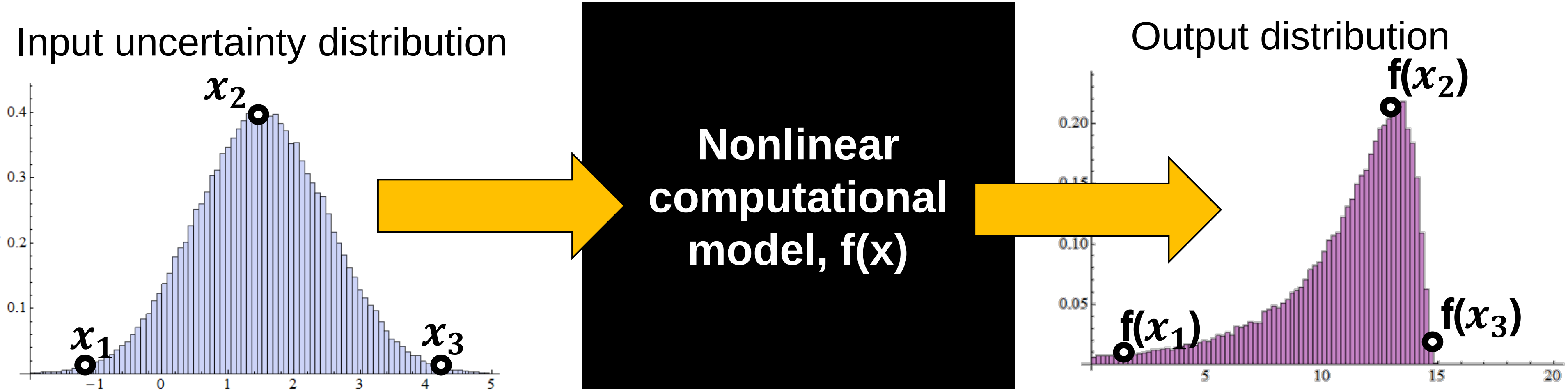
The inherent uncertainty of input parameters for a nonlinear computational model results in variability in the measured outputs.



Due to the complexity of most nonlinear computational models, traditional sampling methods (e.g. factorial, Latin Hypercube, etc.) are too expensive

QUICKER Method Motivation

Meta-model methods allow for rapid sampling from the response surface instead of the full computational simulation



Meta-models ***scale*** poorly with increased dimensionality

Only three sample points are necessary to define the location ($\boldsymbol{\mu}$), shape ($\boldsymbol{\sigma}$), and shift ($\boldsymbol{\theta}$) of the output distribution

The output from QUICKER is always 1-D, and thus scales easily

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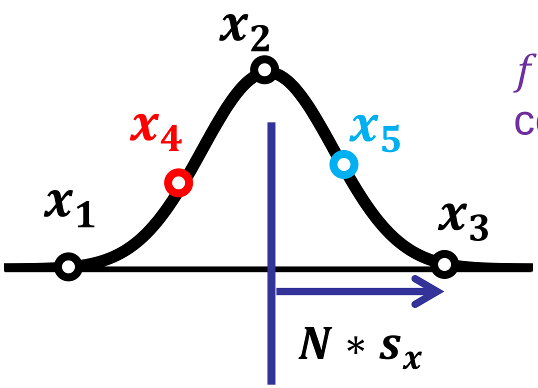
QUICKER Methodology

1. Select input data points



2. Determine ***location*** of output distribution

$$f(x_2) = \boldsymbol{\mu}$$

$f(x)$ represents the nonlinear computational model

3. Determine ***shape*** of output distribution

$$\frac{(\mu - f(x_1)) + (\mu - f(x_3))}{2 * N} = \boldsymbol{\sigma}$$

4. Determine ***shift*** of output distribution

$$f(0) = \boldsymbol{\theta}$$

6. Determine secondary distributions

$$f(x_4) = \boldsymbol{\mu_L}$$

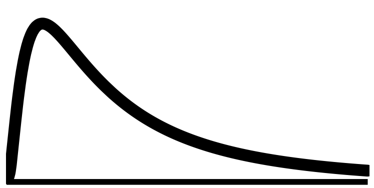
$$\frac{\mu_L - f(x_1) + \mu_L - f(x_2)}{2 * N} = \boldsymbol{\sigma_L}$$

$$f(x_5) = \boldsymbol{\mu_H}$$

$$\frac{\mu_H - f(x_2) + \mu_L - f(x_3)}{2 * N} = \boldsymbol{\sigma_H}$$

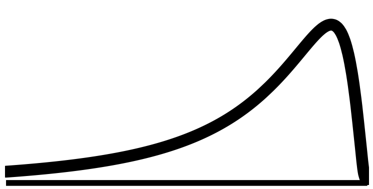
5a. $\sigma < 0$

Lognormal$[\mu - \theta, |\sigma|]$

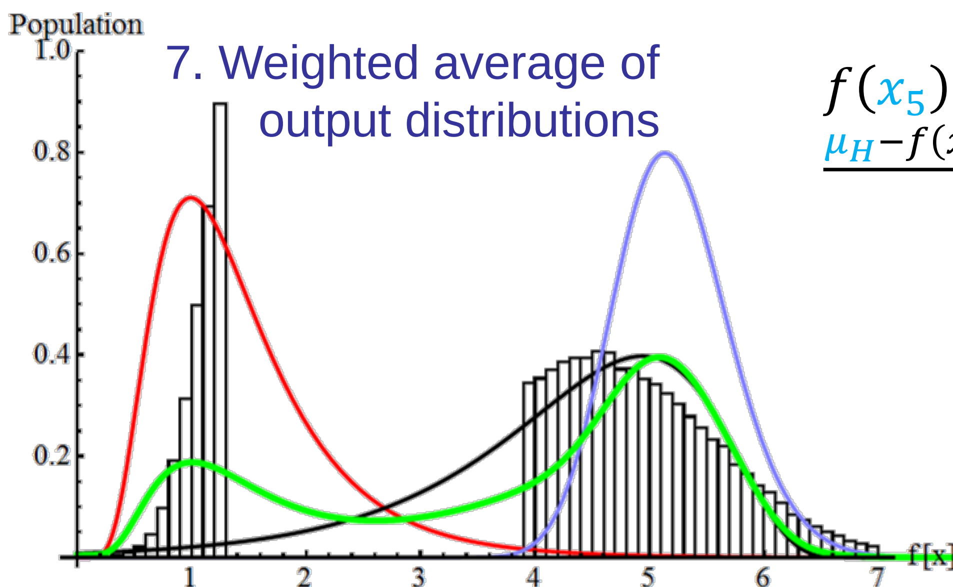


5b. $\sigma > 0$

Lognormal$[f_{max} - \mu, |\sigma|]$



7. Weighted average of output distributions

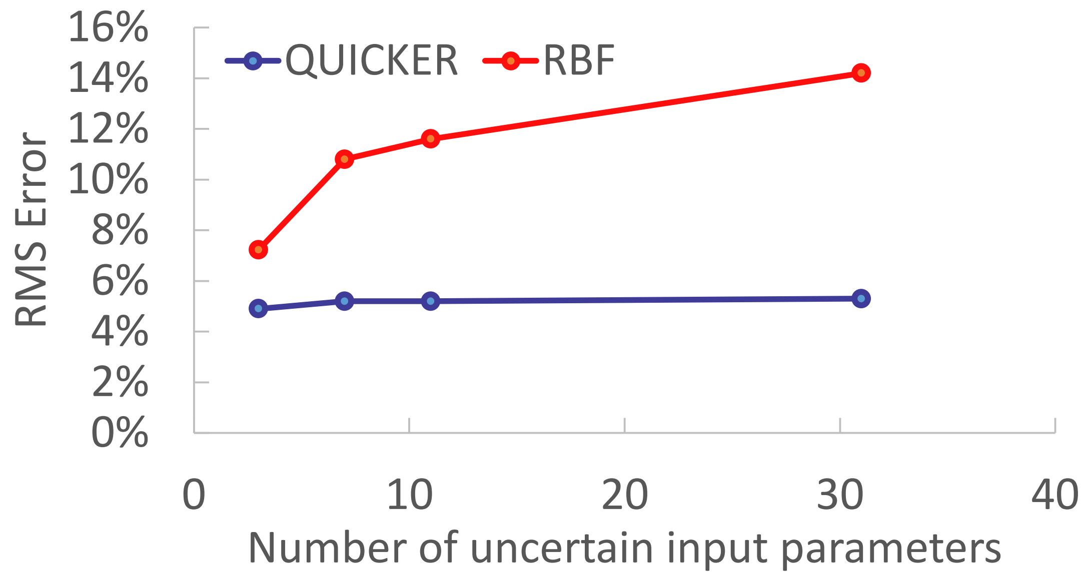


Meta-models vs. QUICKER

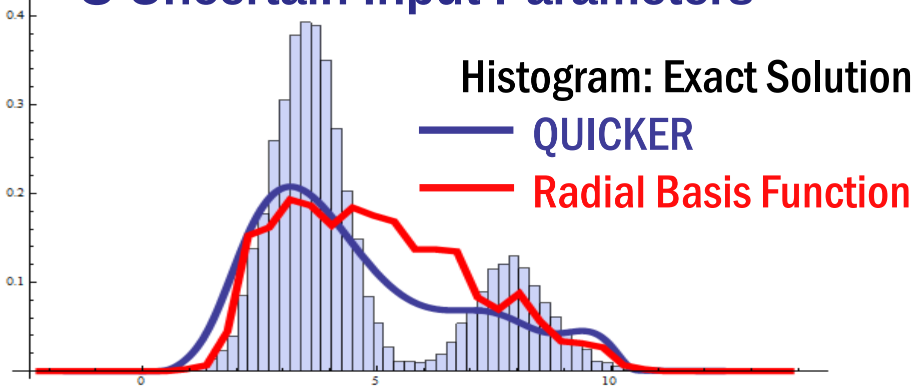
Test Scenario:

$$f(x_1, x_2, x_3 \dots x_n) = x_1 \mathrm{H}(x_1 - 4) + x_2$$

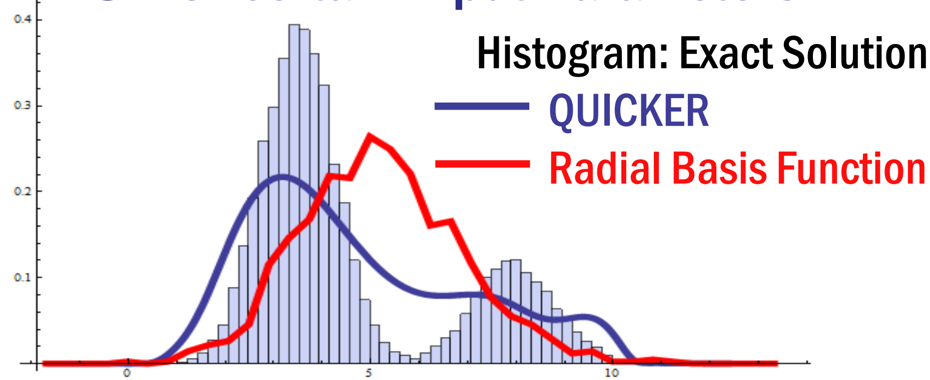
$$\mu_x = 3.5; \sigma_x = 0.75$$



3 Uncertain Input Parameters



31 Uncertain Input Parameters

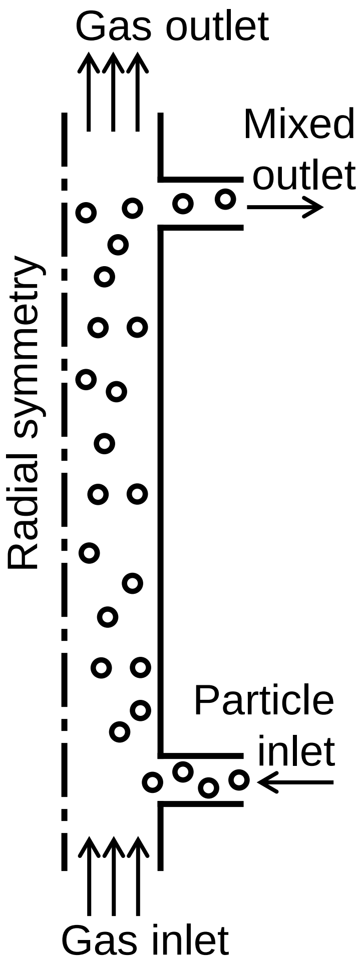


The output from QUICKER remains constant, but the meta-model's output is affected poorly by increased dimensionality

Computational Scenarios

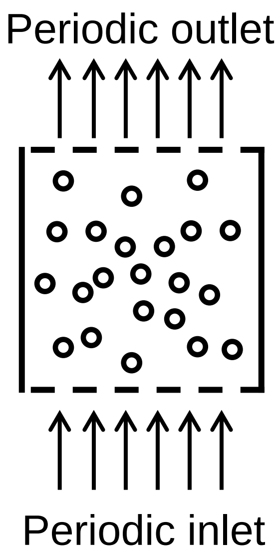
Circulating fluidized bed

3 uncertain input parameters



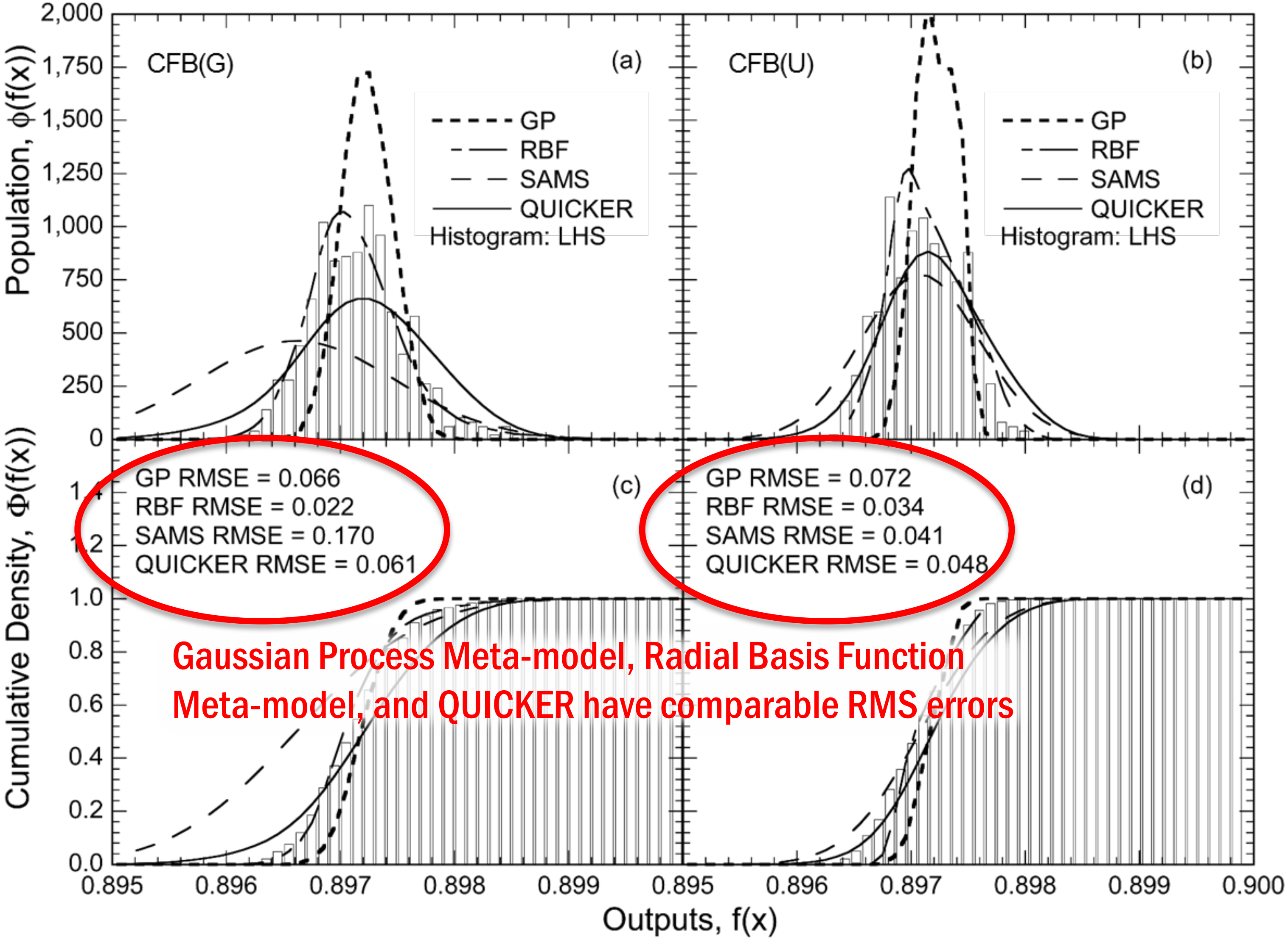
Turbulent fluidized bed

11 uncertain input parameters



Circulating Fluidized Bed Results

3 Uncertain Input Parameters



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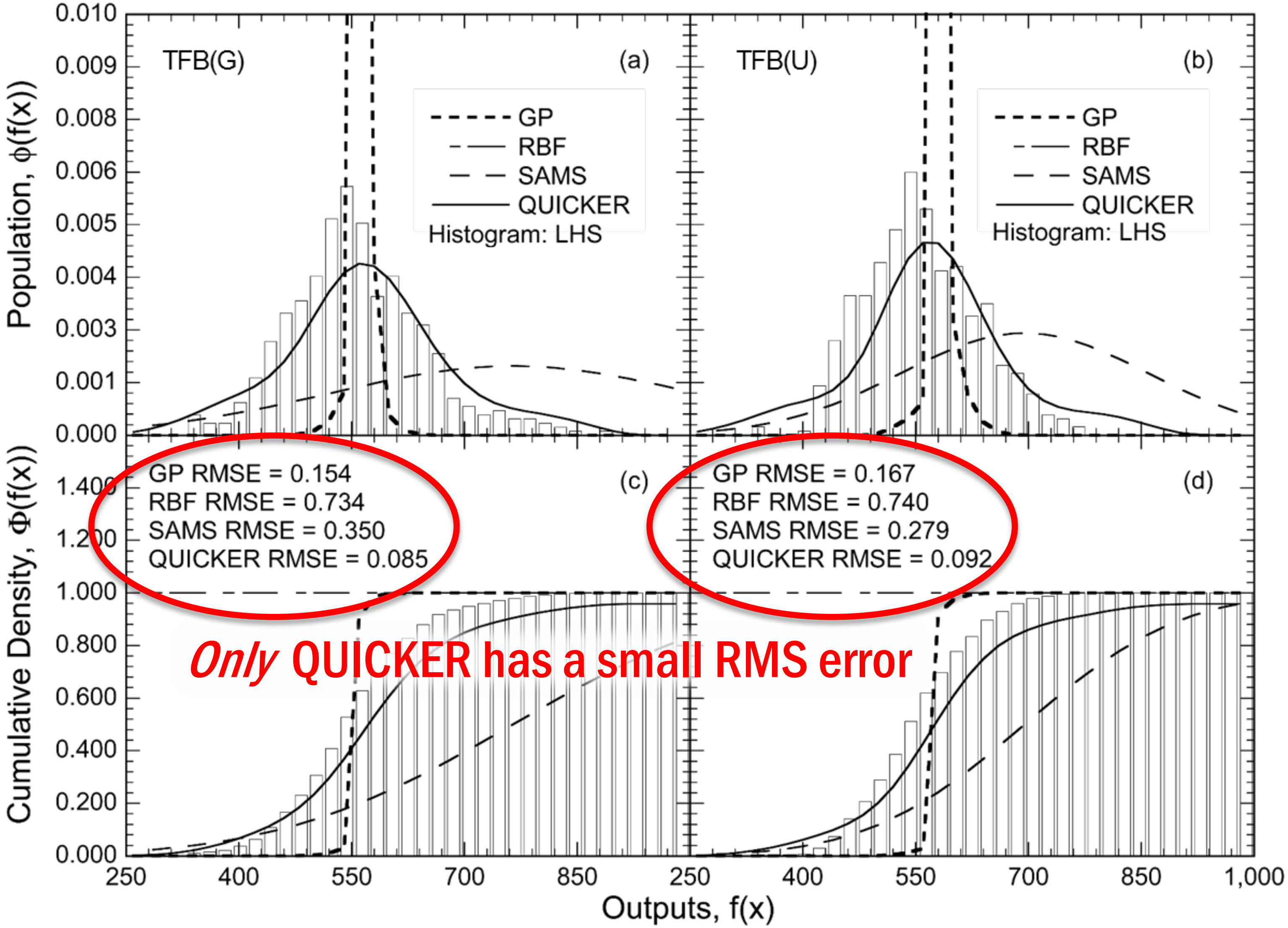
Turbulent Fluidized Bed Results

11 Uncertain Input Parameters



Only **QUICKER has a small RMS error**

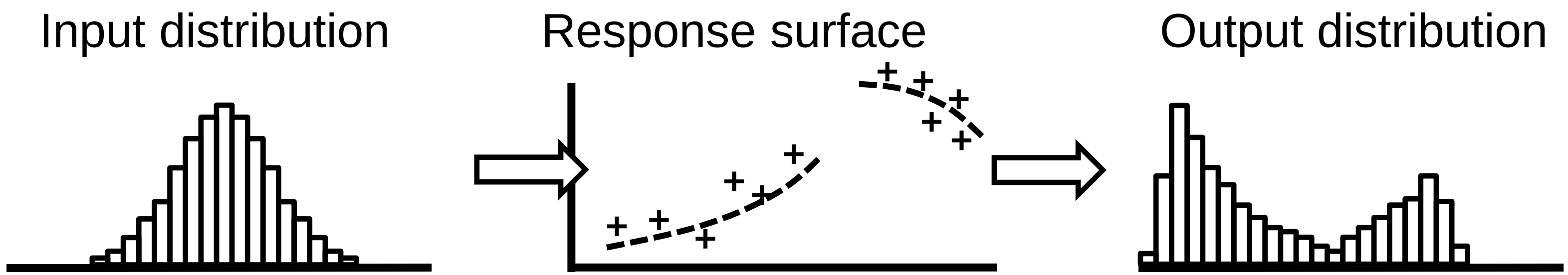
Future Work

- QUICKER for correlated input distributions
  - For when multiple uncertain inputs have some mutual dependence (e.g. proximate and ultimate analysis of coal)

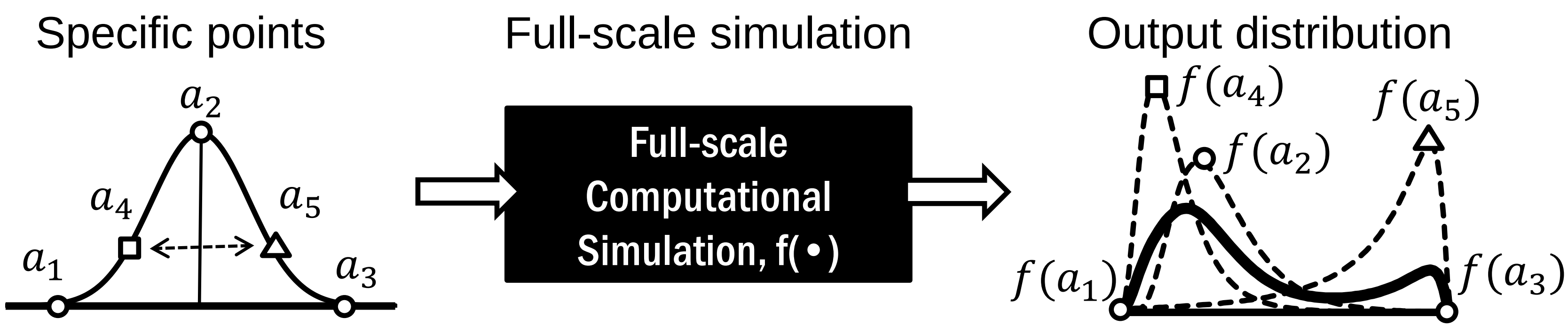
- QUICKER for epistemic uncertainty
  - For when the distribution of a given input distribution is not fully known or defined

Overview and Acknowledgements

Meta-models scale poorly because they require the dimensionality of the response surface to match the number of inputs.



QUICKER scales well because the output distribution is always 1-D.



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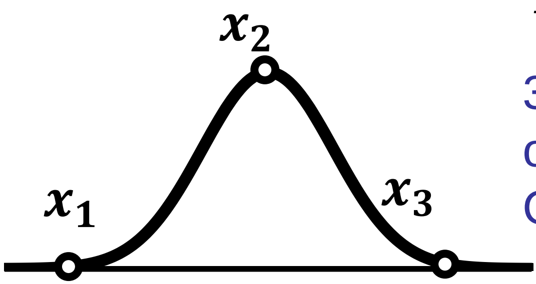
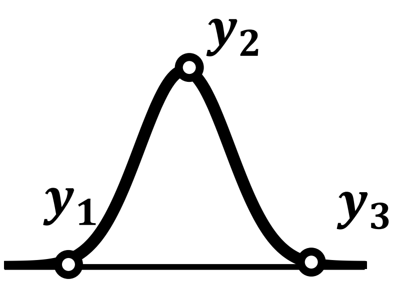
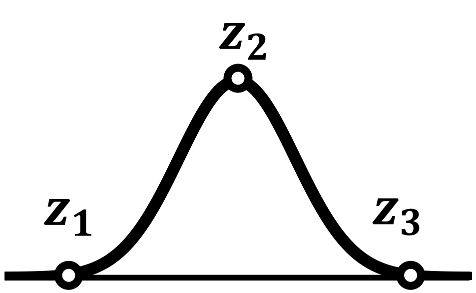
QUICKER Methodology Extended

Multiple input parameters

1. Select input data points







2. Determine ***location*** of output distribution

$$f(x_2, y_2, z_2) = \boldsymbol{\mu}$$

3. Determine ***shape*** of output distribution with Orthogonal Arrays

$$\frac{\mu - f\begin{pmatrix} x_1, y_1, z_1 \\ x_1, y_3, z_3 \\ x_3, y_1, z_3 \\ x_3, y_3, z_1 \end{pmatrix}}{4 * N} = \boldsymbol{\sigma}$$

Cholesky decomposition can be used to correlate inputs

4. Determine ***shift*** of output distribution

$$f(0,0,0) = \boldsymbol{\theta}$$

Non-Gaussian input parameters