A Turbulence Model for Gas-Particle Flows with Favre-Averaging : II. Granular Temperature and Turbulent Kinetic Energy

Dr. Thomas J. O'Brien
NETL-emeritus

1847 Joliet Way
Boulder, CO 80305
TOBBoCo@gmail.com
304-816-6332

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Outline

- **A little rambling**
  - Low-order models
  - What is multiphase turbulence?
  - Previous work
- **Favre-like average**
  - Definitions
  - Identities
- **Averaged equations: continuity, momentum and granular temperature**
  - Derivation
  - Closures
- **Turbulent kinetic energy equations**
  - Derivation
  - Closures
- **Future formal work**
  - Boundary conditions
  - Thermal temperatures
  - Chemistry

Gas-Particle Model Hierarchy

Full Direct Numerical Simulation (DNS)

Hybrid DNS-Discrete Element Model (DNS-DEM)

Two-Fluid Model (*e.g.*, MFIX):

Microscopic equations are averaged (spatial, ensemble) to obtain a granular fluid, characterized by a granular temperature.

TRANSIENT

Thoughts on steady-state calculations

- The vast majority of single-phase CFD calculations are time-steady.
- Transient calculations are very computationally costly … and lengthy even if they are affordably.
- Practically impossible to do scoping studies based on transient calculations.
- Coupling of CFD to process simulations requires a "lower order model," reduced from the full transient model … that is what will be provided.
- **Multiphase flows are inherently transient … the effects of which must be captured faithfully by a turbulence model.**

General comments

- Most work published is based on modifications of gas phase turbulence theory… this probably won't work for dense flows.

- Current approach … turbulence is a property of a fluid, so a granular continuum theory has to be the starting point.

- Utilize the concept of Favre-averaging, introduced for compressible flows, to reduce the closure "burden."

- At this time, this work is purely formal

Multiphase turbulence

Dilute flow – carrier-phase turbulence modulation

Eulerian-Lagrangian approaches

Single average approach
Average microscopic equations to obtain "two-fluid" equations describing interpenetrating continuum, characterized by:
$\varepsilon_g, \varepsilon_s, \mathbf{U}_g, \mathbf{U}_s, P_g, \Theta_s$, all functions of space and time.

Examine "fluctuations" of the microscopic variables.

Double average approach
Average again to obtain mean behavior $\overline{\varepsilon}_g, \overline{\varepsilon}_s, \overline{\mathbf{U}}_g, \overline{\mathbf{U}}_s, \overline{P}_g, \overline{\Theta}_s$

+ "fluctuation," *e.g.*, $\varepsilon_g = \overline{\varepsilon}_g + \varepsilon'_g, \mathbf{U}_g = \overline{\mathbf{U}}_g + \mathbf{U}'_g$, etc.

Single Average Approach

indicator function of fluid/particulate phase $I_{g/s}$

fluid/particulate phase fraction $\varepsilon_{g/s} = \langle I_{g/s} \rangle$

conditional average of based on the gas/particulate phase $\overline{\Psi}_{g/s} \equiv \langle I_{g/s}\Psi \rangle / \langle I_{g/s} \rangle$

turbulent fluctuations $\Psi \equiv \overline{\Psi}_{g/s} + \Psi'$

Previous work: single average

Ahmadi, G., and D. Ma, “A thermodynamic formulation for dispersed multiphase turbulent flows – I Basic theory,” *Int. J. Multiphase Flow,* **16**, 323–340, 1990.

Lhuillier, D., “Macroscopic modelling of multi-phase mixtures,” in U. Schaflinger, ed., Flow of Particles in Suspensions, Springer-Verlag, 39–91, 1996.

Liljegren, L.M., “Fluctuating kinetic energy budget during homogeneous flow of a fluid solid mixture,” *Phys. Fluids* **8**, 84–90, 1996.

Liljegren, L.M., “The dissipation rate of fluctuating kinetic energy in a fluid-solid flow,” *Phys. Fluids* **6**, 3795–3797, 1994.

Kashiwa, B.A., and W.B. VanderHeyden, “Toward a general theory for multiphase turbulence Part I: Development and gauging of the model equations,” LA-13773-MS, Dec. 2000.

MFIX Governing Equations
(Continuity equations)

gas phase, *g*:

$$\frac{\partial}{\partial t}\left(\varepsilon_g \rho_g\right) + \frac{\partial}{\partial x_i}\left(\varepsilon_g \rho_g U_{gi}\right) = 0$$

solids phase, *s*:

$$\frac{\partial}{\partial t}\left(\varepsilon_s \rho_s\right) + \frac{\partial}{\partial x_i}\left(\varepsilon_s \rho_s U_{si}\right) = 0$$

MFIX Governing Equations
(Momentum equations)

gas phase, *g*:

$$\frac{\partial}{\partial t}\left(\varepsilon_g \rho_g U_{gi}\right)+\frac{\partial}{\partial x_j}\left(\varepsilon_g \rho_g U_{gj} U_{gi}\right)$$

$$= -\varepsilon_g \frac{\partial P_g}{\partial x_i} + \frac{\partial \tau_{gij}}{\partial x_j} - I_{gsi} + \varepsilon_g \rho_g g_i$$

solids phase, *s*:

$$\frac{\partial}{\partial t}\left(\varepsilon_s \rho_s U_{si}\right)+\frac{\partial}{\partial x_j}\left(\varepsilon_s \rho_s U_{sj} U_{si}\right)$$

$$= -\varepsilon_s \frac{\partial P_g}{\partial x_i} + \frac{\partial \tau_{sij}}{\partial x_j} + I_{gsi} + \varepsilon_S \rho_S g_i$$

Kinetic Theory of Granular Flow
granular stress

$$\tau_{sij} = \left(-P_s + \frac{(1+e)}{2} \mu_b \frac{\partial U_{si}}{\partial x_i} \right) \delta_{ij} + 2\mu_s S_{sij}$$

$$S_{sij} = \frac{1}{2} \left(\frac{\partial U_{si}}{\partial x_j} + \frac{\partial U_{sj}}{\partial x_i} \right) - \frac{1}{3} \frac{\partial U_{si}}{\partial x_i} \delta_{ij}$$

$$P_s = \varepsilon_s \rho_s \Theta_s \left[1 + 2(1+e) \varepsilon_s g_0 \right]$$

$$\mu_s = C_{\mu_s} f_{\mu_s}(\varepsilon_s) \sqrt{\Theta_s}$$

$$\mu_b = C_{\mu_b} f_{\mu_b}(\varepsilon_s) \sqrt{\Theta_s}$$

Kinetic Theory of Granular Flow granular viscosities

$$\mu_s = \left(\frac{2+\alpha}{3}\right)\left(\frac{5}{96}\rho_s d_p \sqrt{\pi}\right)\left[\frac{1}{\eta(2-\eta)g_0}\left(1+\frac{8}{5}\eta\varepsilon_s g_0\right)\left(1+\frac{8}{5}\eta(3\eta-2)\varepsilon_s g_0\right)+\frac{768}{25\pi}\eta\,\varepsilon_s^2 g_0\right]\sqrt{\Theta_s}$$

$$= \underline{\underline{C_{\mu_s} f_{\mu_s}(\varepsilon_s)\sqrt{\Theta_s}}}$$

$$\mu_b = \frac{8}{3\sqrt{\pi}}\rho_s d_p \left(\varepsilon_s^2 g_0\right)\sqrt{\Theta_s}$$

$$= \underline{\underline{C_{\mu_b} f_{\mu_b}(\varepsilon_s)\sqrt{\Theta_s}}}$$

$\varepsilon_s < \varepsilon_{s,F} \cong 0.497$	fluid – like region
$\varepsilon_{s,F} < \varepsilon_s < \varepsilon_{s,G} \cong 0.56$	meta - stable region
$\varepsilon_{s,G} < \varepsilon_s < \varepsilon_{s,RCP} \cong 0.64$	glass region

$$g_0(\varepsilon_s) = \begin{cases} \text{Carnahan - Starling} & \varepsilon_s < \varepsilon_{s,F} \cong 0.497 \\ \text{Pade Approximant} & \varepsilon_{s,F} < \varepsilon_s < \varepsilon_{s,G} \cong 0.56 \end{cases}$$

$$Z \cong Z_{4,3} = \frac{1+1.024385\,\varepsilon_s + 1.104537\,\varepsilon_s^2 - 0.4611472\,\varepsilon_s^3 - 0.7430382\,\varepsilon_s^4}{1-2.975615\,\varepsilon_s + 3.007000\,\varepsilon_s^2 - 1.097758\,\varepsilon_s^3}$$

$$g_0(\varepsilon_s) = (Z-1)/4\varepsilon_s$$

Ma, D., and Ahmadi G... *J. Chem. Phys.*, **84**, 3449–3... 1986.

Bravo Yuste, López de Haro and Santos, *Phys. Rev. E* **53**, 4820–4826, 1996.

MFIX Governing Equations
(Granular temperature equation)

solids phase, *s*:

$$\frac{3}{2}\varepsilon_s\rho_s\left[\frac{\partial\Theta_s}{\partial t}+U_{sj}\frac{\partial\Theta_s}{\partial x_j}\right]$$

$$=\frac{\partial}{\partial x_i}\left(\kappa_s\frac{\partial\Theta_s}{\partial x_i}\right)+\tau_{sij}\frac{\partial U_{si}}{\partial x_j}+\prod_s-\varepsilon_s\rho_s J_s$$

$$\Pi_s=-3\beta\Theta_s+\frac{81\varepsilon_s\mu_g^2\left|u_g-u_s\right|^2}{g_0 d_p^3\rho_s\sqrt{\pi\Theta_s}}$$

exchange term

$$J_s=\frac{12}{\sqrt{\pi}}\left(1-e^2\right)\frac{\varepsilon_s g_0}{d_p}\Theta_s^{3/2}$$

collisional dissipation

$$\kappa_s=\left(\frac{75\sqrt{\pi}}{12}\right)\left(\frac{\rho_s d_p}{(1+e)(49-33e)}\right)$$

$$\times\left(\frac{1}{g_0}\right)\left[\left(1+\frac{6}{5}(1+e)\varepsilon_s g_0\right)\left(1+\frac{3}{5}(1+e)^2(2e-1)\varepsilon_s g_0\right)+\frac{8}{25\pi}(1+e)^2(49-33e)(\varepsilon_s g_0)^2\right]\sqrt{\Theta_s}$$

conductivity

Previous work: Double average

Elghobashi, S.E., and T.W. Abou-Arab, "A two equation turbulence model for two- phase flows," *Phys. Fluids* **26**, 931-938, 1983.

Bolio, E., and J. Sinclair, "Gas turbulence modulation in the pneumatic conveying of massive particles in vertical tubes," *Int. J. Multiphase Flow* **21**, 985-1001, 1995.

Dasgupta, S., R. Jackson and S. Sundaresan, "Turbulent gas-particle flow in vertical risers," *AIChE J.* **40**, 215-228, 1994.

Hrenya, C., and J. Sinclair, "Effects of particle-phase turbulence in gas-solid flows," *AIChE J.* **43**, 853-869, 1997.

Simonin, O., "Statistical and continuum modelling of turbulent reactive particulate flows. Part I: Theoretical derivation of dispersed phase Eulerian modelling from probability density function kinetic equation," Lecture series - von Karman Institute for Fluid Dynamics **6**, D1-D42, 2000.

Previous work: Double average including some Favre-like averaging

Besnard, D.C., and F.H. Harlow, “Turbulence in multiphase flow,” *Int. J. Multiphase Flow* **14**, 679–699, 1988.

(Dasgupta, S., R. Jackson and S. Sundaresan, “Turbulent gas-particle flow in vertical risers,” *AIChE J.* **40**, 215–228, 1994.)

Krepper, E., D. Lucas and H.-M. Prasser, “On the modelling of bubbly flow in vertical pipes,” *Nucl. Eng. Des.* **235**, 597–611, 2005.

Krepper, E., M. Beyer, T. Frank, D. Lucas and H.-M. Prasser, “CFD modelling of polydispersed bubbly two-phase flow around an obstacle,” *Nucl. Eng. Des.* **239**, 2372–2381, 2009.

Lahey Jr., R.T., and D.A. Drew, “The analysis of two-phase flow and heat transfer using a multidimensional, four-field, two-fluid model,” *Nucl. Eng. Des.* **204**, 29–44, 2001.

Lahey Jr., R.T., “The simulation of multidimensional multiphase flows,” *Nucl. Eng. Des.* **235**, 1043–1060, 2005.

Previous work: miscellaneous

Boelle, A., G. Balzer and O. Simonin, "Second-order prediction of the particle-phase stress tensor of inelastic spheres in simple shear dense suspensions," *ASME-Publications-FED-228*, 9-18, 1995.

Gobin, A., H. Neau, O. Simonin, J.R. Llinas, V. Reiling and J.-L. Selo, "Fluid dynamic numerical simulation of a gas phase polymerization reactor," *Int. J. for Numerical Methods in Fluids* 43, 1199-1220, 2003.

Gevrin, F., O. Masbernat and O. Simonin, "Granular pressure and particle velocity fluctuations prediction in liquid fluidized beds," *Chem. Eng. Sci.* 63, 2450-2464, 2008.

Favre & Favre-like Averaging

Favre average: compressible gas

$$\tilde{U}_{gi} \equiv \overline{\rho_g U_{gi}} \Big/ \overline{\rho}_g$$

Favre, A., "Équations des gaz turbulents compressibles I.- Formes générales," Journal de Mécanique **4**, 361-390, 1965.

Favre, A., "Formulation of the statistical equations of turbulent flows with variable density," in *Studies in Turbulence*, T.B. Gatski, S. Sarkar and C.O. Speziale eds., Springer-Verlag, 324-341, 1992.

Favre-like average: two-fluid

$$\tilde{U}_{si} \equiv \overline{\rho_{s,\mathrm{bulk}} U_{si}} \Big/ \overline{\rho}_{s,\mathrm{bulk}}$$

$$= \overline{\varepsilon_s \rho_s U_{si}} \Big/ \left(\overline{\varepsilon_s \rho_s} \right)$$

$$= \overline{\varepsilon_s U_{si}} \Big/ \overline{\varepsilon}_s$$

$$\tilde{U}_{gi} \equiv \overline{\varepsilon_g U_{gi}} \Big/ \overline{\varepsilon}_g$$

$$\tilde{\Theta}_s \equiv \overline{\varepsilon_s \Theta_s} \Big/ \overline{\varepsilon}_s$$

Physical meaning of Favre velocity

$$\overline{U}_{si} - \widetilde{U}_{si} = \overline{U''_{si}} \qquad \overline{U''_{si}} = -\widetilde{U}'_{si} = -\overline{\varepsilon'_s U''_{si}} \big/ \overline{\varepsilon}_s$$

"The Favre velocity is a mean variable … that includes the velocity fluctuation correlated with density fluctuation. (The Favre velocity fluctuation is) … only the part uncorrelated with density."

Besnard, D.C., and F.H. Harlow, "Turbulence in multiphase flow," *Int. J. Multiphase Flow* **14**, 679-699, 1988.

Notation

Dependent variables $f(\mathbf{x},t)$

$\varepsilon_s, \varepsilon_g, U_{si}, U_{gi}, P_g, \Theta_s$

Reynolds means $f(\mathbf{x})$

Favre means $f(\mathbf{x})$

Reynolds means	Favre means
$\overline{\varepsilon}_s$	$\cdots$
$\overline{\varepsilon}_g$	$\cdots$
$\overline{U}_{si}$	$\widetilde{U}_{si} \equiv \overline{\varepsilon_s U_{si}} / \overline{\varepsilon}_s$
$\overline{U}_{gi}$	$\widetilde{U}_{gi} \equiv \overline{\varepsilon_g U_{gi}} / \overline{\varepsilon}_g$
$\overline{P}_g$	$\cdots$
$\overline{\Theta}_s$	$\widetilde{\Theta}_s \equiv \overline{\varepsilon_s \Theta_s} / \overline{\varepsilon}_s$

Notation

Dependent variables $f(\mathbf{x},t)$

$\varepsilon_s, \varepsilon_g, U_{si}, U_{gi}, P_g, \Theta_s$

Reynolds decomposition

$\varepsilon_s(\mathbf{x},t) = \bar{\varepsilon}_s(\mathbf{x}) + \varepsilon_s'(\mathbf{x},t)$

$\varepsilon_g = \bar{\varepsilon}_g + \varepsilon_g'$

$U_{si} = \overline{U}_{si} + U_{si}'$

$U_{gi} = \overline{U}_{gi} + U_{gi}'$

$P_g = \overline{P}_g + P_g'$

$\Theta_s = \overline{\Theta}_s + \Theta_s'$

Favre decomposition

$\cdots$

$\cdots$

$U_{si} = \widetilde{U}_{si} + U_{si}''$

$U_{gi} = \widetilde{U}_{gi} + U_{gi}''$

$\cdots$

$\Theta_s = \widetilde{\Theta}_s + \Theta_s''$

Identities
… following from the definitions

$$\overline{\varepsilon_s'}(\mathbf{x}) = \overline{\varepsilon_g'} = \overline{U_{si}'} = \overline{U_{gi}'} = \overline{P_g'} = \overline{\Theta_s'} = 0$$

$$\overline{\varepsilon_g U_{gi}''} = 0 \qquad \overline{\varepsilon_s U_{si}''} = 0 \qquad \overline{\varepsilon_s \Theta_s''} = 0$$

$$\left(\overline{U_{gi}''} \neq 0\right) \qquad \left(\overline{U_{si}''} \neq 0\right) \qquad \left(\overline{\Theta_s''} \neq 0\right)$$

$$\overline{U}_{gi} - \tilde{U}_{gi} = \overline{U_{gi}''} \qquad \overline{U}_{si} - \tilde{U}_{si} = \overline{U_{si}''} \qquad \overline{\Theta}_s - \tilde{\Theta}_s = \overline{\Theta_s''}$$

$$\overline{U_{gi}''} = -\tilde{U}_{gi}' = -\overline{\varepsilon_g' U_{gi}''} / \overline{\varepsilon}_g \qquad \overline{U_{si}''} = -\tilde{U}_{si}' = -\overline{\varepsilon_s' U_{si}''} / \overline{\varepsilon}_s \qquad \overline{\Theta_s''} = -\tilde{\Theta}_s' = -\overline{\varepsilon_s' \Theta_s''} / \overline{\varepsilon}_s$$

$$\varepsilon_g'(\mathbf{x}, t) = -\varepsilon_s'(\mathbf{x}, t)$$

Continuity equations

Gas phase

$$\frac{\partial}{\partial t}\left(\varepsilon_g \rho_g\right) + \frac{\partial}{\partial x_i}\left(\varepsilon_g \rho_g U_{gi}\right) = 0$$

$$\cdots \Rightarrow \quad \underline{\underline{\frac{\partial}{\partial t}\left(\rho_g\, \bar{\varepsilon}_g\right) + \frac{\partial}{\partial x_i}\left(\rho_g\, \bar{\varepsilon}_g\, \tilde{U}_{gi}\right) = 0}}$$

Solids phase

$$\frac{\partial}{\partial t}\left(\varepsilon_s \rho_s\right) + \frac{\partial}{\partial x_i}\left(\varepsilon_s \rho_s U_{si}\right) = 0$$

$$\cdots \Rightarrow \quad \underline{\underline{\frac{\partial}{\partial t}\left(\rho_s\, \bar{\varepsilon}_s\right) + \frac{\partial}{\partial x_i}\left(\rho_s\, \bar{\varepsilon}_s\, \tilde{U}_{si}\right) = 0}}$$

Gas Phase Momentum Equations

Instantaneous, local gas phase momentum equations

$$\frac{\partial}{\partial t}\left(\varepsilon_g \rho_g U_{gi}\right)+\frac{\partial}{\partial x_j}\left(\varepsilon_g \rho_g U_{gj} U_{gi}\right)$$

$$= -\varepsilon_g \frac{\partial P_g}{\partial x_i} + \frac{\partial \tau_{gij}}{\partial x_j} - I_{gsi} + \varepsilon_g \rho_g g_i$$

Take the average (commutes with differentiation)

$$\frac{\partial}{\partial t}\left(\rho_g \overline{\varepsilon_g U_{gi}}\right)+\frac{\partial}{\partial x_j}\left(\rho_g \overline{\varepsilon_g U_{gj} U_{gi}}\right)$$

$$= -\overline{\varepsilon_g \frac{\partial P_g}{\partial x_i}} + \frac{\partial \bar{\tau}_{gij}}{\partial x_j} - \bar{I}_{gsi} + \bar{\varepsilon}_g \rho_g g_i$$

Gas phase momentum equation
convective triple product – Favre average

$$\varepsilon_g U_{gj} U_{gi} = \varepsilon_g \left(\tilde{U}_{gj} + U''_{gj}\right)\left(\tilde{U}_{gi} + U''_{gi}\right)$$

$$= \varepsilon_g \tilde{U}_{gj} \tilde{U}_{gi} + \varepsilon_g U''_{gi} \tilde{U}_{gj} + \varepsilon_g U''_{gj} \tilde{U}_{gi} + \varepsilon_g U''_{gj} U''_{gi}$$

Gas phase momentum equation
convective triple product – Favre average

$$\varepsilon_g U_{gj} U_{gi} = \varepsilon_g \left(\tilde{U}_{gj} + U''_{gj}\right)\left(\tilde{U}_{gi} + U''_{gi}\right)$$

$$= \varepsilon_g \tilde{U}_{gj} \tilde{U}_{gi} + \varepsilon_g U''_{gi} \tilde{U}_{gj} + \varepsilon_g U''_{gj} \tilde{U}_{gi} + \varepsilon_g U''_{gj} U''_{gi}$$

average

$$\overline{\varepsilon_g U_{gj} U_{gi}}$$

$$= \bar{\varepsilon}_g \tilde{U}_{gj} \tilde{U}_{gi} + \overline{\varepsilon_g U''_{gi}}\, \tilde{U}_{gj} + \overline{\varepsilon_g U''_{gj}}\, \tilde{U}_{gi} + \overline{\varepsilon_g U''_{gj} U''_{gi}}$$

Gas phase momentum equation
convective triple product – Favre average

$$\overline{\varepsilon_g U_{gj} U_{gi}} = \bar{\varepsilon}_g \tilde{U}_{gj} \tilde{U}_{gi} + \overline{\varepsilon_g U''_{gj} U''_{gi}}$$

$$= \bar{\varepsilon}_g \tilde{U}_{gj} \tilde{U}_{gi} - \bar{\varepsilon}_g \tau_{\tilde{T}gji}$$

gas phase specific Reynolds - ?? stresses

$$\tau_{\tilde{T}gji} \equiv -\overline{\varepsilon_g U''_{gj} U''_{gi}} \Big/ \bar{\varepsilon}_g$$

$$= -\overline{U''_{gj} U''_{gi}} - \overline{\varepsilon'_g U''_{gj} U''_{gi}} \Big/ \bar{\varepsilon}_g$$

Gas phase momentum equation
convective triple product – Reynolds average

$$\overline{\varepsilon_g U_{gj} U_{gi}} = \overline{\left(\bar{\varepsilon}_g + \varepsilon'_g\right)\left(\overline{U}_{gj} + U'_{gj}\right)\left(\overline{U}_{gi} + U'_{gi}\right)}$$

$$= \bar{\varepsilon}_g \overline{U}_{gj} \overline{U}_{gi} + \bar{\varepsilon}_g \overline{U'_{gi}} \overline{U}_{gj} + \bar{\varepsilon}_g \overline{U'_{gj}} \overline{U}_{gi} + \bar{\varepsilon}_g \overline{U'_{gj} U'_{gi}}$$
$$+ \overline{\varepsilon'_g} \overline{U}_{gj} \overline{U}_{gi} + \overline{\varepsilon'_g U'_{gi}} \overline{U}_{gj} + \overline{\varepsilon'_g U'_{gj}} \overline{U}_{gi} + \overline{\varepsilon'_g U'_{gj} U'_{gi}}$$

$$= \bar{\varepsilon}_g \overline{U}_{gj} \overline{U}_{gi} + \bar{\varepsilon}_g \overline{U'_{gj} U'_{gi}} + \overline{\varepsilon'_g U'_{gi}} \overline{U}_{gj} + \overline{\varepsilon'_g U'_{gj}} \overline{U}_{gi} + \overline{\varepsilon'_g U'_{gj} U'_{gi}}$$

$$cf. \quad = \bar{\varepsilon}_g \tilde{U}_{gj} \tilde{U}_{gi} - \bar{\varepsilon}_g \tau_{\tilde{T}gji}$$

$$\tau_{\tilde{T}gji} \equiv - \overline{\varepsilon_g U''_{gj} U''_{gi}} \Big/ \bar{\varepsilon}_g = -\overline{U''_{gj} U''_{gi}} - \overline{\varepsilon'_g U''_{gj} U''_{gi}} \Big/ \bar{\varepsilon}_g$$

Gas phase momentum equation

$$\frac{\partial}{\partial t}\left(\rho_g\,\bar{\varepsilon}_g\,\tilde{U}_{gi}\right)+\frac{\partial}{\partial x_j}\left(\rho_g\,\bar{\varepsilon}_g\,\tilde{U}_{gj}\,\tilde{U}_{gi}\right)-\frac{\partial}{\partial x_j}\left(\rho_g\,\bar{\varepsilon}_g\,\tau_{\tilde{T}gji}\right)$$

$$=-\bar{\varepsilon}_g\,\frac{\partial\bar{P}_g}{\partial x_i}-\overline{\varepsilon'_g\,\frac{\partial P'_g}{\partial x_i}}+\;2\mu_g\,\frac{\partial S_{gij}\left(\tilde{\mathbf{U}}\right)}{\partial x_j}+2\mu_g\,\frac{\partial S_{gij}\left(\overline{\mathbf{U}''}\right)}{\partial x_j}$$

$$-\bar{I}_{gsi}\;+\;\bar{\varepsilon}_g\rho_g g_i$$

$$\bar{I}_{gsi}=\bar{\hat{\beta}}_{gs}\,\bar{\varepsilon}_g\left(\tilde{U}_{gi}-\tilde{U}_{si}\right)+\;\bar{\hat{\beta}}_{gs}\overline{U''_{gi}}$$

$$+\,\bar{\varepsilon}_g\,\overline{\hat{\beta}'_{gs}\left(U''_{gi}-U''_{si}\right)}+\overline{\hat{\beta}'_{gs}\,\varepsilon'_g}\left(\tilde{U}_{gi}-\tilde{U}_{si}\right)+\overline{\hat{\beta}'_{gs}\left(\varepsilon'_g U''_{gi}+\varepsilon'_g U''_{si}\right)}$$

Solids Phase Momentum Equations

$$\frac{\partial}{\partial t}\left(\bar{\varepsilon}_s \rho_s \tilde{U}_{si}\right)+\frac{\partial}{\partial x_j}\left(\bar{\varepsilon}_s \rho_s \tilde{U}_{si}\tilde{U}_{sj}\right)$$

$$= -\bar{\varepsilon}_s \frac{\partial \bar{P}_g}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\bar{\tau}_{sij} + \tau_{\tilde{T}sij}\right) + \bar{I}_{gsi} + \bar{\varepsilon}_s \rho_s g_i - ?\ \overline{\varepsilon'_g \frac{\partial P'_g}{\partial x_i}}$$

solids phase specific Reynolds - ?? stresses

$$\tau_{\tilde{T}sij} \equiv -\overline{\varepsilon_s U''_{si} U''_{sj}} \Big/ \bar{\varepsilon}_s$$

$$= -\overline{U''_{si} U''_{sj}} - \overline{\varepsilon'_s U''_{si} U''_{sj}} \Big/ \bar{\varepsilon}_s$$

$$\bar{I}_{gsi} = \bar{\hat{\beta}}_{gs}\, \bar{\varepsilon}_s \left(\tilde{U}_{gi} - \tilde{U}_{si}\right) + \bar{\hat{\beta}}_{gs} \overline{U''_{gi}}$$

$$+ \overline{\hat{\beta}'_{gs}\, \varepsilon'_s}\left(\tilde{U}_{gi} - \tilde{U}_{si}\right) + \bar{\varepsilon}_s \overline{\hat{\beta}'_{gs}\left(U''_{gi} - U''_{si}\right)} - \overline{\hat{\beta}'_{gs}\left(\varepsilon'_g U''_{gi} + \varepsilon'_s U''_{si}\right)}$$

KTGF Granular Stress

$$\overline{\tau}_{sij} = \left[-\overline{P}_s + \eta\overline{\mu_b\,\partial U_{si}/\partial x_i}\right]\delta_{ij} + 2\overline{\mu_s\,S_{sij}}$$

$$P_s = \rho_s\varepsilon_s\Theta_s\left(1+4\eta\varepsilon_s g_0\right)$$

$$\overline{P}_s = \rho_s\overline{\varepsilon}_s\tilde{\Theta}_s + 2\rho_s\left(1+e_p\right)\overline{\left(\varepsilon_s^2 g_0\right)\Theta_s}$$

$$\overline{\left(\varepsilon_s^2 g_0\right)\Theta_s} = \left(\varepsilon_s g_0\right)_0\overline{\varepsilon}_s\tilde{\Theta}_s + \left(\varepsilon_s g_0\right)_0'\,\overline{\varepsilon}_s\,\overline{\varepsilon_s'\Theta_s''}$$
$$+\left[\left(\varepsilon_s g_0\right)_0' + \frac{1}{2}\left(\varepsilon_s g_0\right)_0''\overline{\varepsilon}_s\right]\tilde{\Theta}_s\overline{\left(\varepsilon_s'\right)^2} + \cdots$$

$$\left(\varepsilon_s g_0\right)_0 \equiv \overline{\varepsilon}_s g_0\left(\overline{\varepsilon}_s\right) \qquad \left(\varepsilon_s g_0\right)_0' \equiv \left.\left(\frac{d\left(\varepsilon_s g_0\right)}{d\varepsilon_s}\right)\right|_{\varepsilon_s=\overline{\varepsilon}_s}$$

KTGF Granular Stress (cont.)

$$\overline{\mu_b \frac{\partial U_{si}}{\partial x_i}} = \overline{\mu}_b \frac{\partial \tilde{U}_{si}}{\partial x_i} + \overline{\mu_b \frac{\partial U''_{si}}{\partial x_i}}$$

$$\overline{f_{\mu_b}(\varepsilon_s)\sqrt{\Theta_s}}^{(1)} = f_{\mu_b}(\overline{\varepsilon}_s)\left(\tilde{\Theta}_s\right)^{1/2} + \frac{1}{2} f_{\mu_b}(\overline{\varepsilon}_s)\left(\tilde{\Theta}_s\right)^{-1/2}\left(\overline{\Theta''_s}\right)$$
$$+\frac{1}{2} f''_{\mu_b}(\overline{\varepsilon}_s)\left(\tilde{\Theta}_s\right)^{1/2}\overline{\left(\varepsilon'_s\right)^2} + \frac{1}{2} f'_{\mu_b}(\overline{\varepsilon}_s)\left(\tilde{\Theta}_s\right)^{-1/2}\overline{\varepsilon'_s\Theta''_s} - \frac{1}{4} f_{\mu_b}(\overline{\varepsilon}_s)\left(\tilde{\Theta}_s\right)^{-3/2}\overline{\left(\Theta''_s\right)^2} + \cdots$$

$$\overline{f_{\mu_b}(\varepsilon_s)\sqrt{\Theta_s}\frac{\partial U''_{si}}{\partial x_i}}^{(1)} = f_{\mu_b}(\overline{\varepsilon}_s)\left(\tilde{\Theta}_s\right)^{1/2}\frac{\partial \overline{U''_{si}}}{\partial x_i}$$
$$+\frac{1}{2} f_{\mu_b}(\overline{\varepsilon}_s)\left(\tilde{\Theta}_s\right)^{-1/2}\overline{\Theta''_s\frac{\partial U''_{si}}{\partial x_i}} + \left(\tilde{\Theta}_s\right)^{1/2} f'_{\mu_b}(\overline{\varepsilon}_s)\overline{\varepsilon'_s\frac{\partial U''_{si}}{\partial x_i}} + \cdots$$

Kinetic Theory of Granular Flow granular temperature equation

$$\frac{3}{2}\frac{\partial\left(\varepsilon_s\rho_s\Theta_s\right)}{\partial t}+\frac{3}{2}\frac{\partial\left(\varepsilon_s\rho_s U_{sj}\Theta_s\right)}{\partial x_j}=\frac{\partial}{\partial x_i}\left(\kappa_s\frac{\partial\Theta_s}{\partial x_i}\right)+\tau_{sij}\frac{\partial U_{si}}{\partial x_j}+\prod_s-\varepsilon_s\rho_s J_s$$

$$\frac{3}{2}\frac{\partial\left(\rho_s\bar{\varepsilon}_s\tilde{\Theta}_s\right)}{\partial t}+\frac{3}{2}\rho_s\frac{\partial}{\partial x_j}\left(\bar{\varepsilon}_s\tilde{U}_{sj}\tilde{\Theta}_s+\bar{\varepsilon}_s q_{\tilde{T}j}\right)$$

$$=\frac{\partial}{\partial x_i}\left(\overline{\kappa_s\frac{\partial\Theta_s}{\partial x_i}}\right)+\overline{\tau_{sij}\frac{\partial U_{si}}{\partial x_j}}+\overline{\prod}_s-\rho_s\overline{\varepsilon_s J_s}$$

$$q_{\tilde{T}j}=\overline{\varepsilon_s U''_{sj}\Theta''_s}\cong-\frac{\mu_{\tilde{T}}}{\Pr_{\tilde{T}}}\frac{\partial\tilde{\Theta}}{\partial x_j}$$

Turbulent Granular Temperature Flux vector

$$\overline{\tau_{sij}\frac{\partial U_{si}}{\partial x_j}}$$

Dilatation & Solenoidal Dissipation

Specific Gas Phase Turbulence Kinetic Energy

$$\bar{\varepsilon}_g \tilde{k}_g = \frac{1}{2}\overline{\varepsilon_g U''_{gi} U''_{gi}}$$

Total kinetic energy of gas flow is nicely partitioned:

$$\frac{1}{2}\overline{\rho_g \varepsilon_g \left(U_g\right)^2} = \frac{1}{2}\overline{\rho_g \varepsilon_g \left(\tilde{\mathbf{U}}_g + \mathbf{U}''_g\right)^2} = \frac{1}{2}\rho_g \bar{\varepsilon}_g \left(\tilde{U}_g\right)^2 + \rho_g \bar{\varepsilon}_g \tilde{k}_g$$

The first term is the energy density of the gas flow due to the <u>combined mean motion and turbulent motion correlated with the void fraction</u>.

The second term is the gas energy density due to the <u>residual turblent motion, *i.e.*, that not correlated with voidage fluctuations</u>.

Besnard, D.C., and F.H. Harlow, 1988.

$$\frac{1}{2}\overline{\rho_g \varepsilon_g \left(U_g\right)^2} = \frac{1}{2}\overline{\rho_g \varepsilon_g \left(\overline{\mathbf{U}}_g + \mathbf{U}'_g\right)^2} = ??$$

Specific Solids Phase Turbulence Kinetic Energy

$$\bar{\varepsilon}_s \tilde{k}_s = \frac{1}{2}\overline{\varepsilon_s U''_{si} U''_{si}}$$

Total kinetic energy of solids flow is nicely partitioned:

$$\frac{1}{2}\overline{\rho_s \varepsilon_s \left(U_s\right)^2} = \frac{1}{2}\overline{\rho_s \varepsilon_s \left(\tilde{\mathbf{U}}_s + \mathbf{U}''_s\right)^2} = \frac{1}{2}\rho_s \bar{\varepsilon}_s \left(\tilde{U}_s\right)^2 + \rho_s \bar{\varepsilon}_s \tilde{k}_s$$

The first term is the energy density of the granular flow due to the <u>combined mean motion and turbulent motion correlated with the solids fraction</u>.

The second term is the solids energy density due to the <u>residual turblent motion, *i.e.*, that not correlated with solids fluctuations</u>.

Besnard, D.C., and F.H. Harlow, 1988.

Gas Phase Turbulence Kinetic Energy

$$\overbrace{\frac{\partial}{\partial t}\left(\rho_g \bar{\varepsilon}_g \tilde{k}_g\right)}^{\substack{\text{Rate of change of} \\ \text{turbulence kinetic energy per unit mass} \\ \text{due to non-stationarity}}} + \overbrace{\frac{\partial}{\partial x_j}\left(\rho_g \bar{\varepsilon}_g \tilde{k}_g \tilde{U}_{gj}\right)}^{\substack{\text{Rate of change of} \\ \text{turbulence kinetic energy per unit mass} \\ \text{due to convection by the mean flow}}}$$

$$- \overbrace{\rho_g \bar{\varepsilon}_g \tau_{\tilde{T}gij} \frac{\partial \tilde{U}_{gi}}{\partial x_j}}^{\substack{\text{Rate of production of} \\ \text{turbulence kinetic energy} \\ \text{from the mean flow gradient}}} + \overbrace{\frac{\partial}{\partial x_j}\left(\frac{1}{2}\rho_g \overline{\varepsilon_g U''_{gj} U''_{gi} U''_{gi}}\right)}^{\substack{\text{Transport of} \\ \text{turbulence kinetic energy} \\ \text{due to the turbulence itself}}}$$

$$= \underbrace{-\bar{\varepsilon}_g \overline{U''_{gi} \frac{\partial P'_g}{\partial x_i}} - \overline{\varepsilon'_g U''_{gi} \frac{\partial P'_g}{\partial x_i}}}_{\substack{\text{Transport of} \\ \text{turbulence kinetic energy} \\ \text{due to the pressure fluctuations}}} + \underbrace{\overline{\varepsilon_g U''_{gi} \frac{\partial \tau_{gij}}{\partial x_j}}}_{\text{Dissipation}} \underbrace{- \overline{U''_{gi} I_{gsi}}}_{\substack{\text{Rate of transfer of} \\ \text{(gas phase) turbulence kinetic energy} \\ \text{to the granular phase}}}$$

Solids Phase Turbulence Kinetic Energy

$$
\overbrace{\frac{\partial}{\partial t}\left(\rho_s \bar{\varepsilon}_s \tilde{k}_s\right)}^{\begin{array}{c}\text{Rate of change of}\\ \text{turbulence kinetic energy per unit mass}\\ \text{due to non-stationarity}\end{array}}
+
\overbrace{\frac{\partial}{\partial x_j}\left(\rho_s \bar{\varepsilon}_s \tilde{k}_s \tilde{U}_{sj}\right)}^{\begin{array}{c}\text{Rate of change of}\\ \text{turbulence kinetic energy per unit mass}\\ \text{due to convection by the mean flow}\end{array}}
$$

$$
-\overbrace{\rho_s \bar{\varepsilon}_s \tau_{\tilde{T}sij} \frac{\partial \tilde{U}_{si}}{\partial x_j}}^{\begin{array}{c}\text{Rate of production of}\\ \text{turbulence kinetic energy}\\ \text{from the mean flow gradient}\end{array}}
+
\overbrace{\frac{\partial}{\partial x_j}\left(\frac{1}{2}\rho_s \overline{\varepsilon_s U''_{sj} U''_{si} U''_{si}}\right)}^{\begin{array}{c}\text{Transport of}\\ \text{turbulence kinetic energy}\\ \text{due to the turbulence itself}\end{array}}
$$

$$
= \underbrace{-\bar{\varepsilon}_s \overline{U''_{si}\frac{\partial P'_g}{\partial x_i}} -/+ \overline{\varepsilon'_{s/g} U''_{si}\frac{\partial P'_g}{\partial x_i}}}_{\begin{array}{c}\text{Transport of}\\ \text{turbulence kinetic energy}\\ \text{due to the pressure fluctuations}\end{array}}
+ \underbrace{\overline{\varepsilon_s U''_{si}\frac{\partial \tau_{sij}}{\partial x_j}}}_{\begin{array}{c}\text{Dissipation}\\ \text{(not inelastic!)}\end{array}}
\underbrace{+\overline{U''_{si} I_{gsi}}}_{\begin{array}{c}\text{Rate of transfer of}\\ \text{(solids phase) turbulence kinetic energy}\\ \text{to the gas phase}\end{array}}
$$

Kinetic Energy Density of the Flow

Kinetic energy density of the gas phase

$$K_g = \frac{1}{2}\varepsilon_g \rho_g U_{gi} U_{gi}$$

Kinetic energy density of the solids phase

$$K_s = \frac{1}{2}\varepsilon_s \rho_s U_{si} U_{si}$$

Total kinetic energy density of the flow

$$K_{tot}(\mathbf{x}, t) = K_g + K_s = \frac{1}{2}\varepsilon_g \rho_g U_{gi} U_{gi} + \frac{1}{2}\varepsilon_s \rho_s U_{si} U_{si}$$

Average Kinetic Energy Density of the Gas Phase

$$\overline{K}_g = \frac{1}{2}\rho_g \overline{\varepsilon_g U_{gi} U_{gi}} = \frac{1}{2}\rho_g \overline{\varepsilon_g \left(\tilde{U}_{gi} + U''_{gi}\right)\left(\tilde{U}_{gi} + U''_{gi}\right)}$$

$$= \frac{1}{2}\rho_g \left(\bar{\varepsilon}_g \tilde{U}_{gi}\tilde{U}_{gi} + 2\overline{\varepsilon_g U''_{gi}}\tilde{U}_{gi} + \overline{\varepsilon_g U''_{gi} U''_{gi}}\right)$$

$$= \frac{1}{2}\rho_g \left(\bar{\varepsilon}_g \tilde{U}_{gi}\tilde{U}_{gi} + \overline{\varepsilon_g U''_{gi} U''_{gi}}\right)$$

… measures the residual turbulent motion.
Besnard et al, 1992

$$\overline{K}_g = \frac{1}{2}\rho_g \bar{\varepsilon}_g \left(\tilde{U}_{gi}\tilde{U}_{gi}\right) + \rho_g \bar{\varepsilon}_g \tilde{k}_g$$

… the energy density due to the combined mean and density-correlated turbulent motion,

gas phase specific turbulence kinetic energy

$$\bar{\varepsilon}_g \tilde{k}_g = \frac{1}{2}\overline{\varepsilon_g U''_{gi} U''_{gi}}$$

Average Kinetic Energy Density

$$\overline{K}_s = \frac{1}{2}\rho_s \overline{\varepsilon_s U_{si} U_{si}} = \frac{1}{2}\rho_g \left(\overline{\varepsilon}_g \tilde{U}_{gi} \tilde{U}_{gi} + \overline{\varepsilon_g U''_{gi} U''_{gi}} \right)$$

$$= \frac{1}{2}\rho_s \overline{\varepsilon}_s \left(\tilde{U}_{si} \tilde{U}_{si} \right) + \frac{1}{2}\rho_s \overline{\varepsilon}_s \tilde{k}_s$$

$$\overline{\varepsilon}_s \tilde{k}_s = \frac{1}{2} \overline{\varepsilon_s U''_{si} U''_{si}}$$

solids phase specific turbulence kinetic energy

$$\overline{K}_{tot} = \overline{K}_g + \overline{K}_s = \frac{1}{2}\left(\rho_g \overline{\varepsilon}_g \tilde{U}_{gi} \tilde{U}_{gi} + \rho_s \overline{\varepsilon}_s \tilde{U}_{si} \tilde{U}_{si} \right) + \left(\rho_g \overline{\varepsilon}_g \tilde{k}_g + \rho_s \overline{\varepsilon}_s \tilde{k}_s \right)$$

Average Kinetic Energy Density of the Gas Phase (alternative approach)

$$\overline{K}_g = \frac{1}{2}\rho_g \overline{\varepsilon_g U_{gi} U_{gi}} = \frac{1}{2}\rho_g \overline{\varepsilon_g \left(\overline{U}_{gi} + U'_{gi}\right)\left(\overline{U}_{gi} + U'_{gi}\right)}$$

$$= \frac{1}{2}\rho_g \left(\overline{\varepsilon}_g \overline{U}_{gi}\overline{U}_{gi} + 2\overline{\varepsilon_g U'_{gi}}\,\overline{U}_{gi} + \overline{\varepsilon_g U'_{gi} U'_{gi}}\right)$$

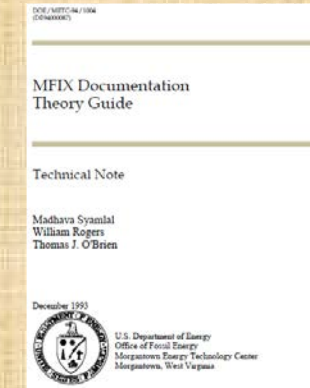
$$= \frac{1}{2}\rho_g \left(\overline{\varepsilon}_g \overline{U}_{gi}\overline{U}_{gi} + 2\overline{\left(\overline{\varepsilon}_g + \varepsilon'_g\right) U'_{gi}}\,\overline{U}_{gi} + \overline{\left(\overline{\varepsilon}_g + \varepsilon'_g\right) U'_{gi} U'_{gi}}\right)$$

$$= \frac{1}{2}\rho_g \left(\overline{\varepsilon}_g \overline{U}_{gi}\overline{U}_{gi} + 2\overline{\varepsilon'_g U'_{gi}}\,\overline{U}_{gi} + \overline{\varepsilon}_g \overline{U'_{gi} U'_{gi}} + \overline{\varepsilon'_g U'_{gi} U'_{gi}}\right)$$

$$= \frac{1}{2}\rho_g \left(\overline{\varepsilon}_g \overline{U}_{gi}\overline{U}_{gi} + 2\overline{\varepsilon'_g U'_{gi}}\,\overline{U}_{gi} + \overline{\varepsilon'_g U'_{gi} U'_{gi}}\right) + \rho_g \overline{\varepsilon}_g \overline{k}_g$$

gas phase specific turbulence kinetic energy $\overline{k}_g = \frac{1}{2}\overline{U'_{gi} U'_{gi}}$

Kinetic Energy Budget

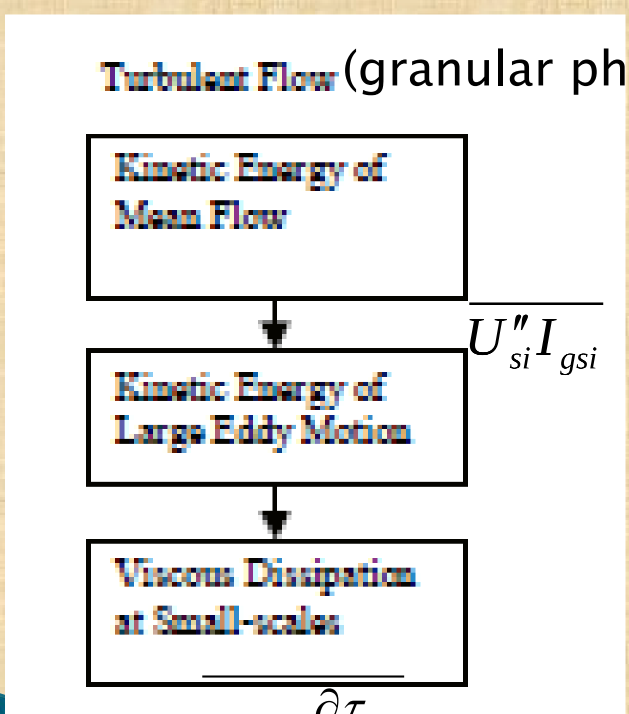

MFIX Documentation
Theory Guide

Technical Note

Madhava Syamlal
William Rogers
Thomas J. O'Brien

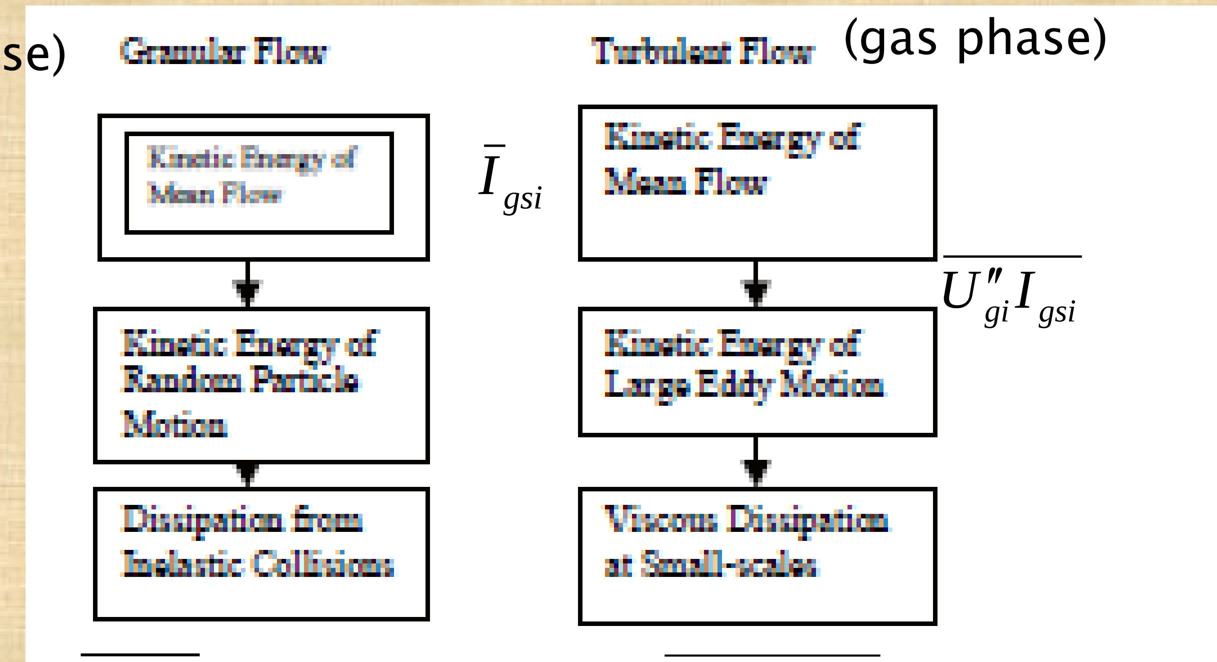
December 1993

U.S. Department of Energy
Office of Fossil Energy
Morgantown Energy Technology Center
Morgantown, West Virginia



(granular phase)

$\overline{U''_{si} I_{gsi}}$



(gas phase)

$\bar{I}_{gsi}$

$\overline{U''_{gi} I_{gsi}}$

Figure 3. Energy Cascade in Granular Flows Compared With That in Turbulent Flows

$$\underbrace{\overline{\varepsilon_s U''_{si} \frac{\partial \tau_{sij}}{\partial x_j}}}_{\text{Dissipation (not inelastic!)}} \qquad - \rho_s \overline{\varepsilon_s J_s} \qquad \underbrace{\overline{\varepsilon_g U''_{gi} \frac{\partial \tau_{gij}}{\partial x_j}}}_{\text{Viscous Dissipation}}$$

Correlations

Volume fraction fluctuation - velocity fluctuation correlations

$$\overline{\varepsilon'_g U'_{gi}} \text{ and } \overline{\varepsilon'_s U'_{si}}$$

Specific gas phase turbulence Reynolds/Favre stresses

$$\bar{\varepsilon}_g \, \tau_{\tilde{T}gij} \equiv -\overline{\varepsilon_g U''_{gi} U''_{gj}} \text{ and } \bar{\varepsilon}_s \, \tau_{\tilde{T}sij} \equiv -\overline{\varepsilon_s U''_{si} U''_{sj}}$$

Drag related correlations

$$\bar{\hat{\beta}}_{gs}, \; \overline{U''_{gi}} = -\tilde{U}'_{gi},$$

$$\overline{\varepsilon'_s \hat{\beta}'_{gs}} = -\overline{\varepsilon'_g \hat{\beta}'_{gs}}, \; \overline{\hat{\beta}'_{gs}\left(U''_{gi} - U''_{si}\right)}, \text{ and } \overline{\hat{\beta}'_{gs}\, \varepsilon'_s \left(U''_{gi} - U''_{si}\right)}$$

Stress related correlation $\overline{\varepsilon'_g \dfrac{\partial P'_g}{\partial x_j}}, \; \overline{U''_{gi}}$

Closures
velocity & volume fraction

Using an eddy - viscosity type turbulence model

$$\overline{\varepsilon_g' U_{gi}'} \approx -\frac{\nu_{tg}}{\sigma_{\varepsilon g}} \nabla \bar{\varepsilon}_g \text{ and } \overline{\varepsilon_s' U_{si}'} \approx -\frac{\nu_{ts}}{\sigma_{\varepsilon s}} \nabla \bar{\varepsilon}_s = +\frac{\nu_{ts}}{\sigma_{\varepsilon s}} \nabla \bar{\varepsilon}_g$$

These correlations are proportional, but opposite in sign

$$\overline{\varepsilon_s' U_{si}'} = -\left(\frac{\nu_{ts}}{\nu_{tg}} \frac{\sigma_{\varepsilon g}}{\sigma_{\varepsilon s}} \right) \overline{\varepsilon_g' U_{gi}'}.$$

$\nu_{tg} \left(\nu_{ts} \right)$ - gas (solids) phase kinematic eddy viscosity

$\sigma_{\varepsilon g} \left(\sigma_{\varepsilon s} \right)$ - turbulent Prandtl number for gas (solids)

Closures
velocity relations

Using the identities

$$\tilde{U}_{gi} = \overline{U}_{gi} + \overline{\varepsilon'_g U'_{gi}} \Big/ \overline{\varepsilon}_g \text{ and } \tilde{U}_{si} = \overline{U}_{si} + \overline{\varepsilon'_s U'_{si}} \Big/ \overline{\varepsilon}_s \,,$$

$$\tilde{U}_{gi} - \overline{U}_{gi} \approx -\frac{\nu_{tg}}{\sigma_{\varepsilon g} \overline{\varepsilon}_g} \nabla \overline{\varepsilon}_g$$

$$\tilde{U}_{si} - \overline{U}_{si} \approx -\frac{\nu_{ts}}{\sigma_{\varepsilon s} \overline{\varepsilon}_s} \nabla \overline{\varepsilon}_g = +\frac{\nu_{ts}}{\sigma_{\varepsilon s} \left(1 - \overline{\varepsilon}_g\right)} \nabla \overline{\varepsilon}_g$$

To Do List

- Extract dissipation, $\epsilon_{g/s}$
- Closures for $k_{g/s}$ & $\epsilon_{g/s}$
- Look at energy cascade
- ~~Add granular temperature equation – currently no dissipation due to inelastic collisions~~
  - ~~Indentify closures~~
- Equation for the full stress tensor, τ
- Compare in detail with Reynolds formalism
- Formulate drag terms for a chosen form

- Include thermal temperatures
- Chemistry (contacting)– hopefully temperature fluctuations will be small

- Develop closure relationships

  … related to specific experimental data

  (A NEVER ENDING TASK)

TOBBoCo@gmail.com, 304-816-6332, 1847 Joliet Way, Boulder, CO 80305

Thank you
Questions?