Using Cylindrical Coordinates in the Simulation of Dense Particle-Gas Multi-Phase Flows

Akhilesh Bakshi, Christos Altantzis, Ahmed F. Ghoniem
Center for Energy and Propulsion Research
Department of Mechanical Engineering
Massachusetts Institute of Technology, USA

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Present Study

Objectives

i. To investigate the accumulation of solid particles at the centerline for the simulation of dense particle-gas multi-phase flows using the cylindrical coordinate system

ii. To compare the quality and computational cost of simulations using the cylindrical coordinate system with those using the Cartesian Cut-Cell approach

Conclusions

i. Better prediction of void fraction with respect to experimental data using averaged solid and gas radial velocities at the centerline

ii. Significantly less computational cost for simulations using the cylindrical coordinates as compared to those employing the Cartesian cut-cell approach
The Two-Fluid Model

- Solid and gas phases fully interpenetrating continua using generalized NS equations
- Computationally efficient
- Conservation equations coupled with constitutive relationships

\[
\frac{\partial}{\partial t} (\varepsilon_{sm} \rho_{sm} \vec{v}_{sm}) + \nabla \cdot (\varepsilon_{sm} \rho_{sm} \vec{v}_{sm} \vec{v}_{sm}) = \nabla \cdot \vec{S}_{sm} + \varepsilon_{sm} \rho_{sm} \vec{g} + \vec{I}_{gm} - \sum_{l=1}^{M} \vec{I}_{ml}
\]

Solid Phase Stress Tensor

Interactions
- Particle-particle and particle-gas
- Kinetic Theory of Granular Flow (KTGF)

The TFM has been implemented using **MFiX** (*Multiphase Flow with Interphase eXchanges*)

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Validation of Numerical Model

Experimental Conditions

Column: D=13.8 cm, H=1.5 m

Particles: Group B; d_p=350 μm, ρ_p=2500 kg/m³

Fluidizing Gas: Air (ambient conditions)

Static Bed Height: 20 cm

Measuring Level: 14.3-18.1 cm

Schematic of the experimental setup by Makkawi et al (Powder Technology 2006)

Time averaged (2-20s) void fraction versus bed height at different superficial velocities using cylindrical coordinates (18 x 160 x 12)
i. Cartesian 2D – Only qualitative analysis

ii. Cartesian Cut-Cell 3D - Expensive!

iii. **Cylindrical 3D - Accurate, Inexpensive**

Simulations using the Cylindrical 3D coordinate system show a characteristic *dip* at the center.

Time averaged (2-20s) void fraction at axial height 14.3-18.1 cm for different radial resolutions.
Radial Velocity

- **No normal flow**
  
  => Accumulation of solid particles at center

- Required for the computation of
  
  (a) Convection terms
  
  (b) Gas-Solid Drag Force

Axial Velocity

- **Free slip boundary condition**

- Numerically, \( v_p' = v_q \) and \( v_q' = v_p \)

- Error (time averaged) =

\[
\left| \frac{\bar{v}_{1jk} - \bar{v}_{0j} k + \frac{N_2}{2}}{\bar{v}_{1jk}} \right| < 2\%
\]

Discretization of the bed cross section for \( y \)-momentum equation

\[ u_1 = u_1(u_{1/2}, u_{3/2}) \]
Centerline Treatment

- Momentum Equations
- Solids Volume Fraction Correction
- Fluids Pressure Correction
- Boundary Conditions
- Scalar Equations

**Multi-Valued Formulation**

\[ u_{r, \frac{1}{2} j k} = u_{r, \frac{3}{2} j k} \]

Different centerline Cartesian velocity in each cell

**Multi-Valued Averaging**

\[ u_{r, \frac{1}{2} j k} = \frac{u_{r, \frac{3}{2} j k} - u_{r, \frac{3}{2} j k + \frac{N\theta}{2}}}{2} \]

Identical centerline Cartesian velocity in diametrically opposite cells

**Single-Valued Averaging**

\[ u_{r, \frac{1}{2} j k} = \bar{u}_x \cos \theta + \bar{u}_y \sin \theta \]

Unique centerline Cartesian velocity in all cells
Solid + Gas injected through a side port into a cylindrical vessel

Comparison along the distributor axis

Numerical Experiment

Gas x-velocity at distributor height

No normal flow boundary condition

Gas x-velocity for (a) Cut-Cell 3D (b) Cylindrical 3D (No Modification) and (c) Cylindrical 3D (Average) at t=0.10s
Resolution

Analysis based on the study by Clemins (1988)

Maximum resolution based on bed, particle size

\[
\frac{R}{L_m} > \bar{\alpha} \left[ \frac{0.5(1-\bar{\alpha})}{|\Delta\bar{\alpha}|_{tol}} \right]^{1/2}
\]

\[(n_r)^2 \times n_{\theta} \times n_{axial} < constant\]
Domain (Cylindrical 3D)

Grid Resolution:
- Low resolution at center for continuum (~ 4.8 mm)
- Sufficient resolution at wall to capture wall effects (~ 3 mm)

Solution:

\[ n_{\text{theta}} = 12 \]
\[ \Delta y = 5 \text{ mm} \]
\[ \Delta r = 3.8 \text{ mm (Uniform Grid)} \]
\[ 3.0 - 4.8 \text{ mm (Non Uniform Grid)} \]

First order accurate
but time averaged void fraction
shows good match (max error < 5%)
Comparison with Cut-Cell – Bubbling Bed

Time averaged (2-20s) plots of (a) Void Fraction and (b) Solid axial velocity at axial height 14.3 – 18.1 cm

<table>
<thead>
<tr>
<th>Approach</th>
<th>Domain</th>
<th>Real Time (s)</th>
<th>CPU Time (hr)</th>
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<tbody>
<tr>
<td>Cylindrical</td>
<td>18 x 120 x 12</td>
<td>20</td>
<td>188</td>
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<tr>
<td>Cut-Cell</td>
<td>36 x 120 x 36</td>
<td>20</td>
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</table>

Simulating bubbling beds using the cylindrical coordinates
(a) shows **better match** with experimental data at the center as well as bed interior and
(b) costs more than **5 times less** as compared to the Cartesian Cut-Cell approach
Comparison with Cut-Cell – Slugging Bed

Simulating slugging beds using the cylindrical coordinates
(a) shows **better match** with experimental data at the center as well as the interiors and
(b) costs **5 times less** as compared to the Cartesian Cut-Cell approach

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<td>Cylindrical</td>
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<tr>
<td>Cut-Cell</td>
<td>$36 \times 160 \times 36$</td>
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</table>

Time averaged (2-20s) plots of (a) Void Fraction and (b) Solid axial velocity at axial height 14.3 – 18.1 cm
Conclusions and Future Work

i. **Single-valued averaging** scheme has been used to predict the centerline gas and solid radial velocities to prevent local accumulation of solids

ii. **Non-uniform grid** employed using cylindrical coordinates; predicted time-averaged void fraction more accurate as compared to that with Cut-Cell approach

iii. Significantly **less computational cost** for simulations using the cylindrical coordinates as compared to those employing the Cartesian cut-cell approach

iv. **Upper limit** on grid resolution based on bed geometry and particle size

v. Free slip boundary condition at the centerline can be used to good approximation

**Future Work**

i. Independent experimental data to validate code modification

ii. In-depth analysis of the continuum limit at the centerline

iii. Investigation of mixing and segregation in 3D Cylindrical beds
References


