Using Cylindrical Coordinates in the Simulation of Dense Particle-Gas Multi-Phase Flows

Akhilesh Bakshi, Christos Altantzis, Ahmed F. Ghoniem
Center for Energy and Propulsion Research
Department of Mechanical Engineering
Massachusetts Institute of Technology, USA

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Present Study

Objectives

i. To investigate the accumulation of solid particles at the centerline for the simulation of dense particle-gas multi-phase flows using the cylindrical coordinate system

ii. To compare the quality and computational cost of simulations using the cylindrical coordinate system with those using the Cartesian Cut-Cell approach

Conclusions

i. Better prediction of void fraction with respect to experimental data using averaged solid and gas radial velocities at the centerline

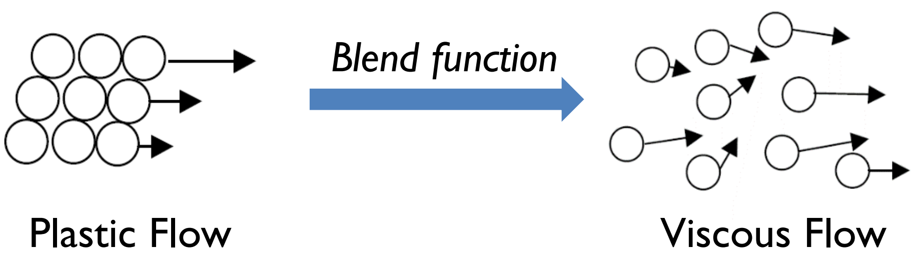
ii. Significantly less computational cost for simulations using the cylindrical coordinates as compared to those employing the Cartesian cut-cell approach

The Two-Fluid Model

- Solid and gas phases fully interpenetrating continua using generalized NS equations
- Computationally efficient
- Conservation equations coupled with constitutive relationships

$$\frac{\partial}{\partial t}(\varepsilon_{sm}\rho_{sm}\vec{v}_{sm}) + \nabla \cdot (\varepsilon_{sm}\rho_{sm}\vec{v}_{sm}\vec{v}_{sm}) = \nabla \cdot \overline{\overline{S}}_{sm} + \varepsilon_{sm}\rho_{sm}\vec{g} + \vec{I}_{gm} - \sum_{\substack{l=1 \\ l\neq m}}^{M} \vec{I}_{ml}$$

Solid Phase Stress Tensor



Interactions

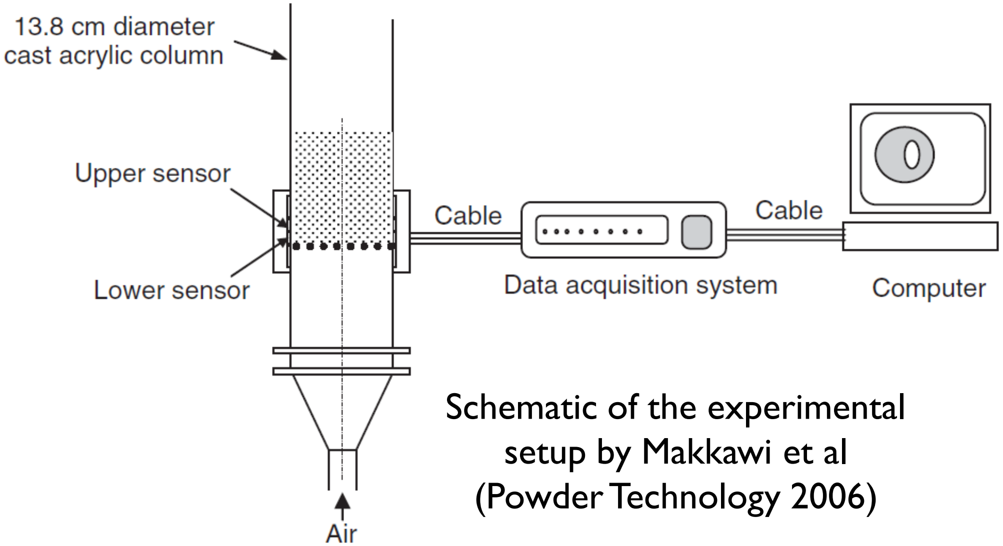
- Particle-particle and particle-gas
- Kinetic Theory of Granular Flow (KTGF)

The TFM has been implemented using **MFiX** *(Multiphase Flow with Interphase eXchanges)*

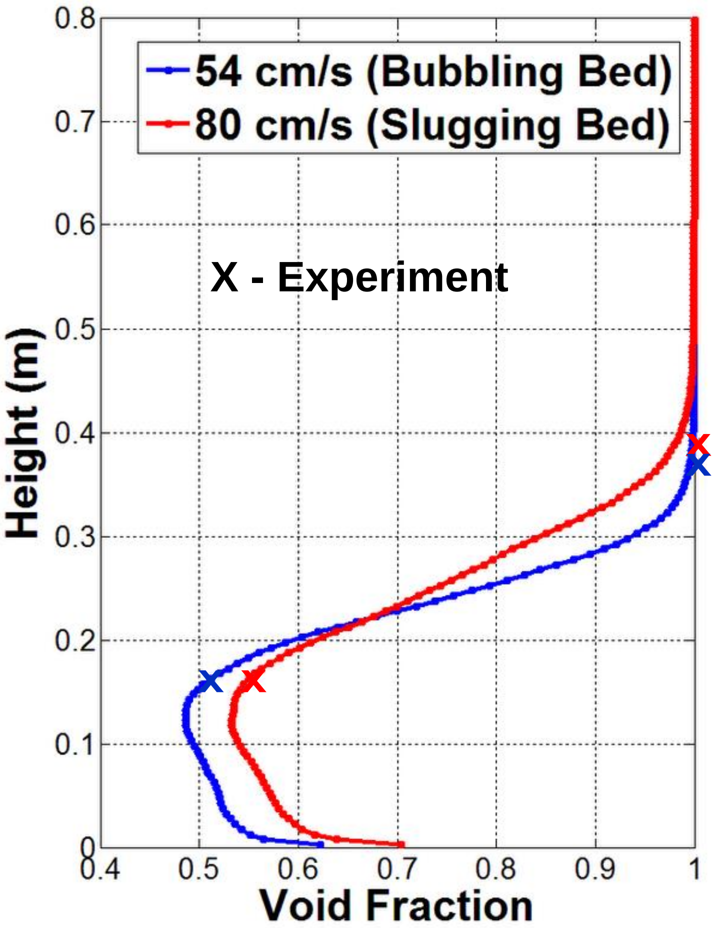
Validation of Numerical Model

Experimental Conditions

Column	D=13.8 cm, H=1.5m
Particles	Group B; d_p=350 μm, ρ_p=2500 kg/m^3
Fluidizing Gas	Air (ambient conditions)
Static Bed Height	20 cm
Measuring Level	14.3-18.1 cm



Schematic of the experimental setup by Makkawi et al (Powder Technology 2006)

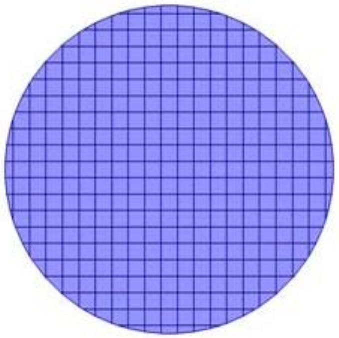


Time averaged (2-20s) void fraction versus bed height at different superficial velocities using cylindrical coordinates (18 x 160 x 12)

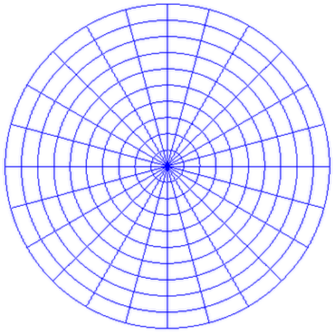
Choice of Coordinate System

i. Cartesian 2D – Only qualitative analysis
ii. Cartesian Cut-Cell 3D - Expensive !
iii. Cylindrical 3D - Accurate, Inexpensive

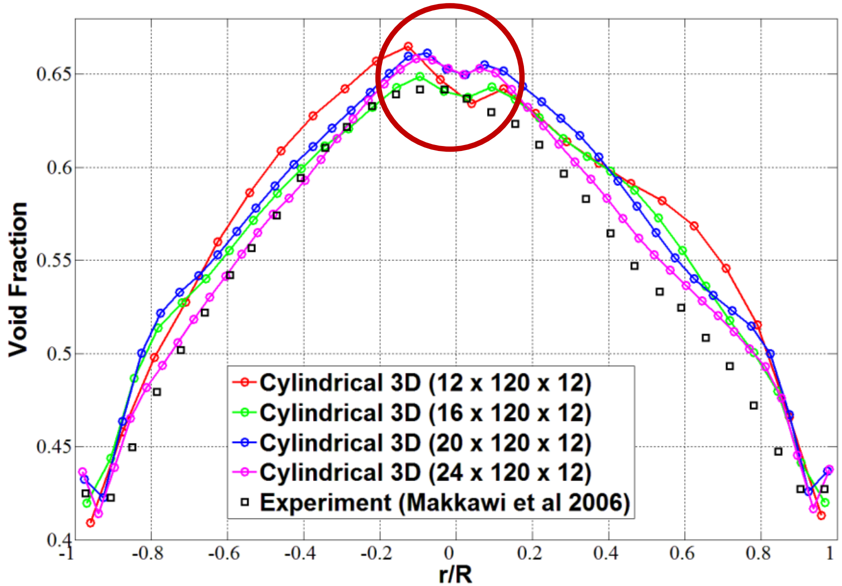
Simulations using the Cylindrical 3D coordinate system show a characteristic *dip* at the center



Cartesian Cut-Cell

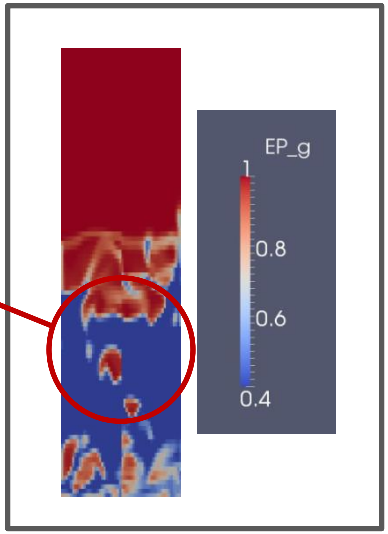


Cylindrical Coordinates



Time averaged (2-20s) void fraction at axial height 14.3-18.1 cm for different radial resolutions

Discontinuity at the center



Time instant snapshot

Centerline Boundary Conditions

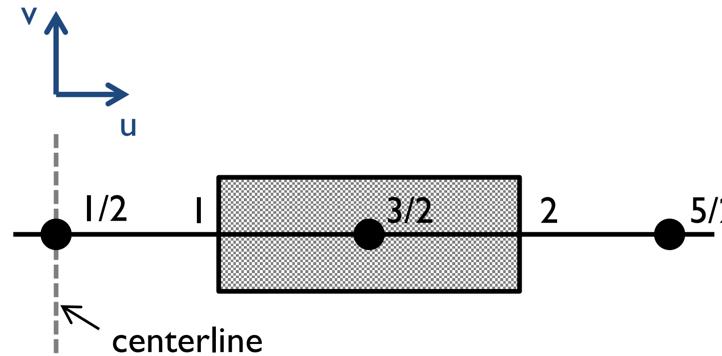
Radial Velocity

- ***No normal flow***

  => Accumulation of solid particles at center
- Required for the computation of

  (a) Convection terms

  (b) Gas-Solid Drag Force



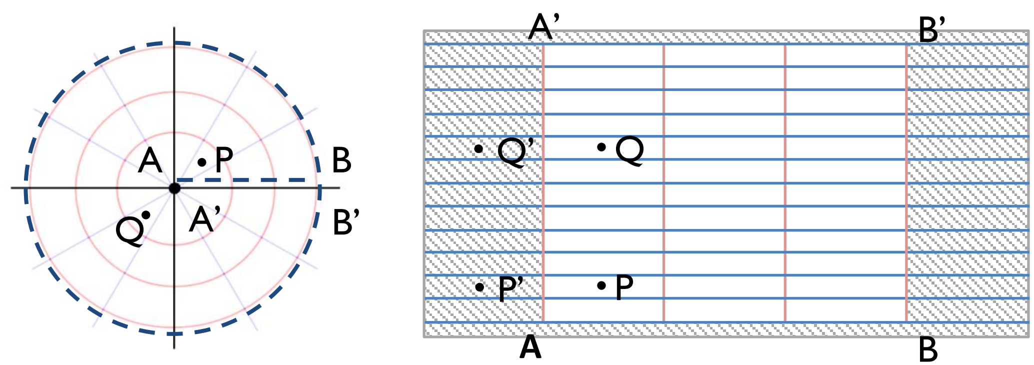
x-momentum equation control volume

$u_1 = u_1(u_{1/2}, u_{3/2})$

Axial Velocity

- ***Free slip boundary condition***
- Numerically, $v_{p'} = v_q$ and $v_{q'} = v_p$
- Error (time averaged) =

$$\left| \frac{\bar{v}_{1jk} - \bar{v}_{0j\,k+\frac{N_\theta}{2}}}{\bar{v}_{1jk}} \right| < 2\%$$



Discretization of the bed cross section for y-momentum equation

Centerline Treatment

Momentum Equations

⇩

Solids Volume Fraction Correction

⇩

Fluids Pressure Correction

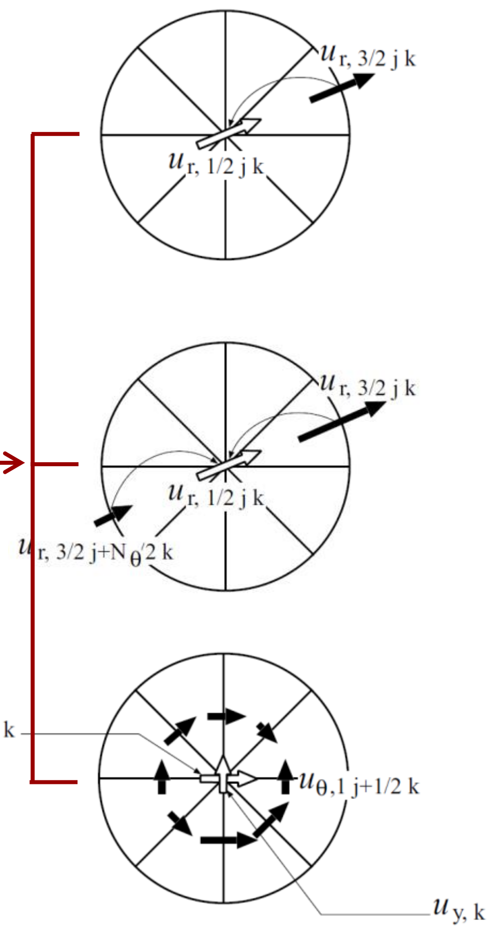
⇩

Boundary Conditions

⇩

Scalar Equations

Code Modification



Multi-Valued Formulation

$$u_{r,\frac{1}{2} j k} = u_{r,\frac{3}{2} j k}$$

Different centerline Cartesian velocity in each cell

Multi-Valued Averaging

$$u_{r,\frac{1}{2} j k} = \frac{u_{r,\frac{3}{2} j k} - u_{r,\frac{3}{2} j k+\frac{N_\theta}{2}}}{2}$$

Identical centerline Cartesian velocity in diametrically opposite cells

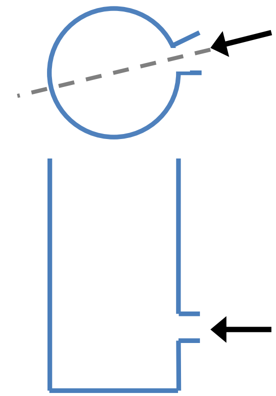
Single-Valued Averaging

$$u_{r,\frac{1}{2} j k} = \bar{u}_x cos\theta + \bar{u}_y sin\theta$$

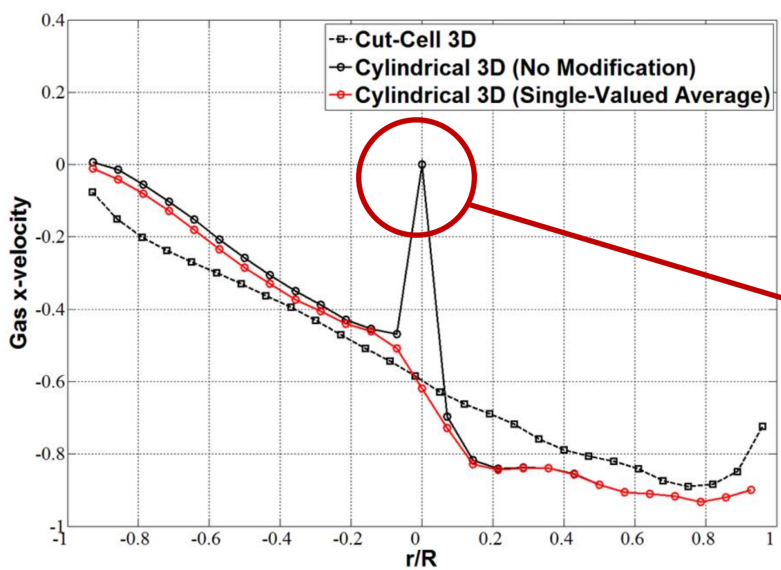
Unique centerline Cartesian velocity in all cells

Numerical Experiment

Solid + Gas injected through a side port into a cylindrical vessel

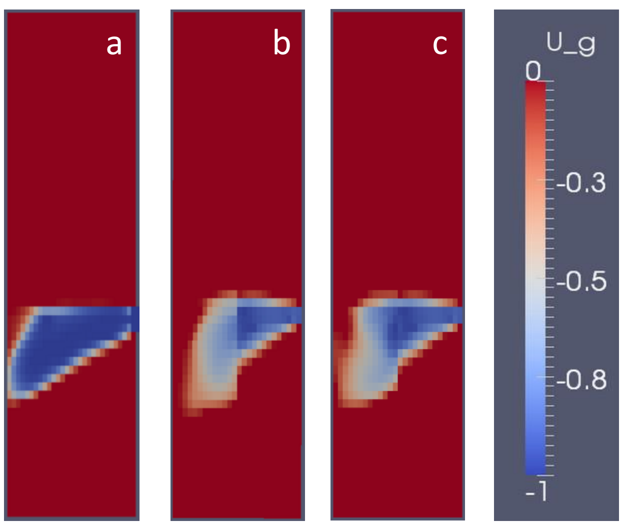


Comparison along the distributor axis



Gas x-velocity at distributor height

No normal flow boundary condition



Gas x-velocity for (a) Cut-Cell 3D (b) Cylindrical 3D (No Modification) and (c) Cylindrical 3D (Average) at t=0.10s

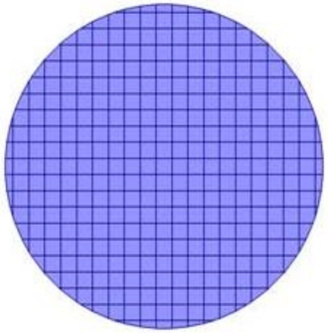
Resolution

Analysis based on the study by **Clemins (1988)**

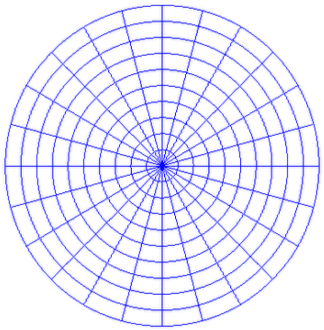
Maximum resolution based on bed, particle size

$$\frac{R}{L_m} > \overline{\alpha}\left[\frac{0.5(1-\overline{\alpha})}{|\Delta\overline{\alpha}|_{tol}}\right]^{1/2}$$

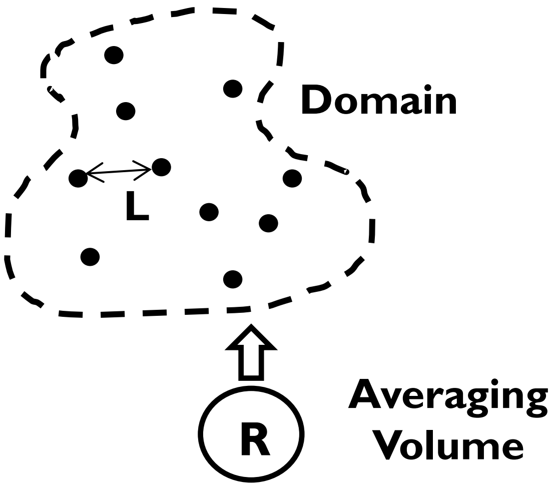
$$(n_r)^2 \times n_{theta} \times n_{axial} < constant$$

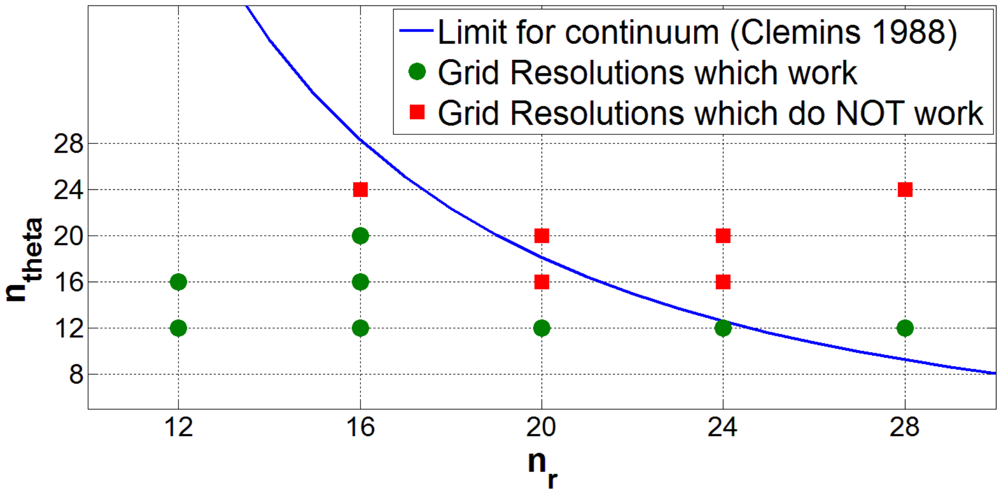


Cartesian Cut-Cell



Cylindrical Coordinates





Grid resolutions tested (axial resolution = 5 mm) with modified code using Cylindrical 3D coordinate system

Domain (Cylindrical 3D)

Grid Resolution:

- Low resolution at center for continuum (~ 4.8 mm)
- *Sufficient* resolution at wall to capture wall effects (~ 3 mm)

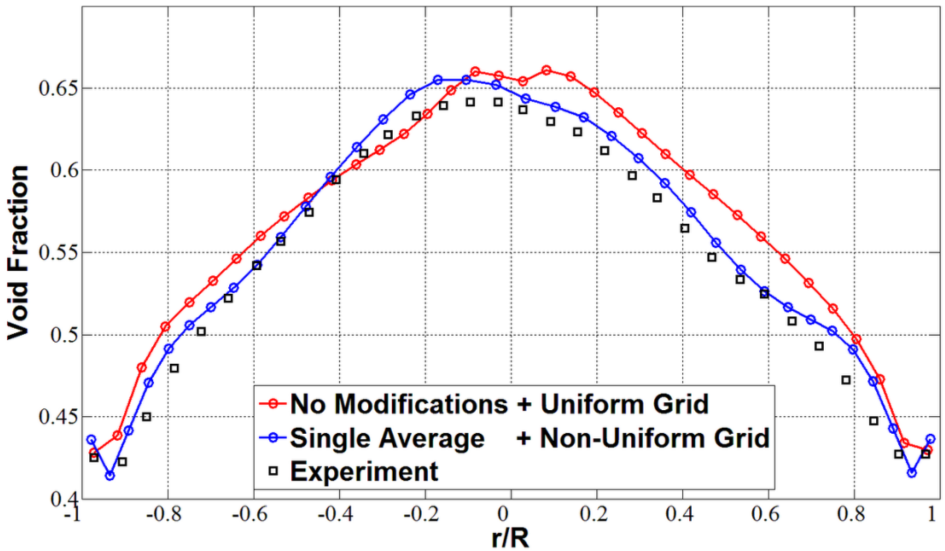
Solution:

n_{theta}	12
Δy	5 mm
Δr	3.8 mm (Uniform Grid)
	3.0 – 4.8 mm (Non Uniform Grid)

First order accurate

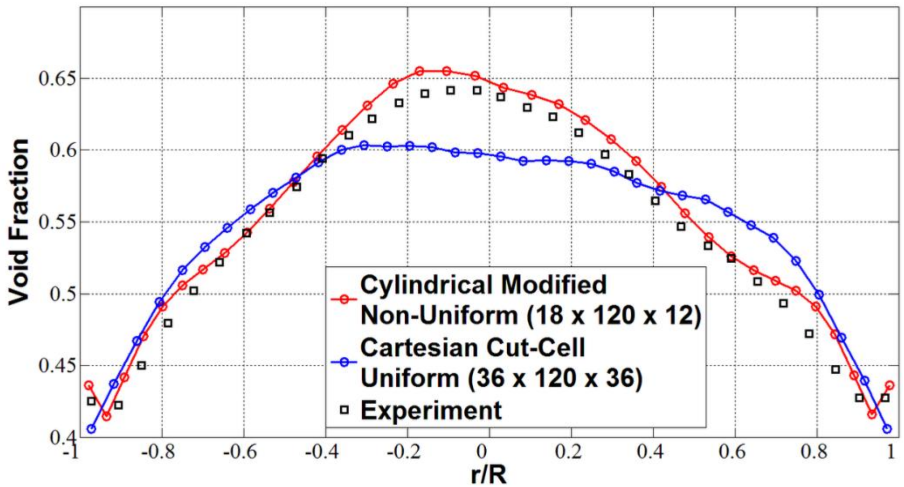
but time averaged void fraction

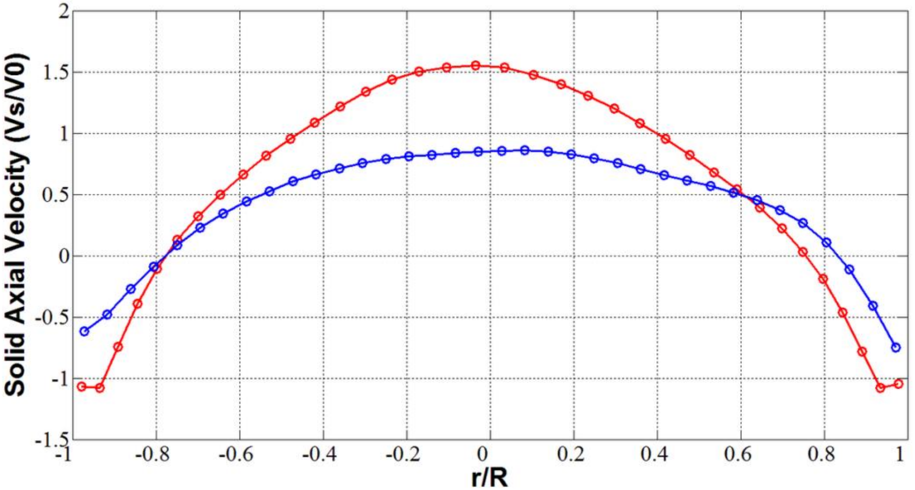
shows good match (max error < 5%)



Time-averaged (2-20s) void fraction at axial height 14.3-18.3 cm

Comparison with Cut-Cell – Bubbling Bed





Time averaged (2-20s) plots of (a) Void Fraction and (b) Solid axial velocity at axial height 14.3 – 18.1 cm

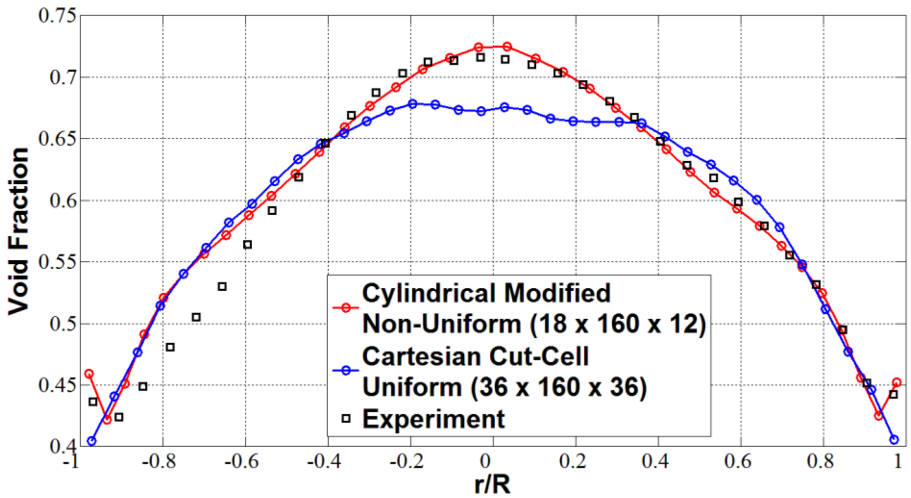
Approach	Domain	Real Time (s)	CPU Time (hr)
Cylindrical	18 x 120 x 12	20	188
Cut-Cell	36 x 120 x 36	20	1116

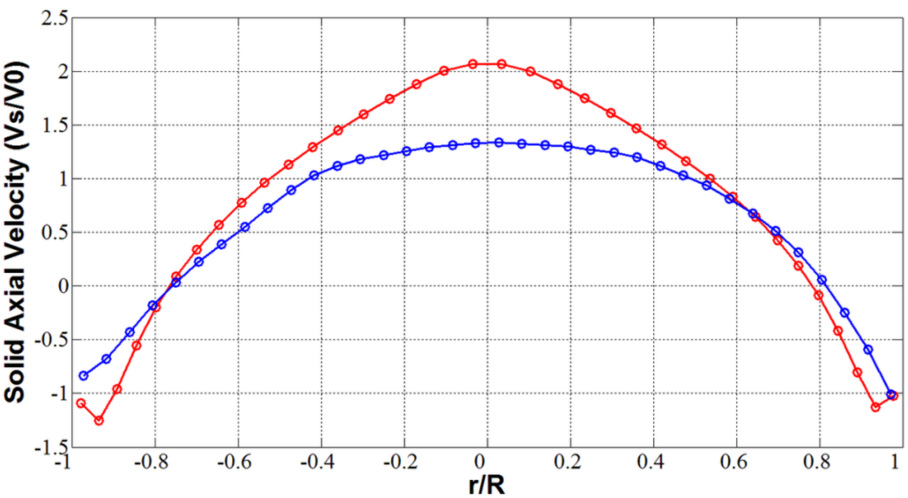
Simulating bubbling beds using the cylindrical coordinates

(a) shows **better match** with experimental data at the center as well as bed interior and

(b) costs more than **5 times less** as compared to the Cartesian Cut-Cell approach

Comparison with Cut-Cell – Slugging Bed





Time averaged (2-20s) plots of (a) Void Fraction and (b) Solid axial velocity at axial height 14.3 – 18.1 cm

Approach	Domain	Real Time (s)	CPU Time (hr)
Cylindrical	18 x 160 x 12	20	309
Cut-Cell	36 x 160 x 36	20	1442

Simulating slugging beds using the cylindrical coordinates

(a) shows **better match** with experimental data at the center as well as the interiors and

(b) costs **5 times less** as compared to the Cartesian Cut-Cell approach

Conclusions and Future Work

i. **Single-valued averaging** scheme has been used to predict the centerline gas and solid radial velocities to prevent local accumulation of solids

ii. **Non-uniform grid** employed using cylindrical coordinates; predicted time-averaged void fraction more accurate as compared to that with Cut-Cell approach

iii. Significantly **less computational cost** for simulations using the cylindrical coordinates as compared to those employing the Cartesian cut-cell approach

iv. **Upper limit** on grid resolution based on bed geometry and particle size

v. Free slip boundary condition at the centerline can be used to good approximation

Future Work

i. Independent experimental data to validate code modification

ii. In-depth analysis of the continuum limit at the centerline

iii. Investigation of mixing and segregation in 3D Cylindrical beds

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