An Arbitrary Lagrangian-Eulerian Method for the Direct Numerical Simulation of Particle-Bubble Interactions

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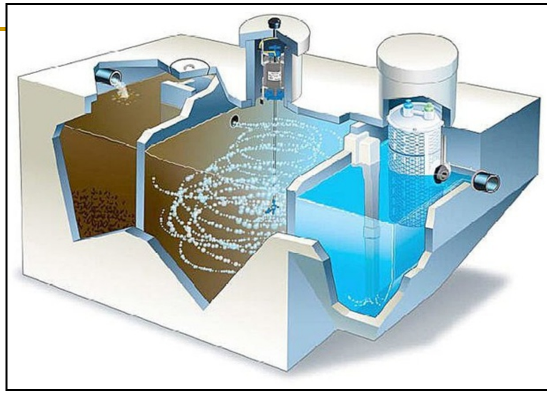
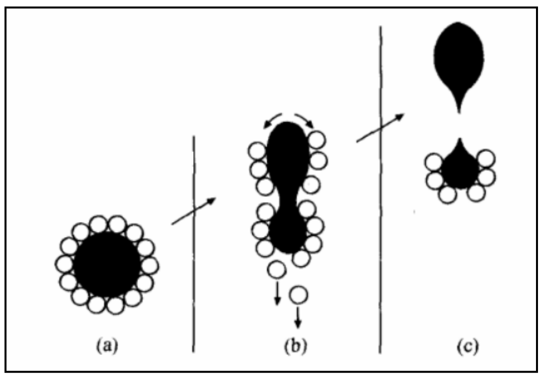
Motivation

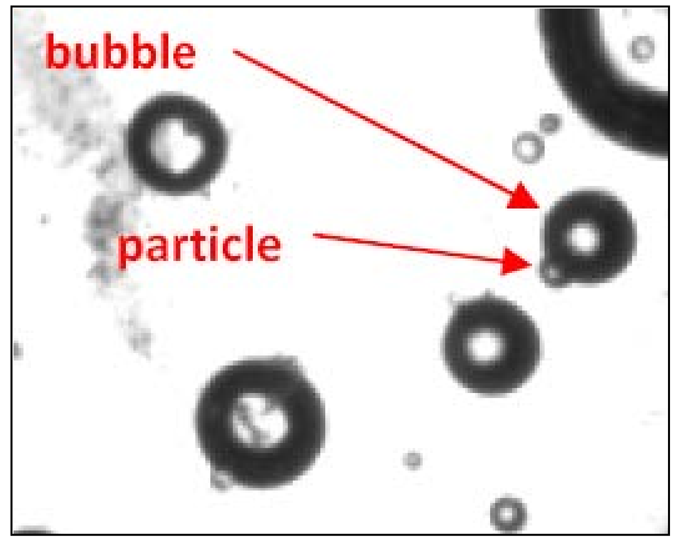


Wastewater treatment

Oil sands extraction

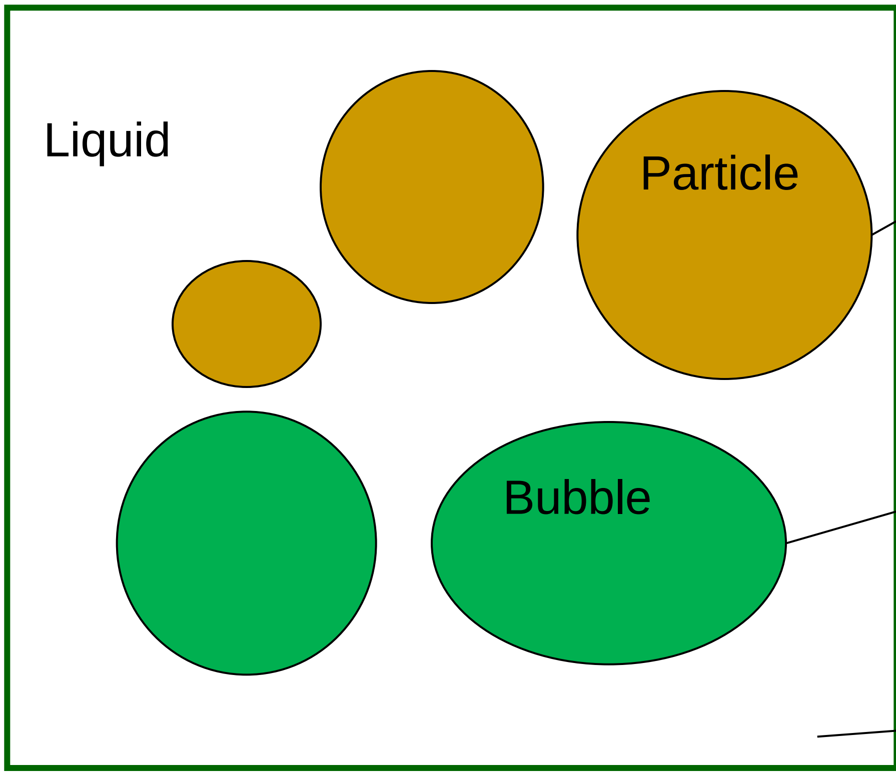
Minerals processing





Upper left: http://www.archiexpo.com/prod/norweco/domestic-wastewater-treatment-plants-59372-139048.html
Lower left: http://greenfriend.summerhost.info/activities/new%20file/29.pdf
Right: experiment of Y. Yang

Problem description



- Immersed Boundary
- Fictitious Domain
- Extended Finite Element
- **ALE**
- …

- Front tracking
- Interface capturing (VoF, level set…)
- **ALE**
- …

- Non-Newtonian fluid rheology

Arbitrary Lagrangian-Eulerian (ALE) method

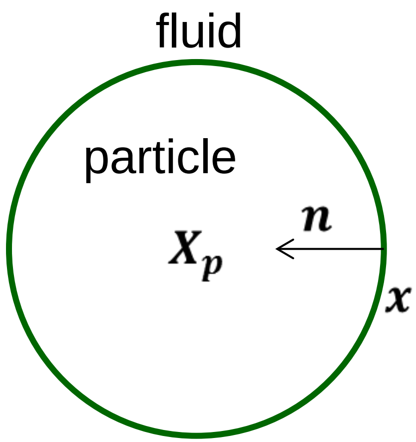
Governing equations

- Fluid

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + \boldsymbol{\tau}$$

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right) = \nabla\cdot\boldsymbol{\sigma} + \rho\boldsymbol{f}$$

$$\nabla\cdot\boldsymbol{u} = 0$$



- Particle

Hydrodynamic force (internal)

$$m_p\frac{d\boldsymbol{U}_p}{dt} = -\int_{\partial\Omega_p}\boldsymbol{\sigma}\cdot\boldsymbol{n}dS + \boldsymbol{G}_p$$

$$\boldsymbol{I}_p\cdot\frac{d\boldsymbol{\omega}_p}{dt} = -\int_{\partial\Omega_p}(\boldsymbol{x}-\boldsymbol{X}_p)\times\boldsymbol{\sigma}\cdot\boldsymbol{n}dS$$

- Bubble

$$p_bV_b = const$$

$\boldsymbol{f}$: body force on fluid

Particle:

m_p: mass

$\boldsymbol{I_p}$: inertia tensor

$\boldsymbol{X_p}$: centroid

$\boldsymbol{G_p}$: body force

$\boldsymbol{U_p}$: translational vel.

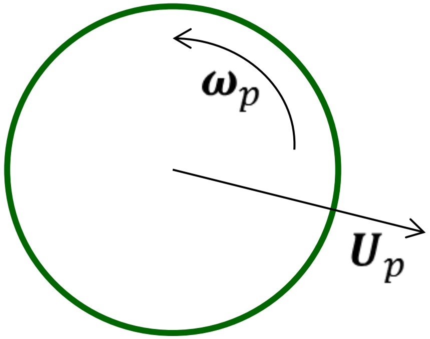
$\boldsymbol{\omega_p}$: angular vel.

Boundary conditions

Particle surface

- No-slip condition

$$\boldsymbol{u} = \boldsymbol{U}_p + \boldsymbol{\omega}_p \times (\boldsymbol{x} - \boldsymbol{X}_p)$$

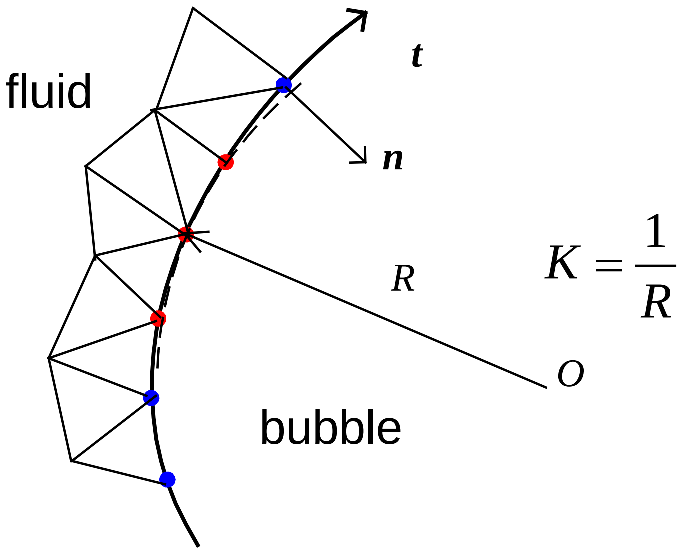


Bubble surface

- Young-Laplace eqn.

$$\boldsymbol{n} \cdot (-p\boldsymbol{I} + \boldsymbol{\tau}) = (-p_b + K\gamma)\boldsymbol{n}$$

curvature Surface tension



Non-Newtonian Rheology

- Viscoelstic fluid (Oldroyd-B)

$$\boldsymbol{\tau} = \boldsymbol{\tau}_v + \boldsymbol{\tau}_p$$

$$\boldsymbol{\tau}_v = \mu_s[\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T]$$

$$\boldsymbol{\tau}_p + \lambda_H \left[\frac{\partial \boldsymbol{\tau}_p}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{\tau}_p - \tau_p \cdot \nabla \boldsymbol{u} - (\nabla \boldsymbol{u})^T \cdot \boldsymbol{\tau}_p \right] = \mu_p[\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T]$$

τ_v: viscous stress; τ_p: polymer stress; λ_H: relaxation time
μ_s: solvent viscosity; μ_p: polymer viscosity

- Generalized Newtonian fluid

$$\boldsymbol{\tau} = \boldsymbol{\tau}(\dot{\boldsymbol{\gamma}})$$

  - Power-law fluid (shear-thinning, shear-thickening)
  - Bingham fluid (yield stress)
  - …

Moving Mesh Scheme

Mesh velocity

$$\nabla \cdot \left(k^e \nabla \boldsymbol{u}_m\right) = 0$$

Boundary conditions

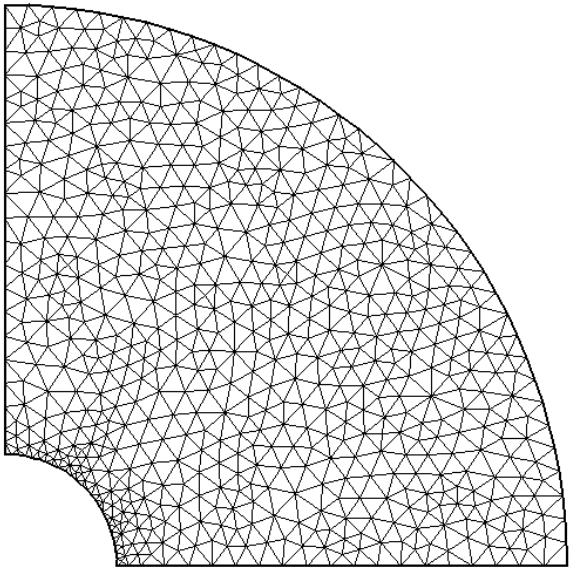
$\boldsymbol{u}_m = \boldsymbol{u}$ on bubble surface

$\boldsymbol{u}_m = \boldsymbol{U}_p$ on particle surface

Mesh position (Crank-Nicolson)

$$\boldsymbol{X}^{n+1} = \boldsymbol{X}^{n} + (\boldsymbol{u}_m^{n+1} + \boldsymbol{u}_m^{n})\frac{\Delta t}{2}$$

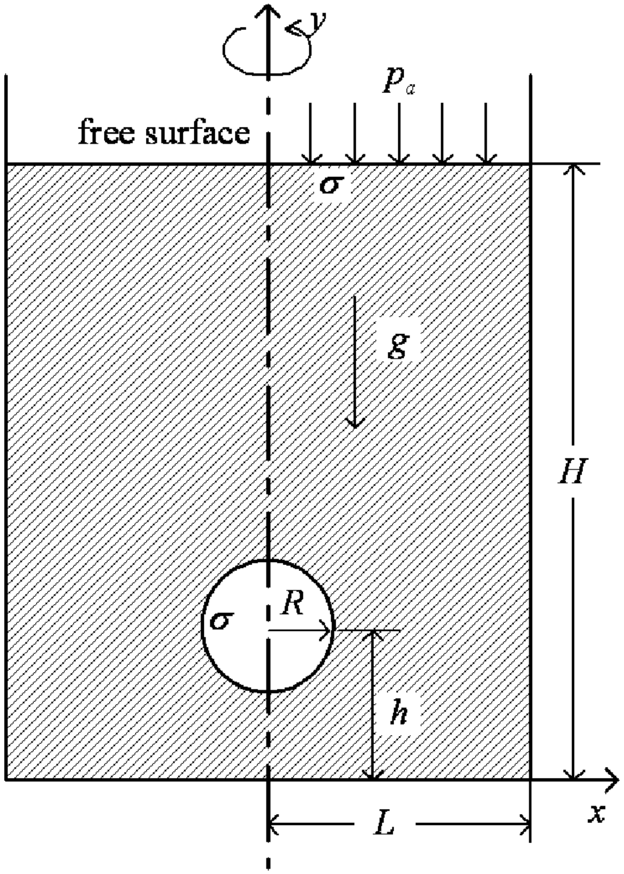
$\boldsymbol{u}_m$: mesh velocity
$\boldsymbol{u}$: fluid velocity
k^e: inverse of element volume
$\boldsymbol{X}$: mesh nodal position



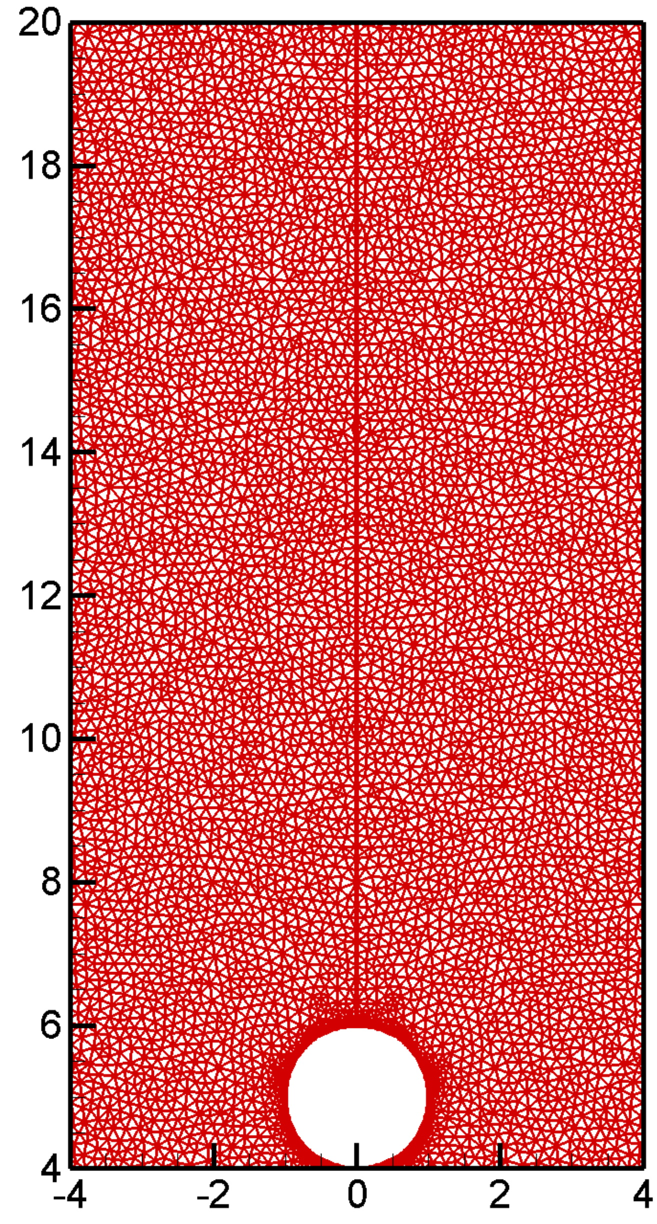
Moving mesh around an oscillating bubble (1/4 shown)

Numerical Results

Bubble Rising



L=10*R*, *H*=30*R*

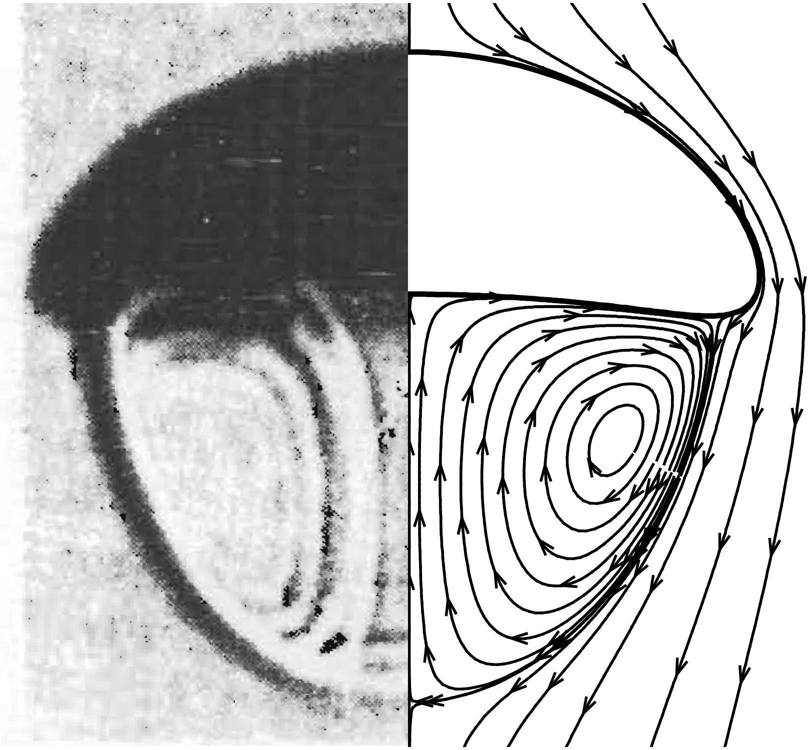


$\rho = 0.8755\times10^3\ \mathrm{Kg/m^3}$ $\sigma = 32.2\times10^{-3}\ \mathrm{J/m^2}$

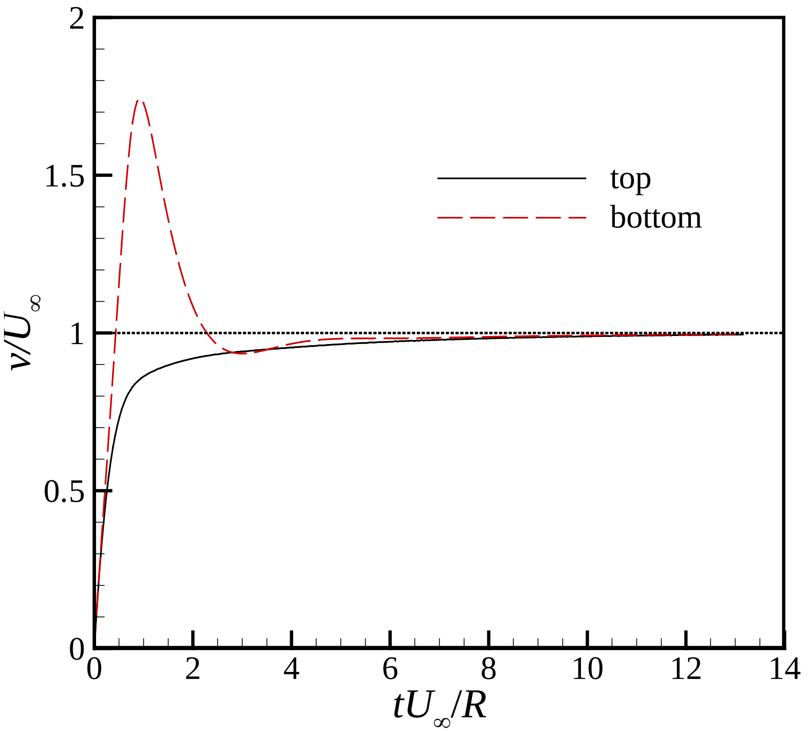
$R = 0.608\ \mathrm{cm}$ $\mu = 0.118\ \mathrm{Pa\ s}$

Experiment by Hnat & Buckmaster, *Phys. Fluids* 1976

Comparisons with Experiment



Bubble shape

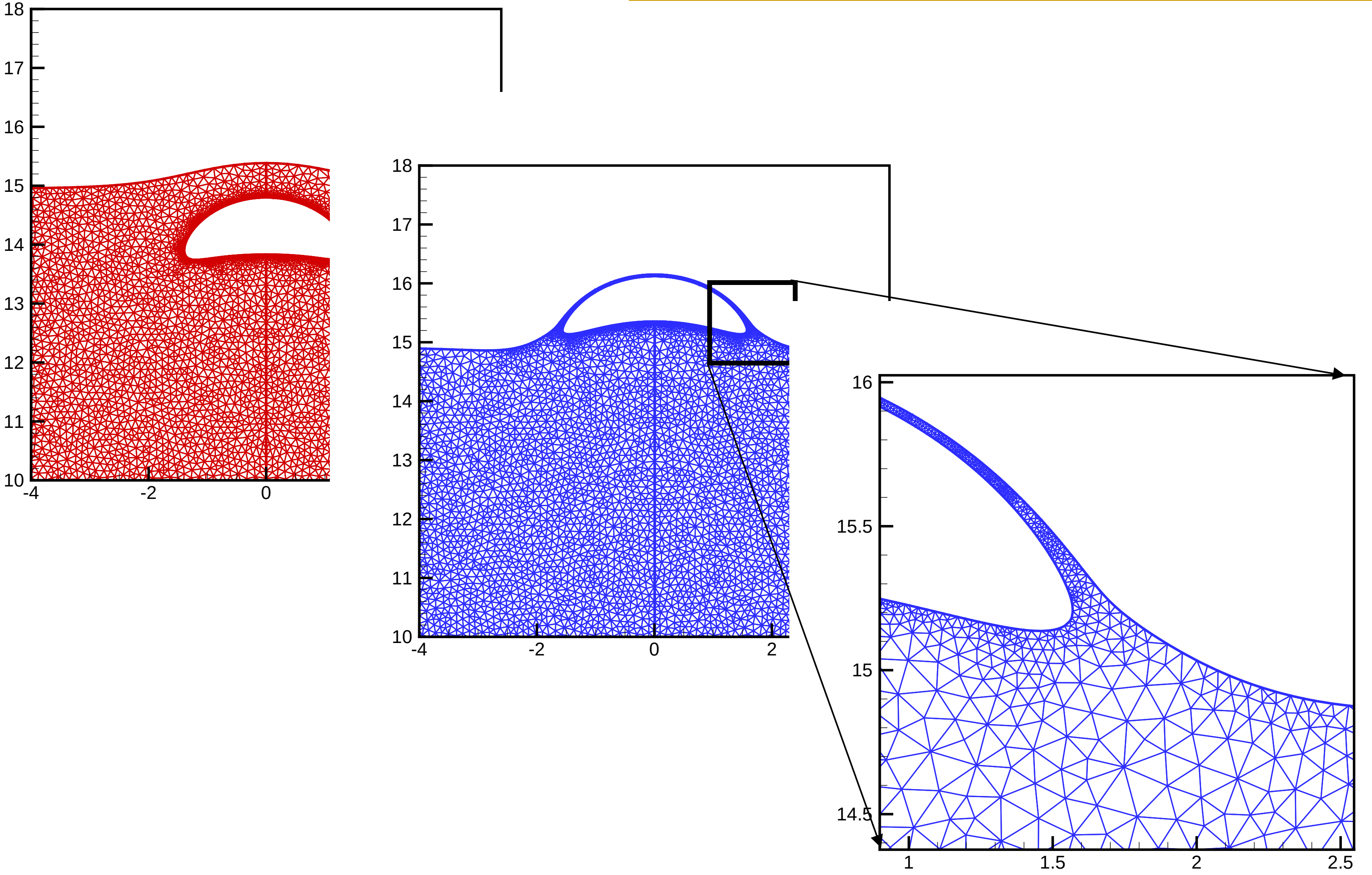


Rising velocity

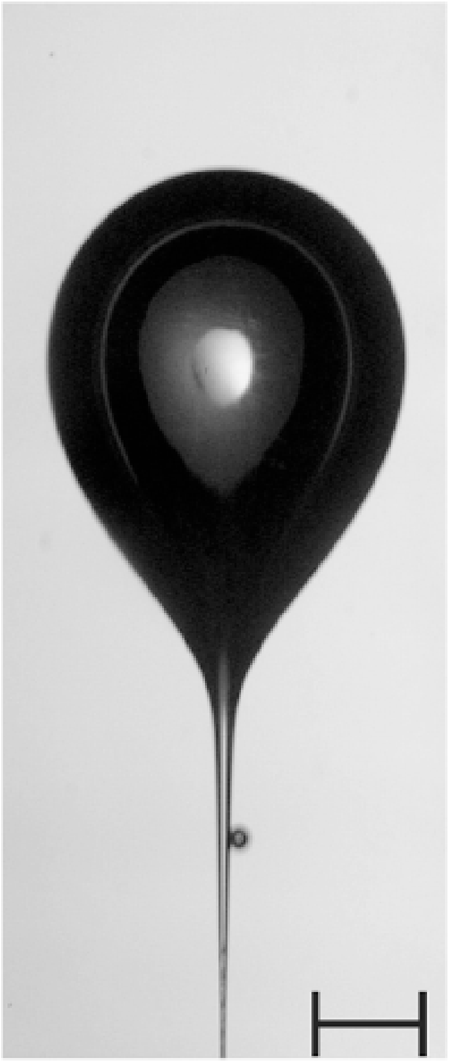
Re=9.69, We=7.64

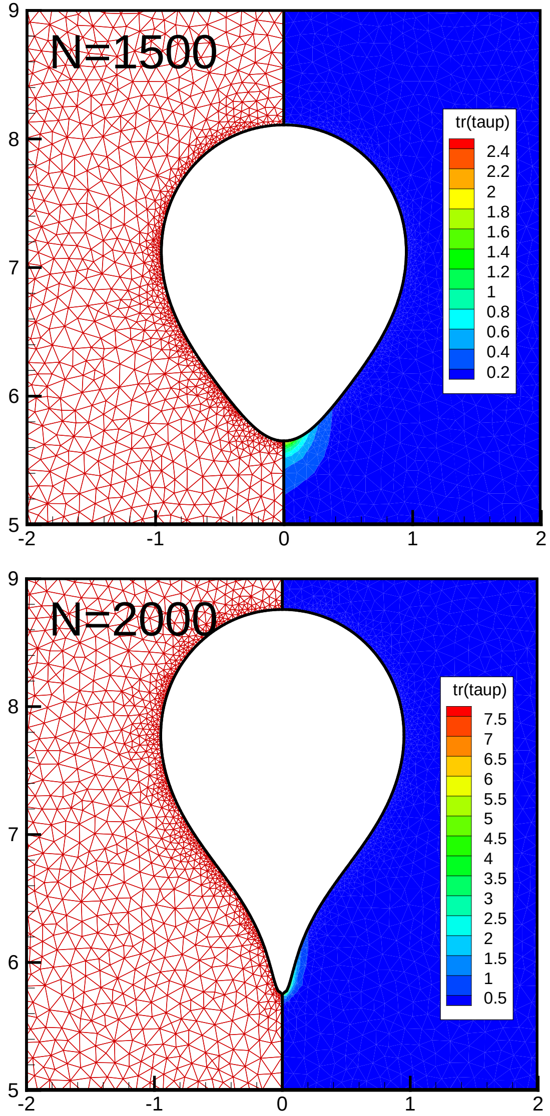
$$U_\infty(numer.) = 0.995 U_\infty(exp.)$$

When bubble approaches surface

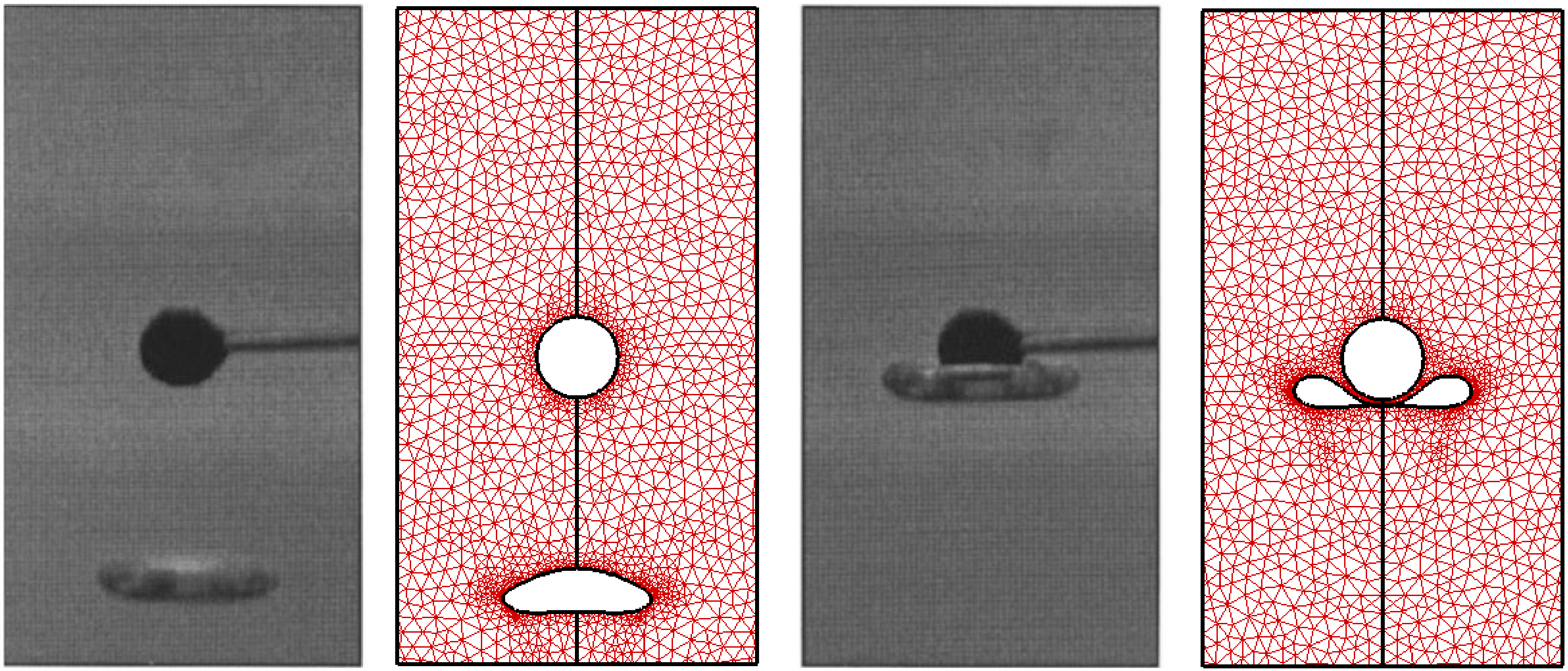


Rising bubble in a viscoelastic liquid





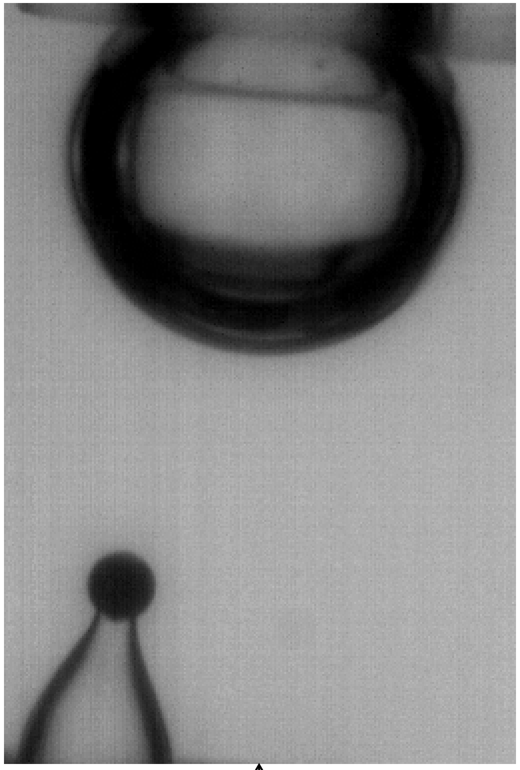
Particle-bubble collision



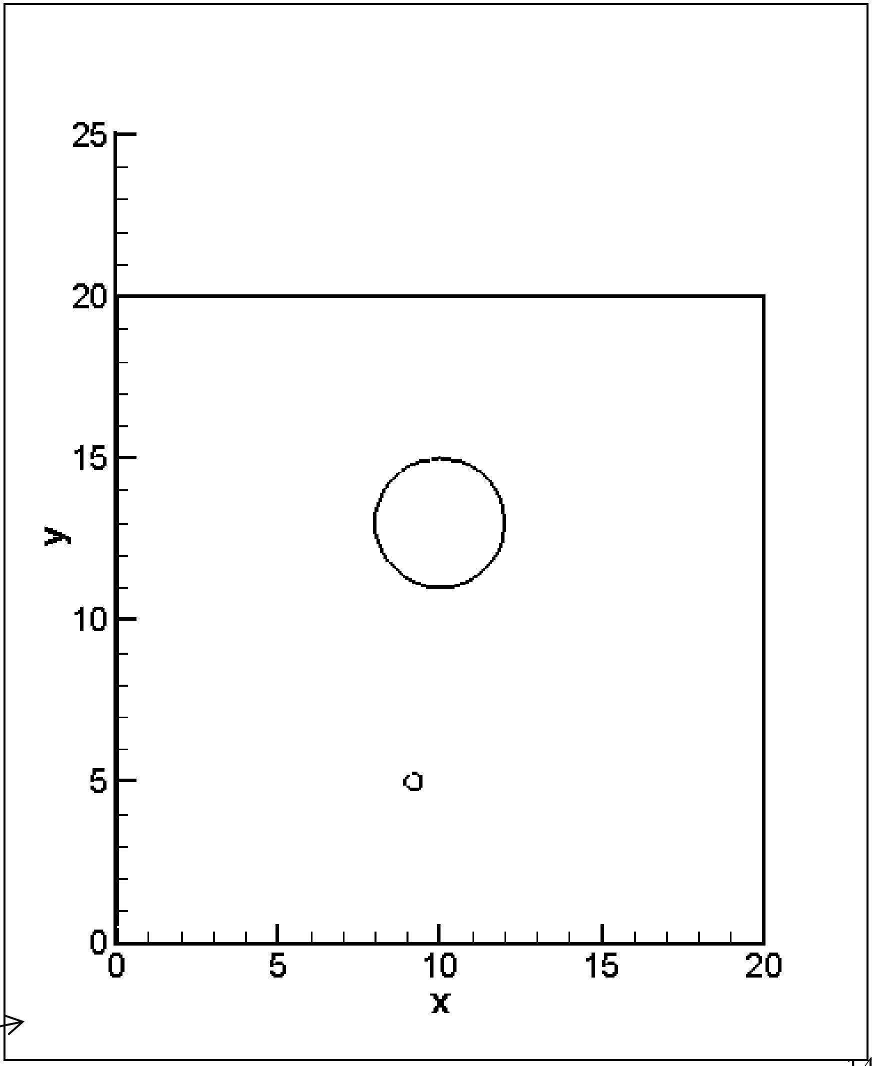
Collision between a rising bubble and a stationary particle
d_p=0.5cm, d_b=1.2cm, 80 wt% glycerin solution

(Experimental by Hong, Fan & Lee, *Int J. Multiphase Flow*,1999)

Oblique collision
(qualitative comparison)

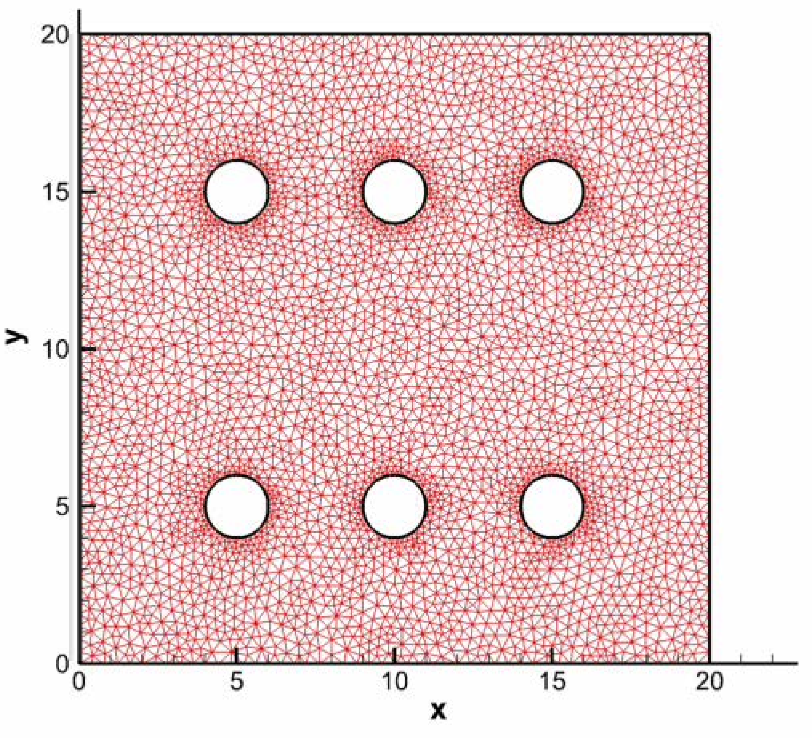


Experiment by Y.Yang (Re =80)

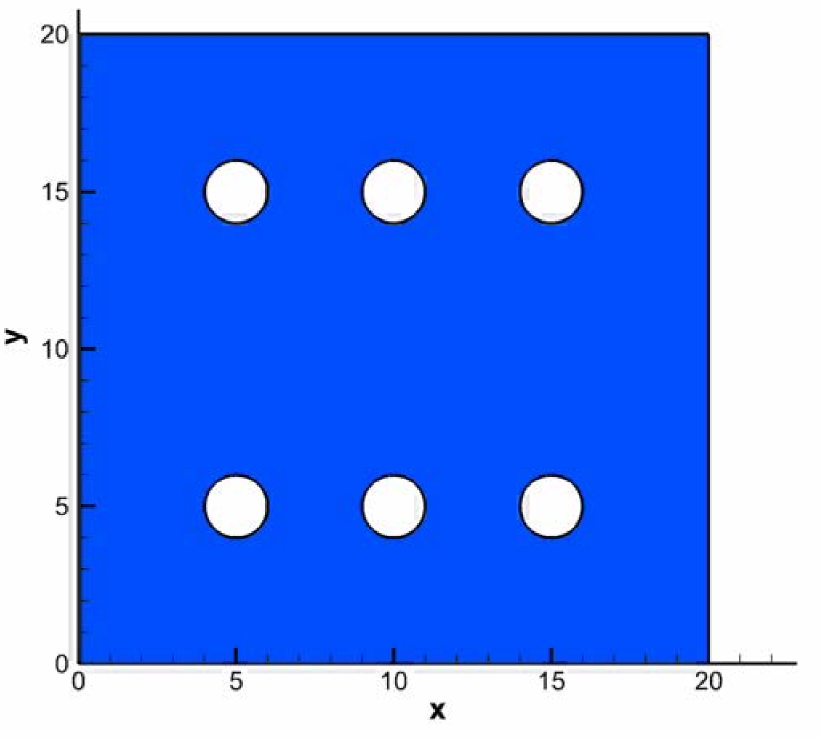


ALE, 2D planar, (Re =20)

Multiple particle-bubble interaction (2D planar)

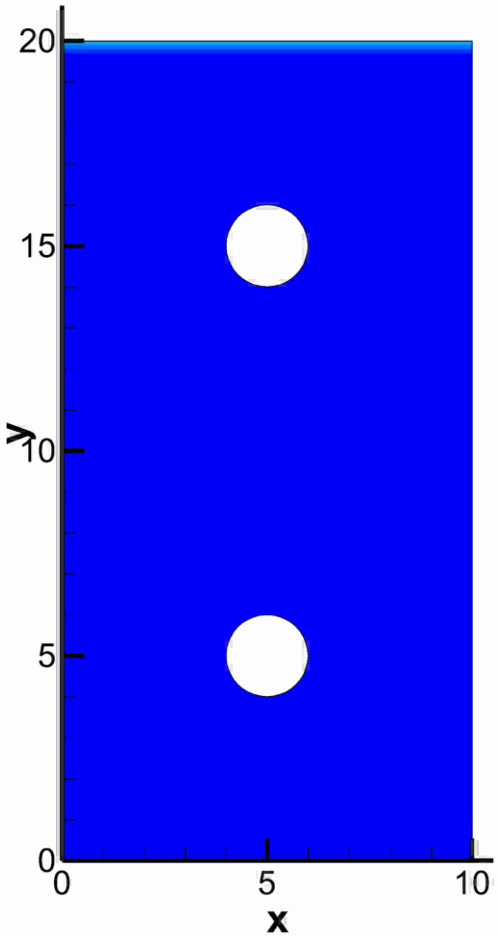


Mesh

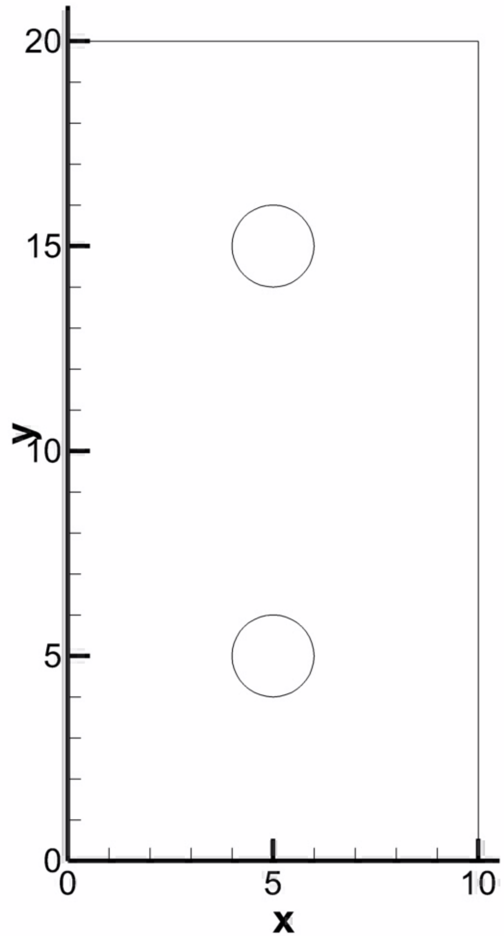


pressure

Head-on collision



pressure
(w.o. hydrostatic)



velocity

Input:

$$\rho_p = 2, \rho = 1$$

$$g_y = -1, \mu = 1$$

$$r_p = r_b = 1$$

$$\sigma = 1$$

Output:

$$V \approx 0.25$$

$$\mathrm{Re} = \frac{\rho V r}{\mu} \approx 0.25$$

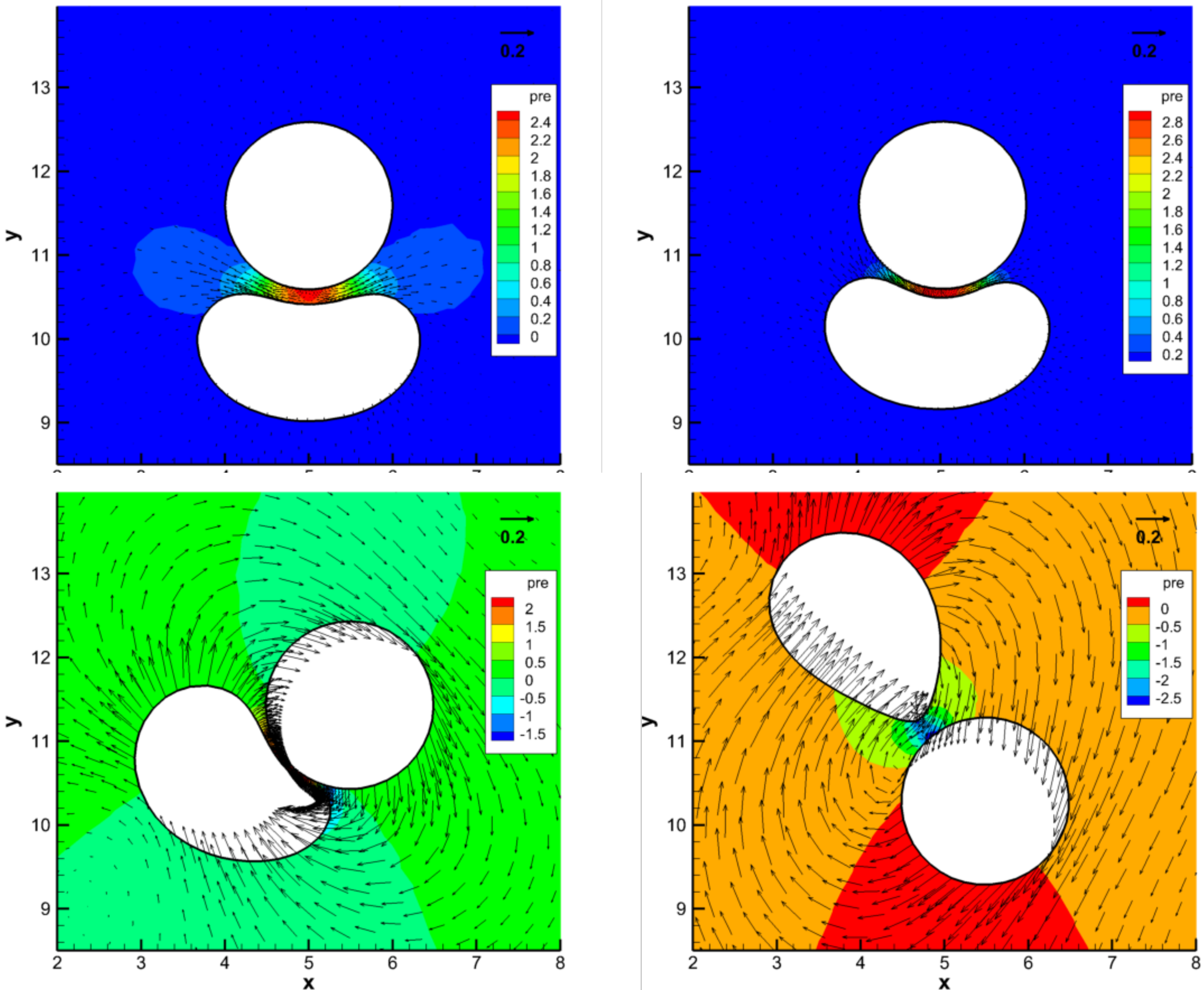
$$Ca = \frac{\mu V}{\gamma} \approx 0.25$$

$$Bo = \frac{\rho g r^2}{\gamma} = 1$$

Pressure & velocity

t=23.48

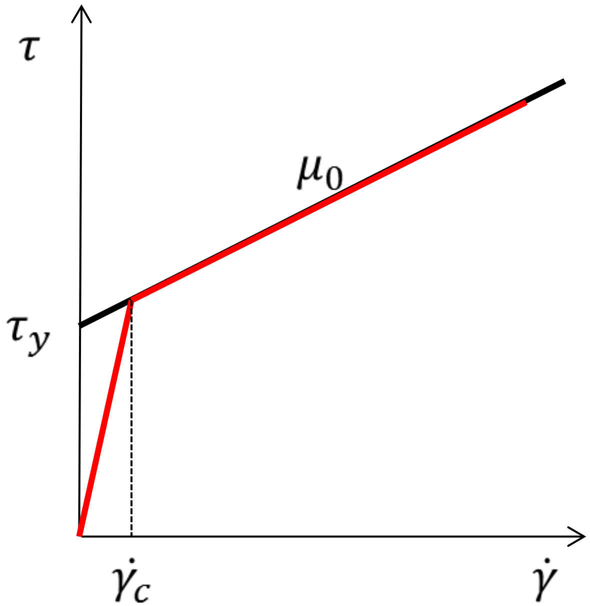
t=33.48



t=43.46

t=49.72

Bingham fluid



$$\tau = \sqrt{\frac{1}{2}\boldsymbol{\tau}:\boldsymbol{\tau}} \qquad \dot{\gamma} = \sqrt{\frac{1}{2}\dot{\boldsymbol{\gamma}}:\dot{\boldsymbol{\gamma}}}$$

$$\dot{\boldsymbol{\gamma}} = \nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T$$

Bingham model

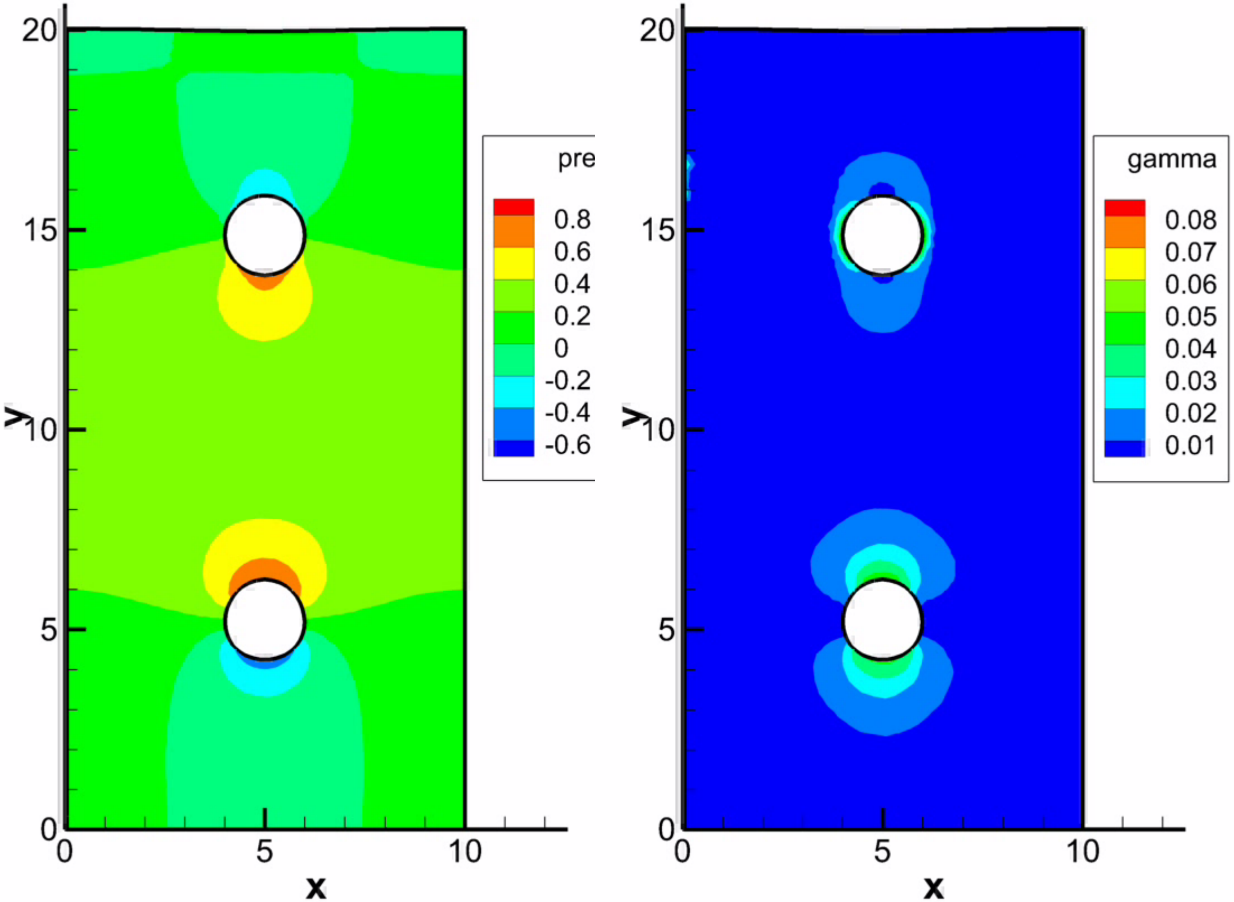
$$\begin{cases} \dot{\gamma} = 0, & \tau \leq \tau_y \\ \tau = \tau_y + \mu_0 \dot{\gamma}, & \tau > \tau_y \end{cases}$$

A simple regularization

↓

$$\mu(\dot{\gamma}) = \begin{cases} \mu_0 + \dfrac{\tau_y}{\dot{\gamma}_c\left(2 - \dfrac{\dot{\gamma}}{\dot{\gamma}_c}\right)}, & \dot{\gamma} \leq \dot{\gamma}_c \\ \mu_0 + \dfrac{\tau_y}{\dot{\gamma}}, & \dot{\gamma} > \dot{\gamma}_c \end{cases}$$

$$\boldsymbol{\tau} = \mu \dot{\boldsymbol{\gamma}}$$



pressure

Rate-of-strain

$\tau_y = 0.1$

$\dot{\gamma}_c = 0.01$

Yielded region:

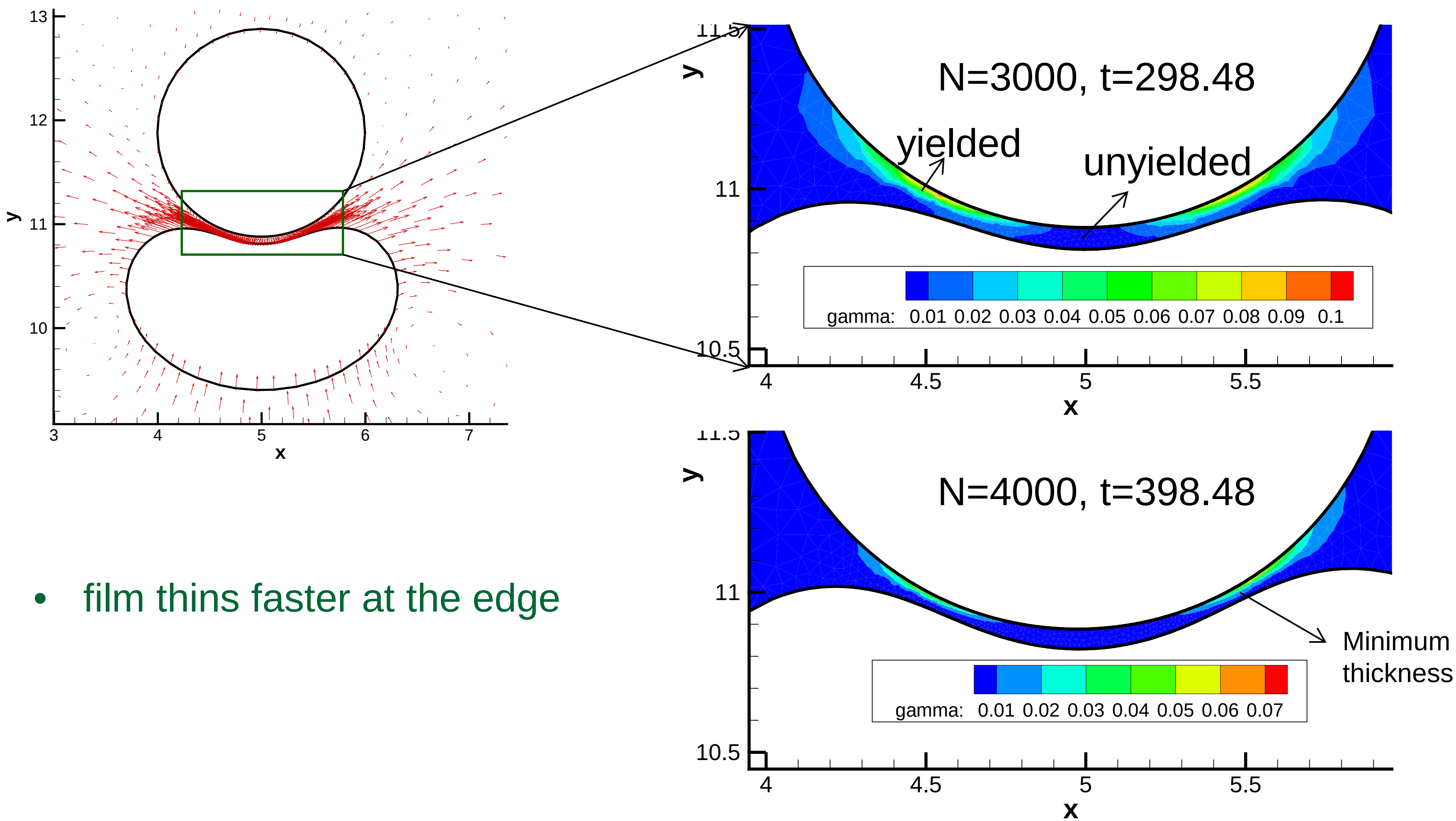
$\dot{\gamma} \geq 0.01$

Bingham number:

$$Bn = \frac{\tau_y R}{\mu_0 V} \sim O(1)$$

- Longer contact time due to unyielded region

Film drainage in Bingham fluid



- film thins faster at the edge

Summary

- Advantages of ALE
  - Accurate tracking of moving boundaries
  - Easy incorporation of matching conditions on moving boundaries
- Disadvantages
  - Topological transition of moving boundaries
  - 3D bubble surface mesh generation and adaptation
- An optimal direction
  - Hybrid ALE-interface capturing method

Thank you!