Numerical Simulations Studying Size Segregation in a Rotating Drum

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Motivation

Investigate the effect on granular flow of a distribution of particle sizes inside the bed thereby, investigate how well theories and correlations developed for mono-dispersed perform for more complex poly-disperse systems





Granular Flows

- Particulate materials exist in many industries including metallurgical, chemical, food, pharmaceuticals, ceramic
- It is sometimes preferable to separate components from mixture whereas, sometimes mixing produces the final product
- Rotary drum is often used as granular mixer, gas/solid reactor, dryer





- Focus on rolling regime which is common for mixing purposes
- Granular bed in rolling mode can divided into two distinct regions:
 - Thin active layer
 - Larger passive layer



Source: Alizadeh et al., AIChE J., 59(6), 2013



Radioactive tracing was used to collect data in plexi-glass drum





Case	Туре	Rotational Speed (RPM)
MDI	Monodisperse (3mm)	5.4
MD2	Monodisperse (3mm)	11.6
PDI	Polydisperse 3mm – 15%, 4mm – 35%; 5mm – 35%; 6mm – 15%	5.4
PD2	Polydisperse 3mm – 15%, 4mm – 35%; 5mm – 35%; 6mm – 15%	11.6

Composition of poly-disperse systems was chosen to ensure rapid segregation





- Particle density, $\rho_s = 2500 kg / m^3$
- Gas density, $\rho_g = 1.18 kg / m^3$
- Gas viscosity, $\mu_g = 1.8 \times 10^{-5} Pa.s$
- Drum diameter, $D = 24 \, cm$
- Drum Length, L=36 cm
- ▶ Bed height, 35% of volume





Model setup in STAR-CCM+

- Algebraic model for granular temperature
- Fluid particle drag modelled using Gidaspow drag
- Inter-particle drag modelled using Gera-Syamlal drag
- Particle kinetic viscosity modelled using Gidaspow model
- Frictional regime modelled using Schaeffer model
- Coefficient of restitution, e = 0.9
- Maximum particle volume fraction set at 0.624



Governing Equations

Continuity

$$\frac{\partial}{\partial t}\alpha_k\rho_k + \nabla \bullet \alpha_k\rho_k u_k = 0$$

Fluid Momentum

$$\frac{\partial}{\partial t}\alpha_k\rho_ku_k + \nabla \bullet \alpha_k\rho_ku_ku_k = -\alpha_k\nabla p + \alpha_k\rho_kg + \nabla \bullet \alpha_k\tau_k + F_I$$

Solid Momentum

$$\frac{\partial}{\partial t}\alpha_{s}\rho_{s}u_{s} + \nabla \bullet \alpha_{s}\rho_{s}u_{s}u_{s} = -\alpha_{s}\nabla p - \nabla p_{s} + \alpha_{s}\rho_{s}g + \nabla \bullet \alpha_{s}\tau_{s} + F_{I}$$



Granular Temperature formulation

 Granular temperature is calculated by an algebraic relation derived by assuming local equilibrium between production and dissipation of fluctuating energy.

$$\begin{split} \sqrt{\theta} &= \begin{cases} \frac{-K_{1}\varepsilon_{s}D_{ii} + \sqrt{K_{1}^{2}D_{ii}^{2}\varepsilon_{s}^{2} + 4K_{4}\varepsilon_{s}\left[K_{2}D_{ii}^{2} + 2K_{3}D_{ij}D_{ij}\right]}{2\varepsilon_{s}K_{4}} \end{cases} \\ K_{1} &= 2\rho_{s}g_{0}(1+e) \\ K_{2} &= \frac{4d_{s}\rho_{s}\varepsilon_{s}g_{0}(1+e)}{3\sqrt{\pi}} - \frac{2}{3}K_{3} \\ K_{3} &= \frac{\rho_{s}d_{s}}{2} \left\{ \frac{\sqrt{\pi}}{3(3-e)} [0.5(3e+1) + 0.4(1+e)(3e-1)\varepsilon_{s}g_{0}] + \frac{8\varepsilon_{s}g_{0}(1+e)}{5\sqrt{\pi}} \right\} \\ K_{4} &= \frac{12(1-e^{2})\rho_{s}g_{0}}{d_{s}\sqrt{\pi}} \end{aligned}$$

Strain rate tensor D_{ii}



Kinetic theory stress tensor

$$S_{s} = \left[-P_{s} + \left(\xi_{s} - \frac{2}{3}\mu_{s}\right)\nabla \cdot u_{s} \right]I$$
$$P_{s} = \rho_{s}\varepsilon_{s}\theta + P_{s}^{C}$$
$$P_{s}^{C} = 2\rho_{s}\varepsilon_{s}^{2}\theta g_{0}(1+e) \qquad \text{(Lun et al.)}$$

Solid pressure,

- Collisional solid pressure,
- Particle shear viscosity,

Particle kinetic viscosity,

$$\mu_{s} = \frac{4}{5} \varepsilon_{s}^{2} \rho_{s} d_{s} g_{0} (1+e) \sqrt{\frac{\theta}{\pi}} + \mu_{s}^{K}$$

$$\mu_{s}^{K} = \frac{10 \rho_{s} d_{s} \sqrt{\pi \theta}}{96(1+e)g_{0}} \left[1 + \frac{4}{5} (1+e)g_{0} \varepsilon_{s} \right]^{2} \qquad \text{(Gidaspow et al.)}$$

Particle bulk viscosity,

 $\mu_s = \frac{4}{3} \varepsilon_s^2 \rho_s d_s g_0 (1+e) \sqrt{\frac{\theta}{\pi}} \qquad \text{(Lun et al.)}$



Frictional stress tensor – Schaeffer

$$S_{s}^{f} = \begin{bmatrix} -P_{s}^{f} + \left(\xi_{s}^{f} - \frac{2}{3}\mu_{s}^{f}\right)\nabla \cdot u_{s} \end{bmatrix} I$$

Frictional Solid pressure,
$$P_{s}^{f} = 10^{25} \left(\varepsilon_{s} - \varepsilon_{s}^{\max}\right)^{10} \qquad \varepsilon_{s} > \varepsilon_{s}^{\max}$$
$$= 0 \qquad \varepsilon_{s} \le \varepsilon_{s}^{\max}$$
Frictional viscosity,
$$\mu_{s}^{f} = \min\left(\frac{P_{s}^{f}\sin\phi}{\sqrt{4I_{2D}}}, \mu_{s,\max}^{f}\right) \qquad \varepsilon_{s} > \varepsilon_{s}^{\max}$$
$$= 0 \qquad \varepsilon_{s} \le \varepsilon_{s}^{\max}$$

$$I_{2D} = \frac{1}{6} \left[\left(D_{s,11} - D_{s,22} \right)^2 + \left(D_{s,22} - D_{s,33} \right)^2 + \left(D_{s,33} - D_{s,11} \right)^2 \right] + D_{s,12}^2 + D_{s,23}^2 + D_{s,31}^2$$

 $\xi_s^f = 0$ Particle bulk viscosity,





Velocity vectors in transverse plane of drum for MD2





Velocity vectors in transverse plane of drum for PD2





Velocity vectors in transverse plane of drum for MD2





Stream wise velocity profile in transverse plane of drum along x = 0







Results – Active layer thickness

Case	Experiments	Simulations
MDI	2.4	2.9
PDI	2.6	2.7
MD2	3.3	2.9
PD2	3.5	3.2



Results – PD1



Results – PD2





Conclusions

- Qualitative trends in velocity profiles and void fraction distribution are captured
- Active layer thickness predicted reasonably well except for MDI
- Small difference between velocity profiles of polydispersed and mono-dispersed cases
- Small particles exist in core (3 mm), larger particles (5 and 6 mm) surround them while ones with 4 mm are spread across whole volume

