Numerical simulation of polydisperse gas-particle flow in a vertical riser using a size-velocity quadrature-based moment method

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Tasks accomplished in FY14:

Quadrature-based moment method (QBMM)

- Advanced library to manage moments in CFD context, based on Hash-Table method
- Comprehensive moment inversion library (univariate/multivariate), including QMOM, EQMOM, TP, CQMOM, ECQMOM
- New explicit disperse-phase volume fraction limit scheme used in general QBMM
- New high-order moment kinetic flux algorithm to improve accuracy of continuous QBMM (EQMOM)

Polydisperse gas-particle flow

- Variable-particle-size model with size-conditioned particle velocity, based on EQMOM
- Polydisperse particle collision model, including granular pressure term

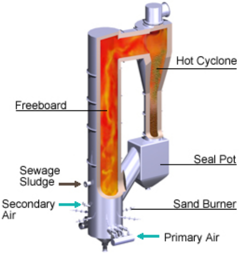
Outline

1. Introduction
2. Governing Equations for Polydisperse Gas Particle Flows
3. Numerical Methods
   - Moment inversion
   - Kinetic flux and forces
   - Polydisperse collision model
4. Test Case: Wall-bounded Vertical Riser
5. Summery

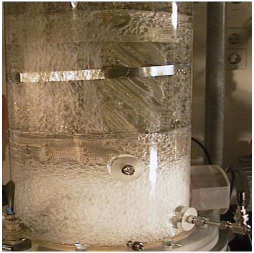
Motivation

Polydisperse multiphase flows: continuous phase and disperse phase

Fluidized Bed



Bubble column



Fuel Injection

In many commonly encountered applications

- polydispersity (e.g., size, density, shape) is present
- "size" and velocity of disperse phase are closely coupled

Thus, to accurately model polydisperse multiphase flows, joint number density function of "size" and velocity has to be properly described

Existing models for polydisperse bubbly flows

Euler-Lagrange Models

- Discrete Element Method (DEM)

  Limitation: Computationally expensive for industrial applications

Euler-Euler Models

- Population Balance Equation (PBE) carried by fluid velocity

  Limitation: Spatial fluxes do not depend on size
- Class method with separate class velocities

  Limitation: Computationally expensive for continuous size distribution
- Direct Quadrature Method of Moments (DQMOM) with a multi-fluid model

  Limitation: Weights and abscissas are not conserved quantities

Objective

Develop a robust and accurate moment-based polydisperse flow solver that incorporates microscale physics with reasonable computation cost

Governing equations for polydisperse gas-particle flow

Gas phase: Continuity and momentum transport equations

$$\frac{\partial}{\partial t}\rho_{\mathrm{g}}\varepsilon_{\mathrm{g}} + \nabla\cdot\rho_{\mathrm{g}}\varepsilon_{\mathrm{g}}\mathbf{U}_{\mathrm{g}} = 0$$

$$\frac{\partial}{\partial t}\rho_{\mathrm{g}}\varepsilon_{\mathrm{g}}\mathbf{U}_{\mathrm{g}} + \nabla\cdot\rho_{\mathrm{g}}\varepsilon_{\mathrm{g}}\mathbf{U}_{\mathrm{g}}\otimes\mathbf{U}_{\mathrm{g}} = \nabla\cdot\varepsilon_{\mathrm{g}}\boldsymbol{\tau}_{\mathrm{g}} - \nabla p + \rho_{\mathrm{g}}\varepsilon_{\mathrm{g}}\mathbf{g} + \mathbf{M}_{\mathrm{lb}}$$

Particle phase: Kinetic equation for joint size-velocity NDF $f(\xi, \mathbf{v})$

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{x}} + \frac{\partial}{\partial \mathbf{v}}\cdot f\mathbf{A} = \mathbb{S}$$

where $\mathbf{A}$ represents acceleration due to forces acting on each particle, $\mathbb{S}$ represents other possible source terms (e.g., collisions, aggregation, break up, and chemical reaction in particles)

Moments method for solving Kinetic Equation

Moments for joint size-velocity NDF

$$m_{p,i,j,k} = \int \xi^p v_x^i v_y^j v_y^k f(\xi, \mathbf{v})\, d\xi d\mathbf{v}$$

Lower-order moments have particular physical significance:

$m_{1,0,0,0} = \rho_\mathrm{p}\varepsilon_\mathrm{p}$, $m_{1,1,0,0} = \rho_\mathrm{p}\varepsilon_\mathrm{p} U_{\mathrm{b}x}$, $m_{1,0,1,0} = \rho_\mathrm{p}\varepsilon_\mathrm{p} U_{\mathrm{b}y}$, $m_{1,0,0,1} = \rho_\mathrm{p}\varepsilon_\mathrm{p} U_{\mathrm{b}z}$

Moments transport equation:

$$\frac{\partial m_{p,i,j,k}}{\partial t} + \frac{\partial m_{p,i+1,j,k}}{\partial x} + \frac{\partial m_{p,i,j+1,k}}{\partial y} + \frac{\partial m_{p,i,j,k+1}}{\partial z} = F_{p,i,j,k}$$

$$F_{p,i,j,k} = \int \xi^p v_x^i v_y^j v_z^k \left[i v_x^{-1}(A_x) + j v_y^{-1}(A_y) + k v_z^{-1}(A_z) \right] f(\xi, \mathbf{v})\, d\xi d\mathbf{v}$$

Quadrature-Based Moment Methods (QBMM) are introduced to attain closure of higher-order moments (spatial fluxes) and $F_{p,i,j,k}$

Representation of joint size-velocity NDF

Joint size-velocity NDF

$$n(\xi, \mathbf{v}) = f(\xi)\ g(\mathbf{v}|\xi)$$

Comparison with traditional particle size population transport method

Kinetic equation for particle size number density (with $\mathbb{S} = 0$) can be rewritten

$$\frac{\partial n}{\partial t} + \nabla \cdot \mathbf{U}n = 0$$

$$\frac{\partial n\mathbf{U}}{\partial t} + \nabla \cdot \mathbf{U} \otimes \mathbf{U}n = n\mathbf{g} + \mathbf{A}(\mathbf{U})n$$

$\mathbf{U}$ is often assumed to be independent of size ξ, however, a model with a continuous particle velocity conditioned on size $\mathbf{U}(\xi)$ needed for polydisperse particles

Particle size distribution $f(\xi)$: Extended quadrature method of moments (EQMOM)

Reconstructed size NDF

$$n(\xi) = \sum_{\alpha=1}^{N} w_\alpha \delta_\sigma(\xi, \xi_\alpha)$$

with parameters found from size moments:

- Gamma ($0 < \xi < \infty$)

$$\delta_\sigma(\xi, \xi_\alpha) \equiv \frac{\xi^{\lambda_\alpha - 1} e^{-\xi/\sigma}}{\Gamma(\lambda_\alpha)\, \sigma^{\lambda_\alpha}}$$

  with $\lambda_\alpha = \xi_\alpha / \sigma$

- Beta ($0 < \xi < 1$)

$$\delta_\sigma(\xi, \xi_\alpha) \equiv \frac{\xi^{\lambda_\alpha - 1} (1 - \xi)^{\mu_\alpha - 1}}{B(\lambda_\alpha, \mu_\alpha)}$$

  with $\lambda_\alpha = \xi_\alpha / \sigma$ and $\mu_\alpha = (1 - \xi_\alpha)/\sigma$

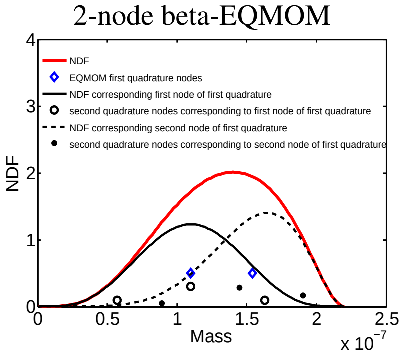
First $2N$ moments always exact

Converges to exact NDF as $N \to \infty$

Dual-quadrature form

$$\delta_\sigma(\xi, \xi_\alpha) \approx \sum_{\beta=1}^{M} w_{\alpha\beta} \delta(\xi - \xi_{\alpha\beta})$$

with known weights $w_{\alpha\beta}$ and abscissas $\xi_{\alpha\beta}$



Size-conditioned particle velocity distribution $g(\mathbf{v}|\xi)$

Monokinetic: *PTC effect and Brownian fluctuation are both negligible*

$$n(\xi,\mathbf{u}) = \sum_{\alpha=1}^{N_s} \rho_\alpha K(\xi;\xi_\alpha,\sigma_s)\ \delta(\mathbf{u}-\mathbf{U}(\xi))$$

$\mathbf{U}(\xi)$ is size-conditioned particle mean velocity

Anisotropic Gaussian: *PTC small but Brownian fluctuation non-negligible*

$$n\,(\xi,\mathbf{u}) = \sum_{\alpha=1}^{N_s} \rho_\alpha K(\xi;\xi_\alpha,\sigma_s)\ g(\mathbf{u}-\mathbf{U}(\xi);\sigma^2(\xi)\underline{R})$$

$\sigma^2(\xi)$ is size-conditioned granular temperature, $\underline{R}$ is normalized velocity covariance tensor

ECQMOM method: *PTC significant and Brownian fluctuation non-negligible*

$$n(\xi,u,v,w) = \sum_{\alpha=1}^{N_s} \rho_\alpha K(\xi;\xi_\alpha,\sigma_s)\Bigg\{\sum_{\beta=1}^{N_1} \rho_{\alpha\beta}\ g(u-U(\xi);u_{\alpha\beta},\sigma_{1\alpha})$$

$$\Bigg[\sum_{\gamma=1}^{N_2} \rho_{\alpha\beta\gamma}\ g(v-V(\xi,u);v_{\alpha\beta\gamma},\sigma_{2\alpha\beta})\left(\sum_{\kappa=1}^{N_3} \rho_{\alpha\beta\gamma\kappa}\ g(w-W(\xi,u,v);w_{\alpha\beta\gamma\kappa},\sigma_{3\alpha\beta\gamma})\right)\Bigg]\Bigg\}$$

Solving size-conditioned velocity

Anisotropic Gaussian velocity distribution

$$g(\mathbf{u}-\mu(\xi),\sigma^2(\xi)\underline{R}) = \frac{1}{(2\pi)^{3/2}\sqrt{|\sigma^2(\xi)\underline{R}|}}\exp\left[-\frac{1}{2\sigma^2(\xi)}\left(\mathbf{u}-\mathbf{U}(\xi)\right)^T\underline{R}^{-1}\left(\mathbf{u}-\mathbf{U}(\xi)\right)\right]$$

Approximation method

$\mathbf{U}(\xi)$ and $\sigma^2(\xi)$ can be approximated as

$$\mathbf{U}(\xi) = \sum_{n=0}^{2N_s}\mathbf{u}_n g_n(\xi) \qquad \sigma^2(\xi) = \sum_{n=0}^{2N_s}\sigma_n g_n(\xi)$$

where $\mathbf{u}_n(\xi)$ and σ_n are constant coefficients, and $g_n(\xi)$ are basis functions, which can be defined using various kinds of **orthogonal polynomial functions** and **piecewise functions**

Conditional mean velocity

Vector function $\mathbf{U}(\xi)$ is defined to have following properties, which can be used to solve for $\mathbf{u}_n(\xi)$

$$\sum_{\alpha=1}^{N_s}\rho_\alpha\int_\Omega \xi^s\mathbf{U}(\xi)K(\xi;\xi_\alpha,\sigma_s)d\xi = \begin{bmatrix} M_{s,1,0,0}\\ M_{s,0,1,0}\\ M_{s,0,0,1}\end{bmatrix} \qquad M_{s,i,j,k} = \int_\Omega\int_{\Re^3}\xi^s u^i v^j w^k\, n\left(\xi,\mathbf{u}\right)d\xi d\mathbf{u}$$

for $s = 0, ..., d_s$ with $d_s \leq 2N_s$

Solving size-conditioned velocity

Conditional granular temperature

Similarly, the conditional granular temperature $\sigma^2(\xi)$ has the following properties, which is used to solve for σ_n:

$$\sum_{\alpha=1}^{N_s} \rho_\alpha \int_\Omega \xi^s \left(\mathbf{U}(\xi) \cdot \mathbf{U}(\xi) + 3\sigma^2(\xi) \right) K(\xi; \xi_\alpha, \sigma_s) d\xi = M_{s,2,0,0} + M_{s,0,2,0} + M_{s,0,0,2}$$

$$\text{for } s = 0, ..., d_s \text{ with } d_s \leq 2N_s$$

Normalized velocity covariance tensor

Finally, normalized velocity covariance tensor (size-independent) is found using its definition:

$$\sum_{\alpha=1}^{N_s} \rho_\alpha \int_\Omega \xi^s \left[\mathbf{U}(\xi) \otimes \mathbf{U}(\xi) + \sigma^2(\xi) \underline{\boldsymbol{R}} \right] K(\xi; \xi_\alpha, \sigma_s) d\xi = \begin{bmatrix} M_{s,2,0,0} & M_{s,1,1,0} & M_{s,1,0,1} \\ M_{s,1,1,0} & M_{s,0,2,0} & M_{s,0,1,1} \\ M_{s,1,0,1} & M_{s,0,1,1} & M_{s,0,0,2} \end{bmatrix}$$

Sample size and sample velocity

Sample size: Jacobi quadrature for Beta kernel function

$$K(\xi;\xi_\alpha,\sigma_s) = \sum_{\beta=1}^{N_{jq}} \tilde{\rho}_{\alpha\beta}\delta(\xi,\tilde{\xi}_{\alpha,\beta})$$

Sample velocity: 3-D Hermite quadrature for anisotropic Gaussian kernel function

$$g(\mathbf{u}-\mathbf{U}(\xi),\sigma^2(\xi)\underline{R}) = \sum_{\gamma=1}^{N_{hq}} \tilde{\rho}_\gamma\delta(\mathbf{u},\tilde{\mathbf{u}}_\gamma) = \sum_{\gamma=1}^{N_{hq}} \tilde{\rho}_\gamma\delta\left(\mathbf{u},\begin{bmatrix}\tilde{u}_\gamma\\ \tilde{v}_\gamma\\ \tilde{w}_\gamma\end{bmatrix}\right)$$

Using spectral decomposition scheme with triple 1-D Hermite quadratures

Moments calculation using sample size and velocity

Now moments can be calculated as

$$\begin{aligned} M_{s,i,j,k} &= \sum_{\alpha=1}^{N_s} \rho_\alpha\ \xi_\alpha^s u^i v^j w^k\ K(\xi;\xi_\alpha,\sigma_s)\ g(\mathbf{u}-\mathbf{U}(\xi),\sigma^2(\xi)\underline{R}) \\ &= \sum_{\alpha=1}^{N_s} \rho_\alpha \sum_{\beta=1}^{N_{jq}} \tilde{\rho}_{\alpha\beta}\tilde{\xi}_{\alpha\beta}{}^s \sum_{\gamma=1}^{N_{hq}} \tilde{\rho}_{\alpha\beta\gamma}\ \tilde{u}_{\alpha\beta\gamma}{}^i \tilde{v}_{\alpha\beta\gamma}{}^j \tilde{w}_{\alpha\beta\gamma}{}^k \end{aligned}$$

Kinetics-based finite-volume method: spatial fluxes

Spatial moment fluxes are decomposed into two contributions corresponding to positive and negative velocity in each spatial direction:

$$F^{x}_{s,i,j,k} = Q^{x,+}_{s,i,j,k} + Q^{x,-}_{s,i,j,k}$$

$$\begin{aligned}
Q^{x,+}_{s,i,j,k} &= \int_{\mathbb{R}} \left(\int_{0}^{\infty} \xi^{i} u^{i+1} v^{j} w^{k} f_{sv}\left(\xi, \mathbf{u}\right) d\mathbf{u} \right) d\xi \\
&= \sum_{\alpha=1}^{N_s} \sum_{\beta=1}^{N_{jq}} \sum_{\gamma=1}^{N_{hq}} \mathbf{max}(\tilde{u}_{\alpha\beta\gamma}, 0) \rho_{\alpha} \tilde{\rho}_{\alpha\beta} \tilde{\rho}_{\alpha\beta\gamma} \tilde{\xi}^{s}_{\alpha\beta} \tilde{u}^{i}_{\alpha\beta\gamma} \tilde{v}^{j}_{\alpha\beta\gamma} \tilde{w}^{k}_{\alpha\beta\gamma} \\
Q^{x,-}_{s,i,j,k} &= \int_{\mathbb{R}} \left(\int_{-\infty}^{0} \xi^{i} u^{i+1} v^{j} w^{k} f_{sv}\left(\xi, \mathbf{u}\right) d\mathbf{u} \right) d\xi \\
&= \sum_{\alpha=1}^{N_s} \sum_{\beta=1}^{N_{jq}} \sum_{\gamma=1}^{N_{hq}} \mathbf{min}(\tilde{u}_{\alpha\beta\gamma}, 0) \rho_{\alpha} \tilde{\rho}_{\alpha\beta} \tilde{\rho}_{\alpha\beta\gamma} \tilde{\xi}^{s}_{\alpha\beta} \tilde{u}^{i}_{\alpha\beta\gamma} \tilde{v}^{j}_{\alpha\beta\gamma} \tilde{w}^{k}_{\alpha\beta\gamma}
\end{aligned}$$

Realizablity condition:

$$\Delta t = CFL \min_{\alpha\beta\gamma} \left(\frac{\Delta x}{\left|\tilde{u}_{\alpha\beta\gamma}\right|}, \frac{\Delta y}{\left|\tilde{v}_{\alpha\beta\gamma}\right|}, \frac{\Delta z}{\left|\tilde{w}_{\alpha\beta\gamma}\right|} \right)$$

Forces: drag and gravity

Contributions to evolution of moments due to drag force acting on each particle are directly computed, operating on sample velocities $\tilde{\mathbf{u}}$ from quadrature approximation by solving an ODE:

$$\frac{d\tilde{\mathbf{u}}}{dt} = \mathbf{A}_d + \mathbf{g} = K_D \left(\mathbf{u}_g - \tilde{\mathbf{u}} \right) + \mathbf{g}$$

The sample velocity at next time step is

$$\tilde{\mathbf{u}}^* = \tilde{\mathbf{u}} e^{-K_D \Delta t} + \left(1 - e^{-K_D \Delta t} \right) \left(\mathbf{u}_g + \frac{\mathbf{g}}{K_D} \right)$$

And overall drag force received by entire particle phase is

$$\mathbf{F}_{D,pg} = \sum m_p \cdot A_d = \frac{\rho_p \pi}{6} \sum_{\alpha=1}^{N_s} \sum_{\beta=1}^{N_{jq}} \rho_\alpha \tilde{\rho}_{\alpha\beta} \tilde{\xi}^3_{\alpha\beta} \sum_{\gamma=1}^{N_{hq}} \tilde{\rho}_{\alpha\beta\gamma} K_{D,\alpha\beta\gamma} \left(\mathbf{u}_g - \tilde{\mathbf{u}}_{\alpha\beta\gamma} \right)$$

So using Newton's first law, drag force received by gas phase is

$$\mathbf{F}_{D,gp} = -\mathbf{F}_{D,pg}$$

Polydisperse collision model

Moment transport equation

$$\frac{\partial M_{s,i,j,k}}{\partial t} + \frac{\partial M_{s,i+1,j,k}}{\partial x} + \frac{\partial M_{s,i,j+1,k}}{\partial y} + \frac{\partial M_{s,i,j,k+1}}{\partial z} = \mathbb{A}_{s,i,j,k} + \boxed{\mathbb{C}_{s,i,j,k}}$$

Using operator splitting, collision term can be conveniently updated by solving

$$\frac{\partial M_{s,i,j,k}}{\partial t} = \boxed{\mathbb{C}_{s,i,j,k}}$$

$$M_{s,i,j,k} = \sum_{\alpha=1}^{N_s} \sum_{\beta=1}^{N_{jq}} \rho_\alpha \; \tilde{\rho}_{\alpha\beta} \tilde{\xi}^s_{\alpha\beta} \sum_{\gamma=1}^{N_{hq}} \tilde{\rho}_{\alpha\beta\gamma} \; \tilde{u}^i_{\alpha\beta\gamma} \tilde{v}^j_{\alpha\beta\gamma} \tilde{w}^k_{\alpha\beta\gamma} = \sum_{a=1}^{N_t} w_a \xi^s_a G_{ijk}(\xi_a)$$

Since we can assume that size does not change due to collisions, and also that collisions are binary

$$\boxed{\mathbb{C}_{s,i,j,k}} = \frac{\partial}{\partial t} \sum_{a=1}^{N_t} w_a \xi^s_a G_{ijk}(\xi_a) = \sum_{a=1}^{N_t} w_a \xi^s_a \frac{\partial G_{ijk}(\xi_a)}{\partial t} = \sum_{a=1}^{N_t} w_a \xi^s_a \sum_{b=1}^{N_t} \mathbb{C}_{i,j,k}(\xi_a, \xi_b)$$

$$\mathbb{C}_{i,j,k}(\xi_a, \xi_b) = C_{i,j,k}(\xi_a, \xi_b) + \nabla \cdot \mathbf{G}_{i,j,k}(\xi_a, \xi_b)$$

Collision source term

BGK model (valid to second order)

$$C_{ijk}(\xi_a,\xi_b) = \kappa_{ab}\left(G^*_{ijk,ab} - G_{ijk,a}\right) = \frac{24 g_{0,ab}\alpha_b \chi_{ab}^3 \sqrt{\sigma_{ab}^2}}{\sqrt{\pi} d_{ab}}\left(G^*_{ijk,ab} - G_{ijk,a}\right)$$

Zero-order moments	$C_{i+j+k=0}(\xi_a,\xi_b) = 0$
First-order moments	$C_{i+j+k=1}(\xi_a,\xi_b) = \kappa_{ab}\left(\mathbf{U}_{ab} - \mathbf{U}_a\right)$
Second-order moments	$C_{i+j+k=2}(\xi_a,\xi_b) = \kappa_{ab}\left(\mathbf{U}_{ab}\otimes\mathbf{U}_{ab} + \underline{\underline{\Sigma}}_{ab} - \mathbf{U}_a\otimes\mathbf{U}_a - \underline{\underline{\Sigma}}_a\right)$

For equilibrium Gaussian distribution

Mean velocity

$$\mathbf{U}_{ab} = \mathbf{U}_a + \frac{1}{4}(1+e_{ab})\mu_{ab}(\mathbf{U}_b - \mathbf{U}_a)$$

Covariance tensor

$$\underline{\underline{\Sigma}}_{ab} = \underline{\underline{\Sigma}}_a + \frac{1}{2}(1+e_{ab})\mu_{ab}\left[\frac{1}{4}(1+e_{ab})\mu_{ab}\underline{\underline{\boldsymbol{S}}}_{ab} - \underline{\underline{\Sigma}}_a\right]$$

where $\underline{\underline{\boldsymbol{S}}}_{ab} = 1/2(\underline{\underline{\Sigma}}_a + \underline{\underline{\Sigma}}_b + \sigma_{ab}^2\boldsymbol{I})$, $\underline{\underline{\Sigma}}_a = \sigma_a^2\underline{\underline{\boldsymbol{R}}}$, $\underline{\underline{\Sigma}}_b = \sigma_b^2\underline{\underline{\boldsymbol{R}}}$

Collisional flux and Granular pressure

Collisional flux

m^{th} component of collisional-flux term caused by collisions between particles of size ξ_a and ξ_b can be calculated as

$$\mathbf{G}_{m,ijk}(\xi_a,\xi_b) = \mathbf{G}^{(0)}_{m,ijk}(\xi_a,\xi_b) + \cancel{\mathbf{G}^{(1)}_{m,ijk}(\xi_a,\xi_b)}$$

$$\mathbf{G}^{(0)}_{m,ijk}(\xi_a,\xi_b) = \frac{3\chi^2_{ab}\xi_a g_{0,ab}}{\xi_b}\int_{\mathbb{R}^6} I^{(m)}_{ijk}\left(\omega_{ab},\mathbf{v}_a,\mathbf{v}_a-\mathbf{v}_b\right) f(\mathbf{v}_a) f(\mathbf{v}_b) d\mathbf{v}_a d\mathbf{v}_b$$

This term can be explicitly closed by using sample size and sample velocity, which can make system unstable in simulations

Granular pressure

For first-order velocity moments $M_{s,i+j+k=1}$, collisional flux can be treated as **part of solid stress tensor** $\underline{\underline{T}}_s$ in particle-phase momentum equation, which can be decomposed as

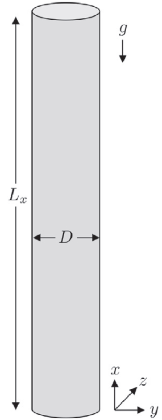
$$\underline{\underline{T}}_{s,k} = p_{s,k}\mathbf{I} + \underline{\underline{\tau}}_{s,k}$$

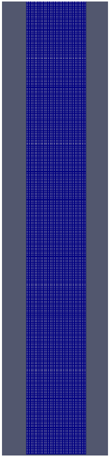
Granular pressure due to particle collisions for particle phase can be calculated as

$$p_{s,k} = \rho_s \sum_{a=1}^{N_t} w_a \left[\sigma_a^2 + \sum_{b=1}^{N_t} w_b \frac{\chi^3_{ab}\mu_{ab}(1+e_{ab})g_{0,ab}E_{ab}}{3\chi_{ba}}\right]$$

Test case: Wall-bounded vertical riser

Geometry and mesh (2D)

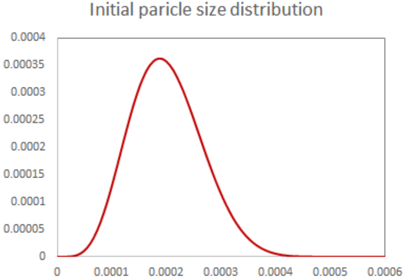




Periodic in X-direction

Gas phase is statistically stationary with zero volume flow rate

gas density (ρ_p)	1 kg/m^3
gas viscosity (ν_g)	1.84e-5 m^2/s
particle density (ρ_g)	2000 kg/m^3
mean diameter (d_p)	0.0002 m
restitution coeff (e)	0.9



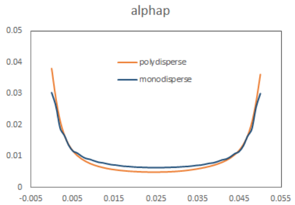
Animation, velocity fields, 2-5 second

Particle-phase velocity ,gas-phase velocity, and granular temperature

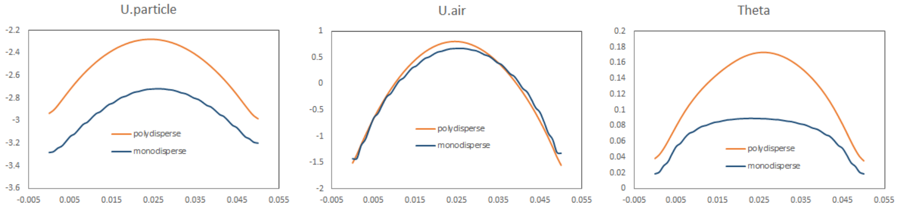
Animation, scalar fields, 2-5 second

Volume fraction, mean particle diameter, and standard deviation

Particle volume fraction and mean diameter



Particle velocity, gas-phase velocity, and granular temperature



Summary

Conclusions

- Novel approach to model polydisperse gas-particle flows with quadrature-based moment methods using kinetic equation for joint size-velocity number density function
- Quasi-2D wall-bounded vertical riser simulated with continuous particle size distribution initial condition
- Solver includes explicit representation of joint NDF using EQMOM that directly incorporates effects of polydispersity
- Size segregation is captured in simulations, and results demonstrate our approach is effective way to model complicated polydisperse gas-particle flows

Plans for future work in FY15

- Detailed validation with 3-D experimental/Lagrangian simulation data for polydisperse gas-particle flows
- Implementation of new multiphase turbulence model, and validation with Lagrangian simulation of Capecelatro et al.

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Thanks for your attention!

Questions?