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Spherical and Non-Spherical Particle Transport and Deposition in Turbulent Flows

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Outline

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- Introduction
- Spherical and Fiber Transport and Deposition
- RANS Computational Models
- DNS Model
- Deposition Results
- Conclusions

Turbulence Modeling

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- Direct Numerical Simulation
- Large Eddy Simulation
- RANS Models
  - Stress Transport Model
  - Two-Equation Models

Instantaneous Turbulent Fluctuation Velocity

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- Direct Numerical Simulation
- Subgrid Scale Simulation
- Gaussian Models
  - Filtered White Noise
  - Eddy Life Time
- Pdf – Based Model

Instantaneous Turbulent Fluctuation Velocity

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Instantaneous Velocity

Thompson (1987)

$$\frac{d\mathbf{u}_i}{dt} = \frac{\mathbf{u}_i - \bar{\mathbf{u}}_i}{T_L} + \sqrt{\frac{2\,\overline{\mathbf{u}_i'^{\,2}}}{T_L}}\,\zeta(t)$$

Lagrangian Time Macro-Scale

$$T_L = \int_0^\infty \frac{\overline{u'^p(t)\,u'^p(t+\tau)}}{\overline{u'^p u'^p}}\,d\tau$$

Instantaneous Turbulent Fluctuation Velocity

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Instantaneous Velocity

Iliopoulos et al. (2003) and Dehbi (2008) included the drift term

$$\frac{d\mathbf{u}_2}{dt} = \frac{\mathbf{u}_2 - \bar{\mathbf{u}}_2}{T_L} + \sqrt{\frac{2\,\overline{\mathbf{u}_2'^2}}{T_L}}\,\zeta(t) + \frac{1}{2(1+Stk)}\frac{\partial \overline{\mathbf{u}_2'^2}}{\partial x_2}$$

$$Stk = \frac{\tau_p}{T_L}$$

Particle Equation

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$$\frac{du_i^p}{dt} = \underbrace{\frac{C_D\, Re_P}{24}\frac{1}{\tau}\left(u_i - u_i^p\right)}_{\text{Drag force}} + \underbrace{f_i^L}_{\text{Lift force}}$$

$$+ \underbrace{g_i}_{\text{Gravitational force}} + \underbrace{f_i^E}_{\text{Electric force}} + \underbrace{n_i(t)}_{\text{Brownian force}}$$

Assumptions: Dilute Flows, One-Way Interaction, Neglect Particle Collisions

$$C_D = \frac{24[1 + 0.15\, Re^{0.687}]}{Re}$$

$$1 < Re < 1000$$

Brownian Motion

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Spectral Intensity

White Noise Model

$$S_{nn} = \frac{2kT}{\pi\tau}$$

Particle Relaxation Time

$$\tau = \frac{d^2 \rho^p C_c}{18\mu} = \frac{S d^2 C_c}{18\nu}$$

Brownian Motion

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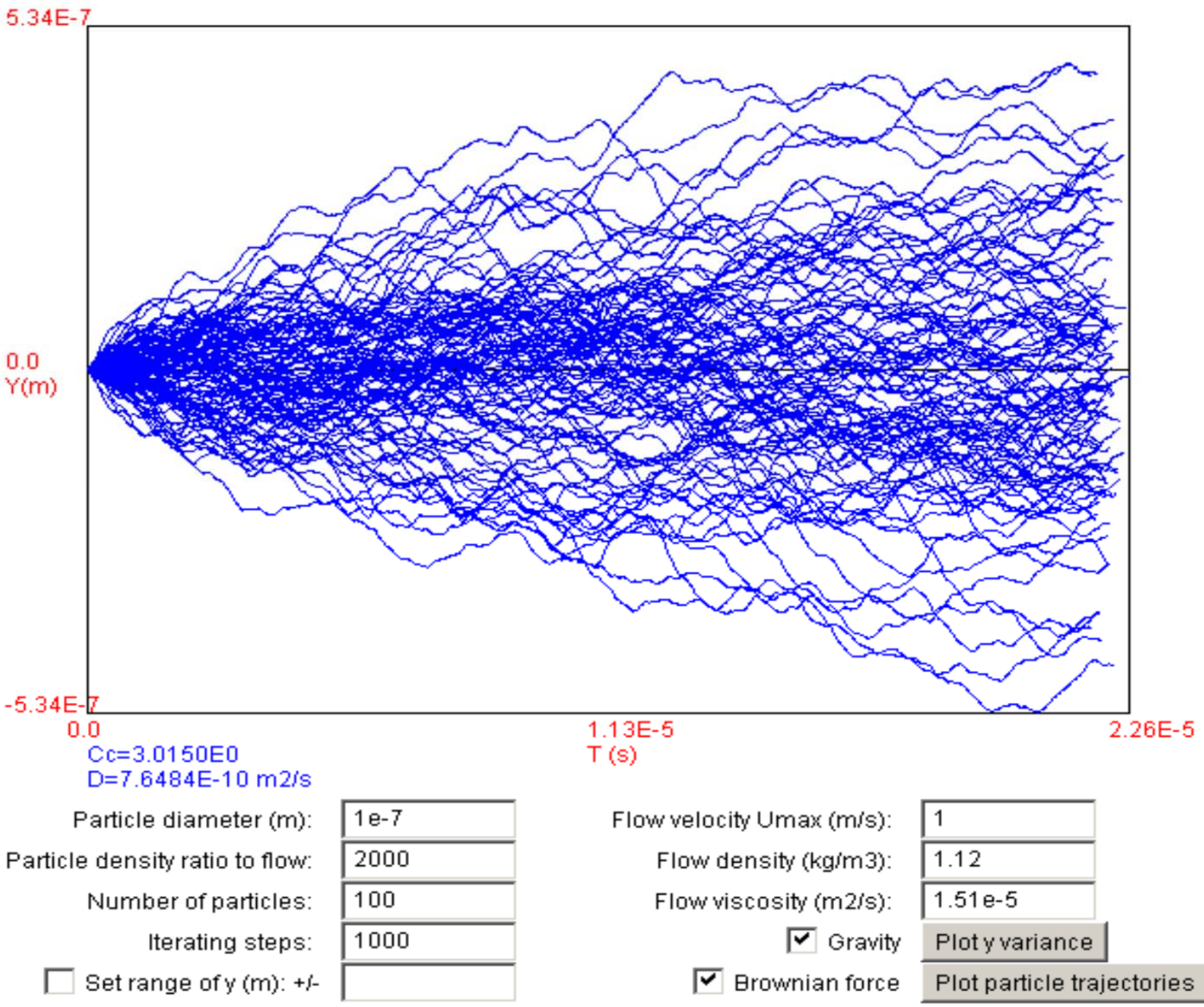
CRCD Web-Based Course Module



Sample Brownian Dispersioin

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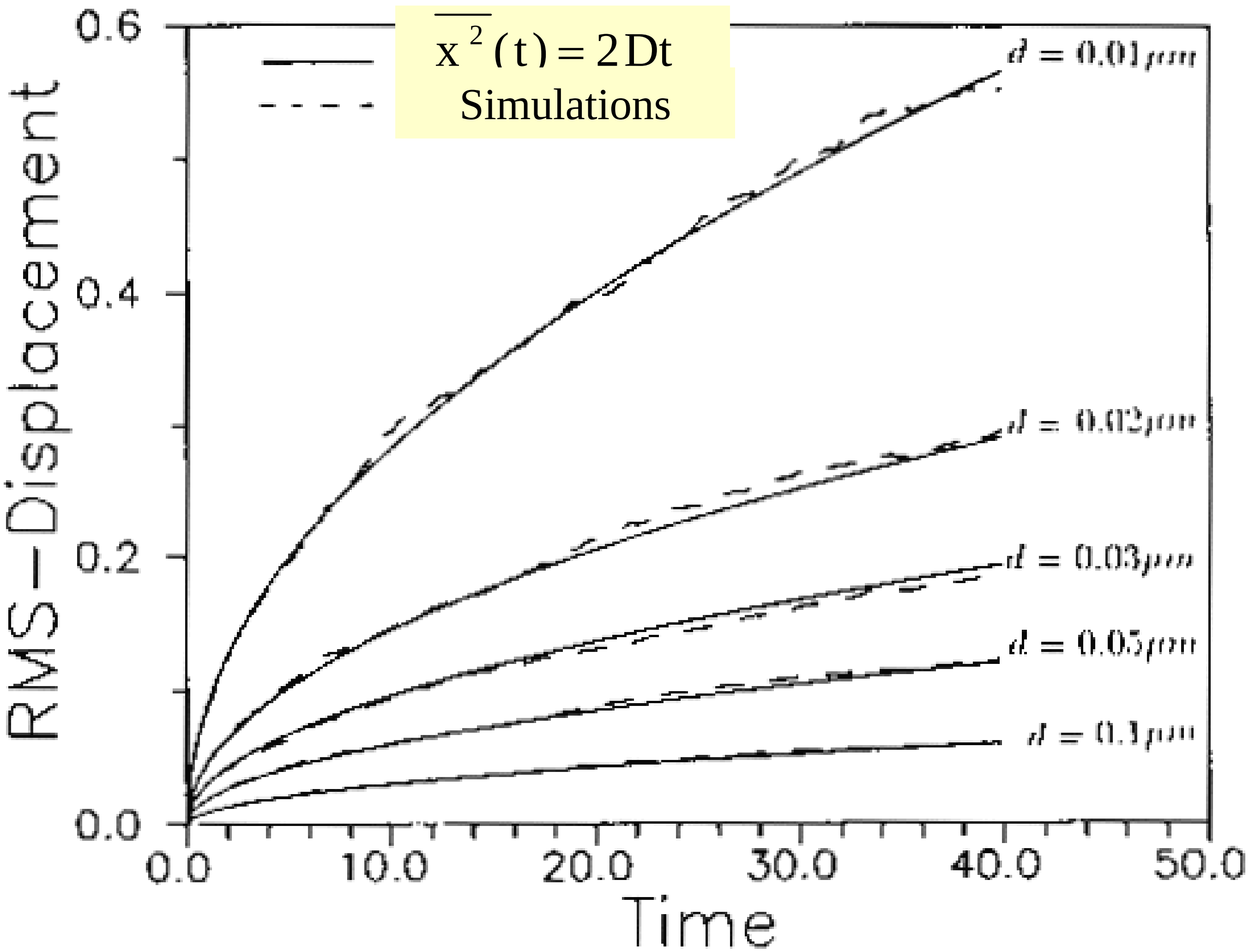

Particle Deposition in a Duct

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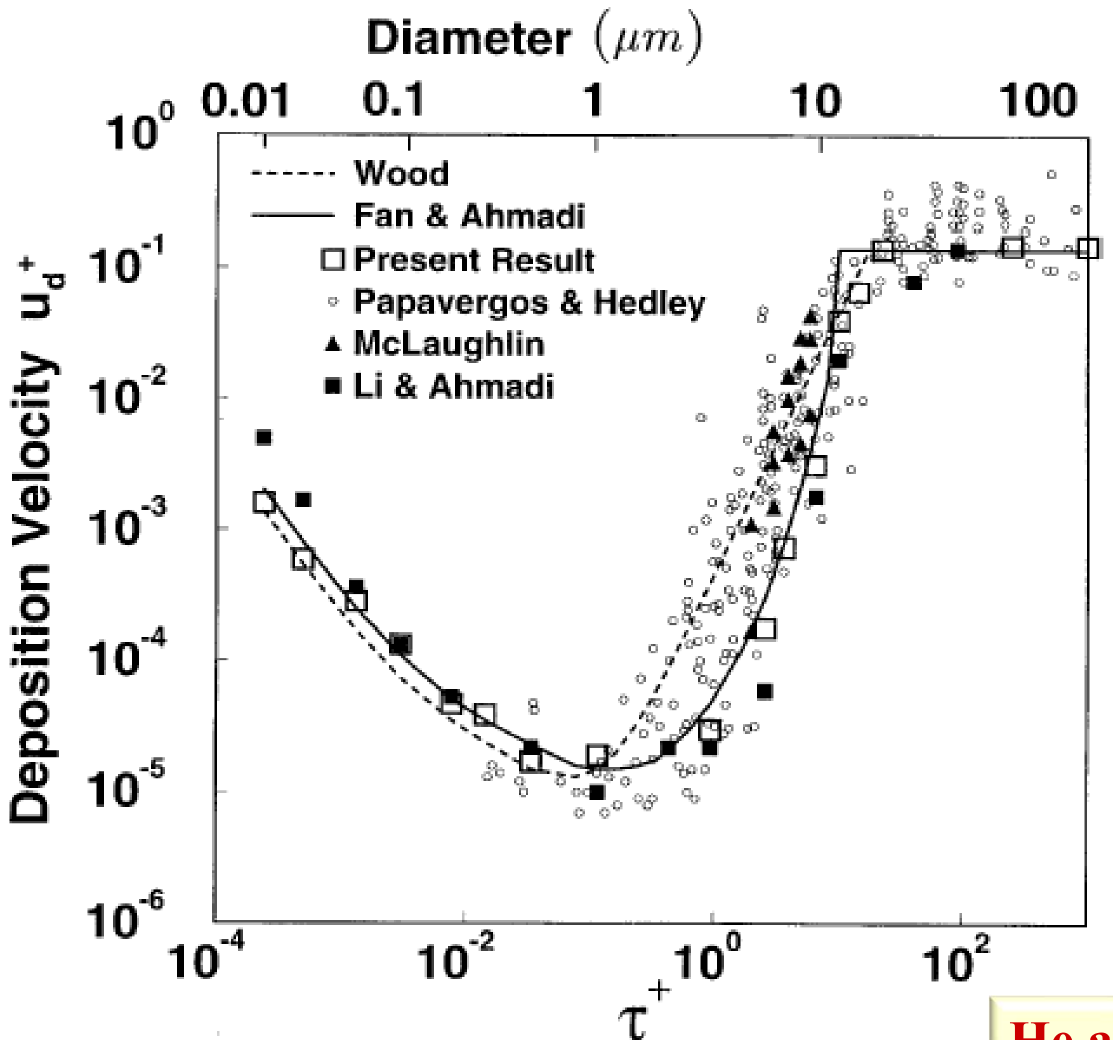

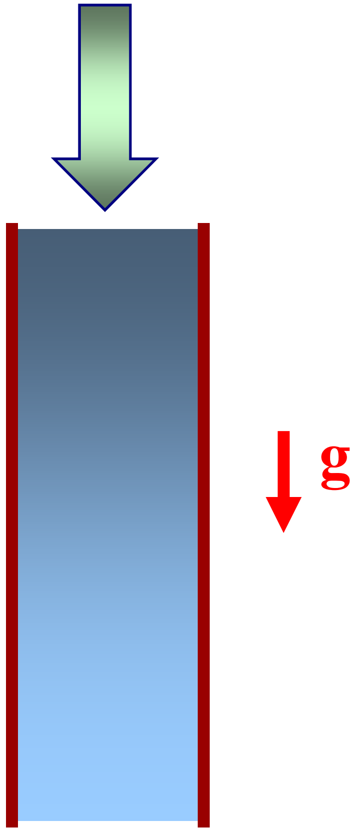

He and Ahmadi (1999)

Particle Deposition in a Duct

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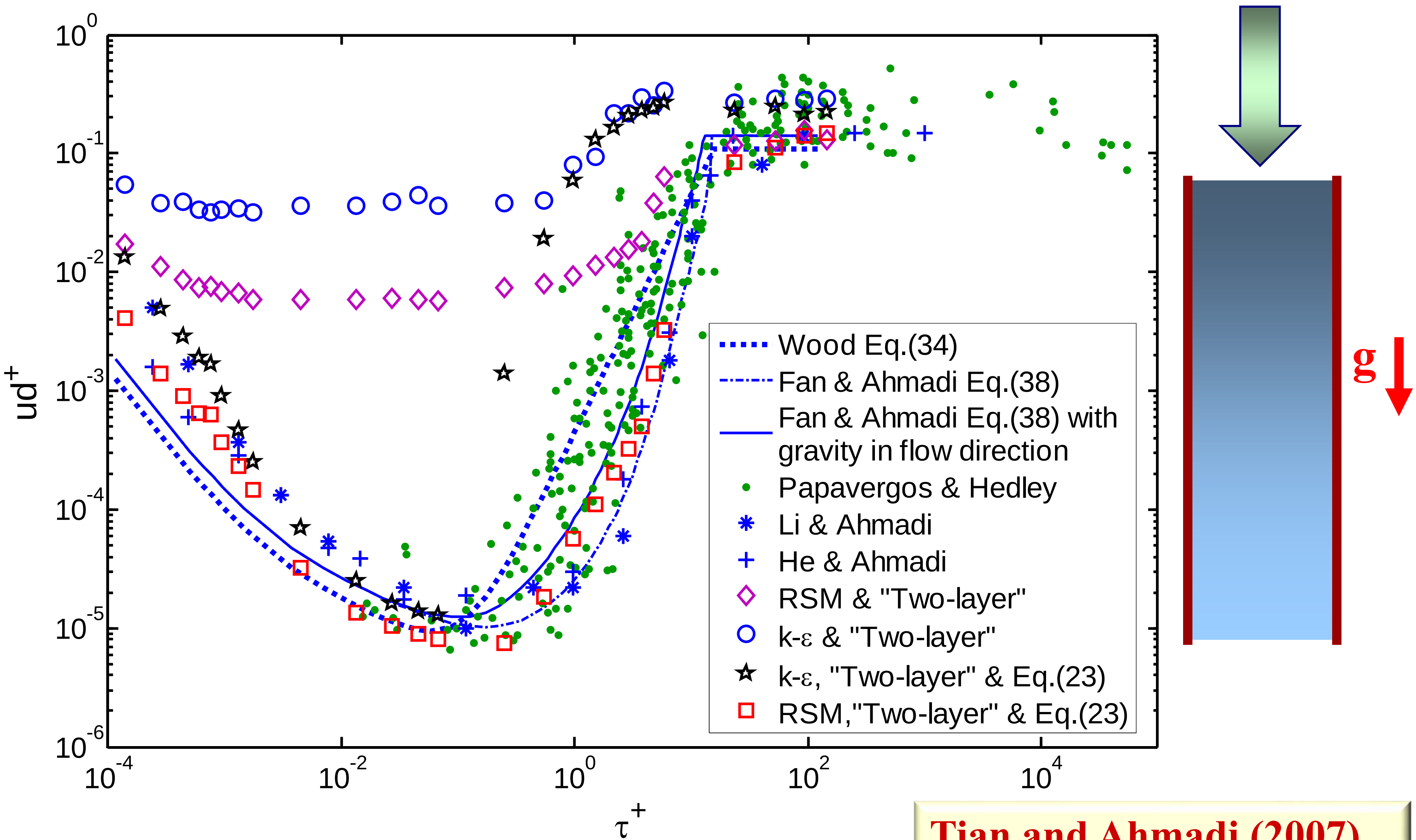

Tian and Ahmadi (2007)

Turbulence Near Wall Model

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Quadratic Variation Near Wall

Hinze, 1975 $\sqrt{\overline{v'^2}} \propto y^2$ $\quad y \to 0$

$$v^+ = Ay^{+2} \qquad y^+ < 4$$

$$y^+ = \frac{yu^*}{\nu} \qquad v^+ = \frac{\sqrt{\overline{v'^2}}}{u^*} \qquad A = 0.008$$

Li and Ahmadi, 1993

Ounis, et al. 1993 (DNS)

Turbulence Near Wall Model

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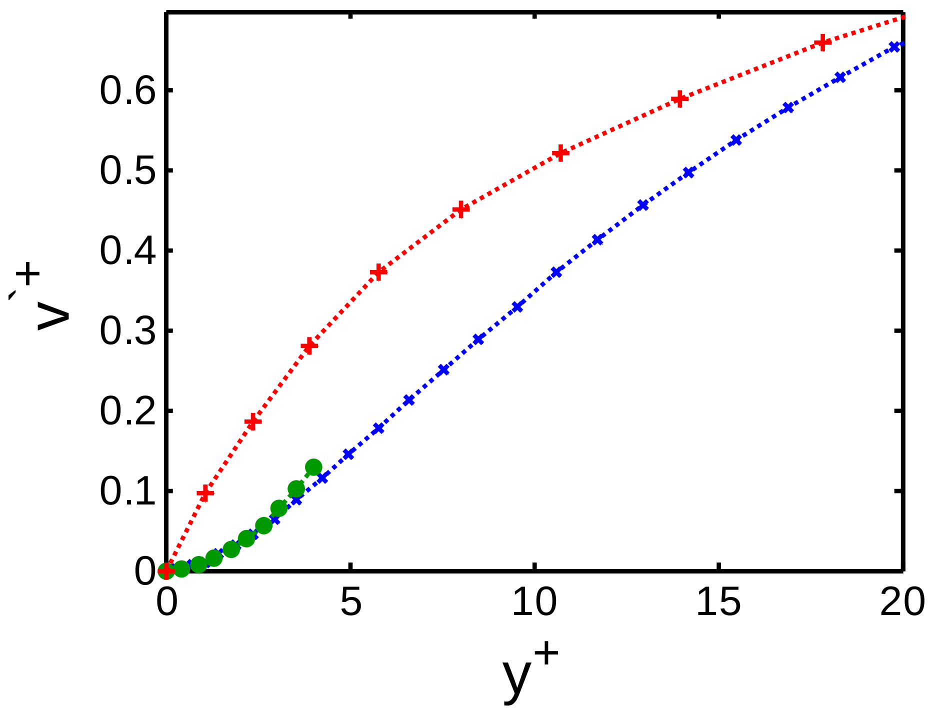
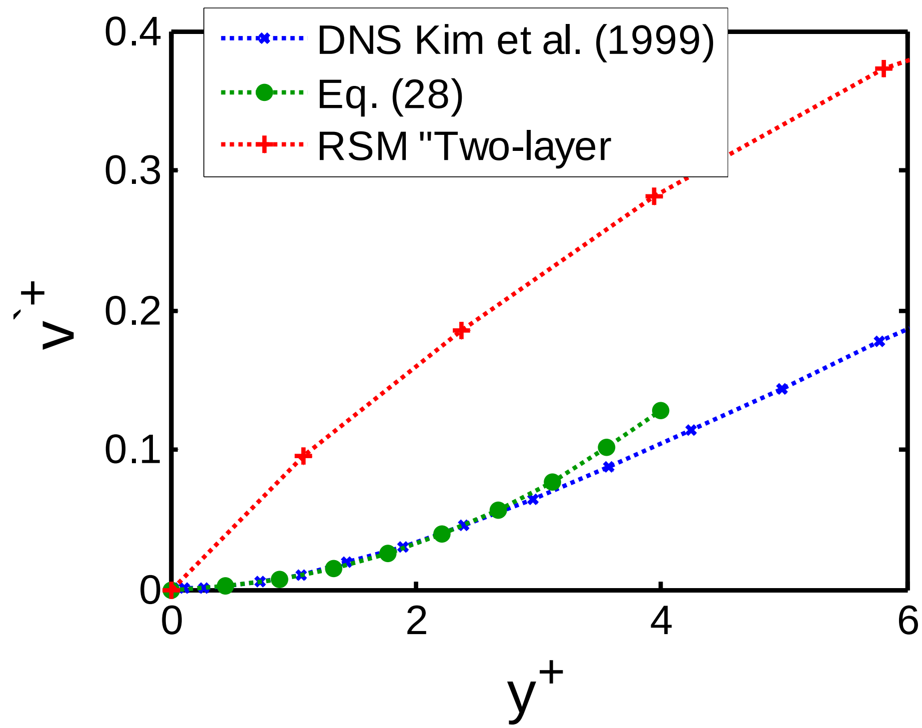
RSM with two-layer vs. near wall correction by Li & Ahmadi (1993)





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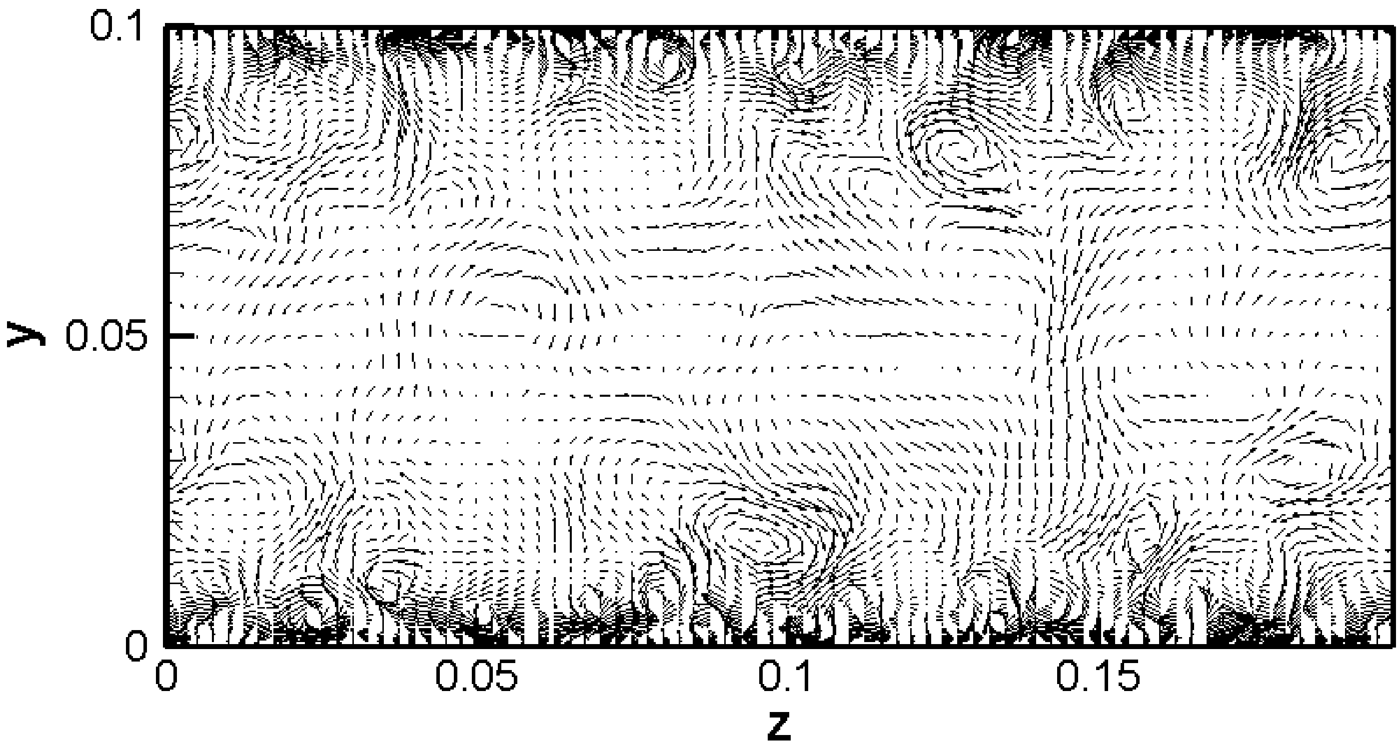

LES

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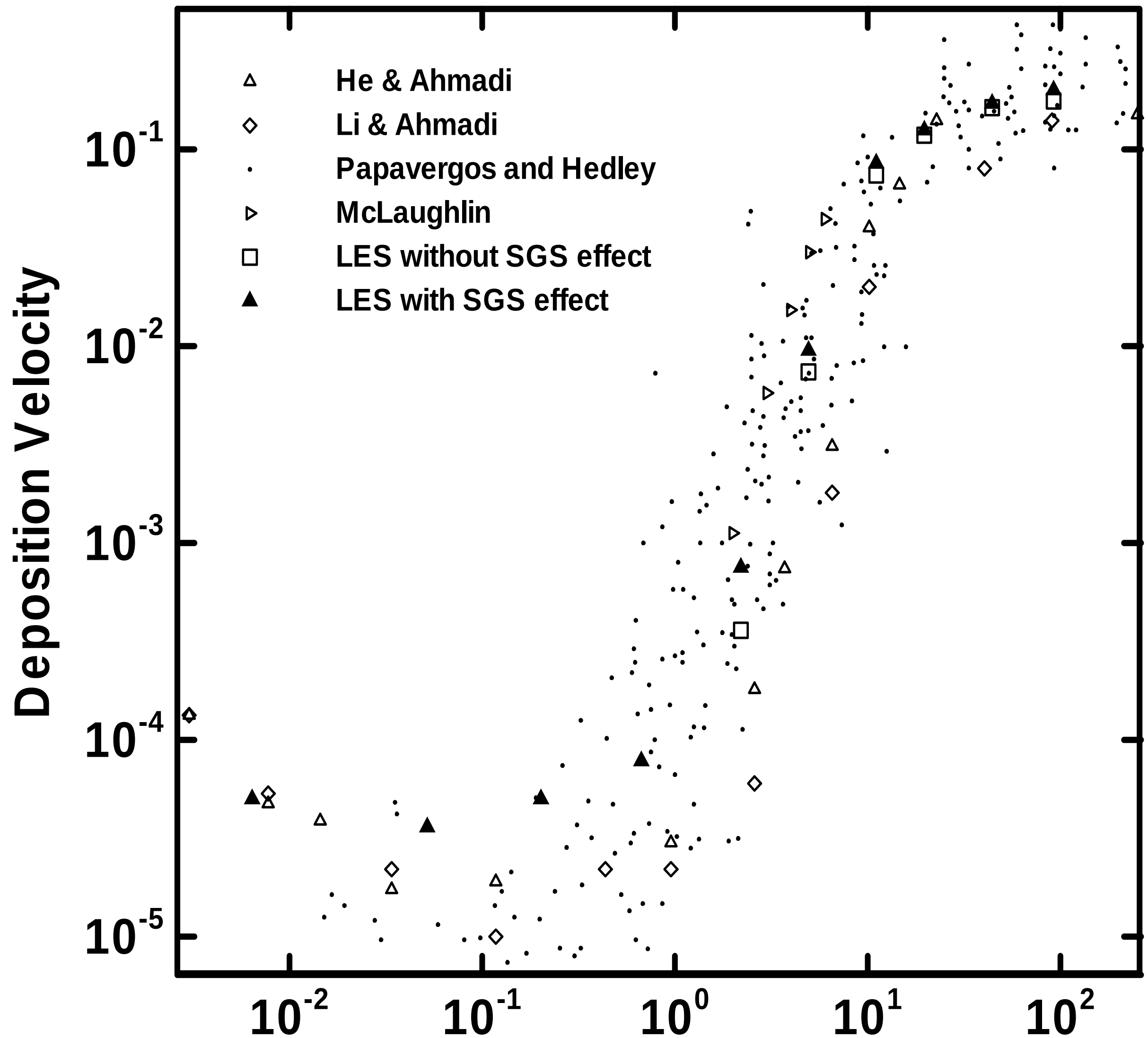
Vertical Ducts

Effects of Subgrad Scale



Deposition Velocity

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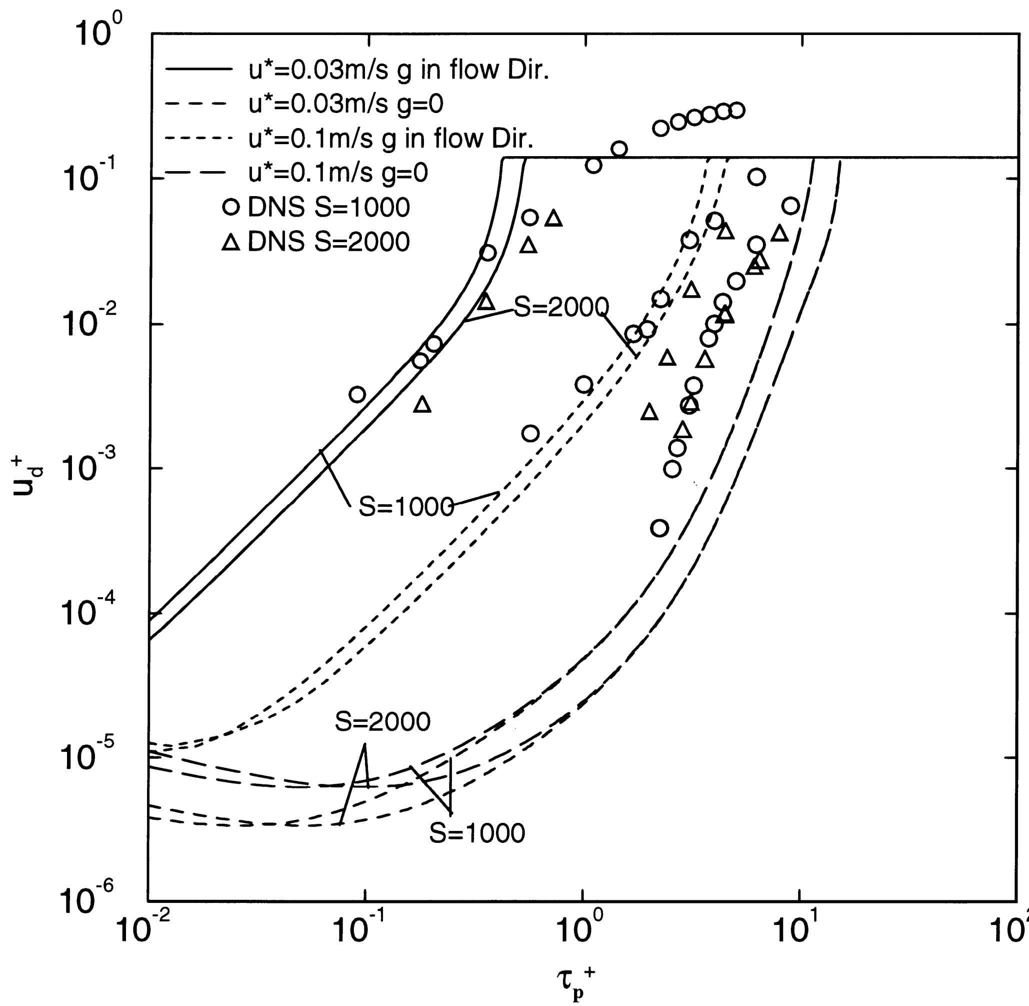
Turbulent Flow

DNS

Vertical Ducts

Effects of Shear Velocity and Density Ratio



Deposition Velocity

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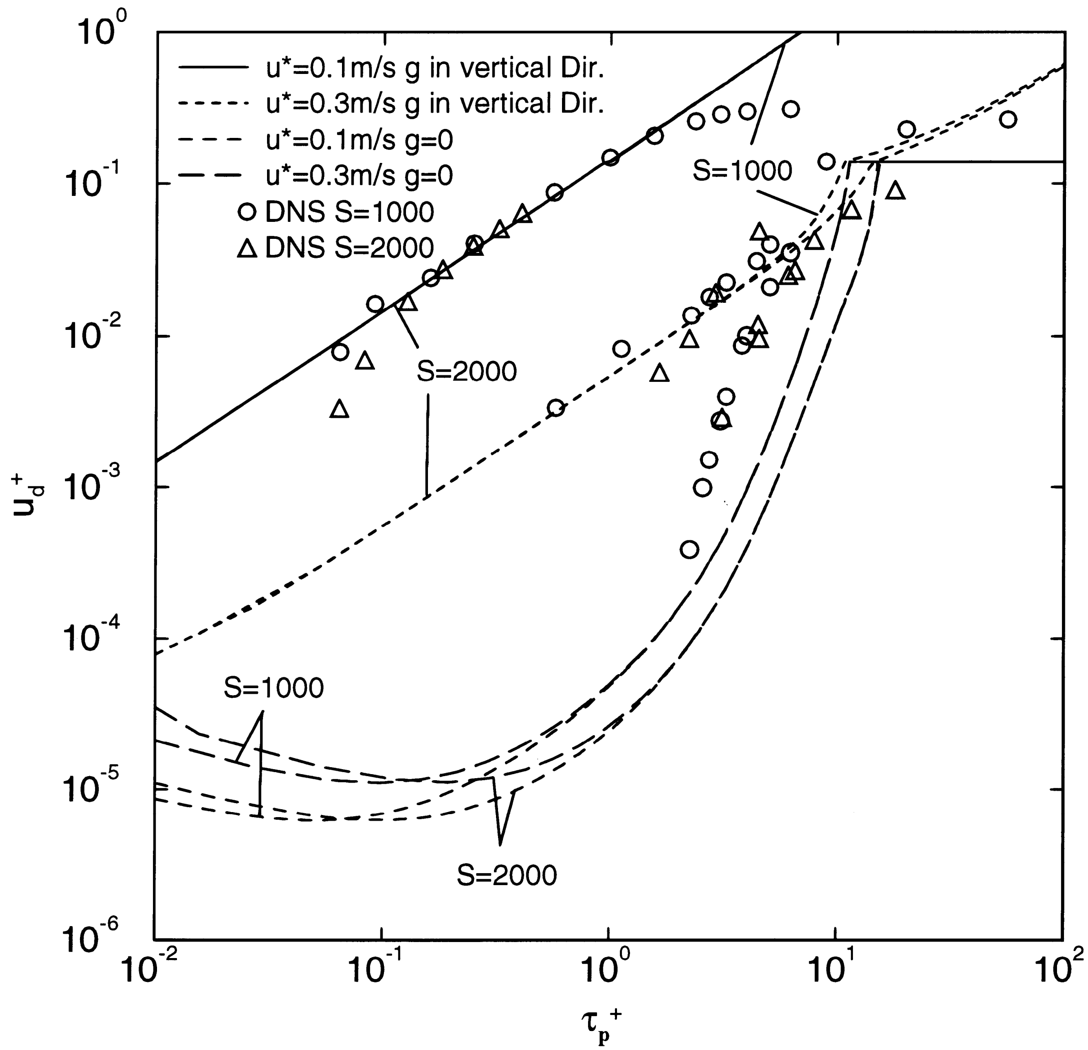
Turbulent Flow

DNS

Horizontal Ducts

Effects of Shear Velocity and Density Ratio



Particle Deposition Velocity

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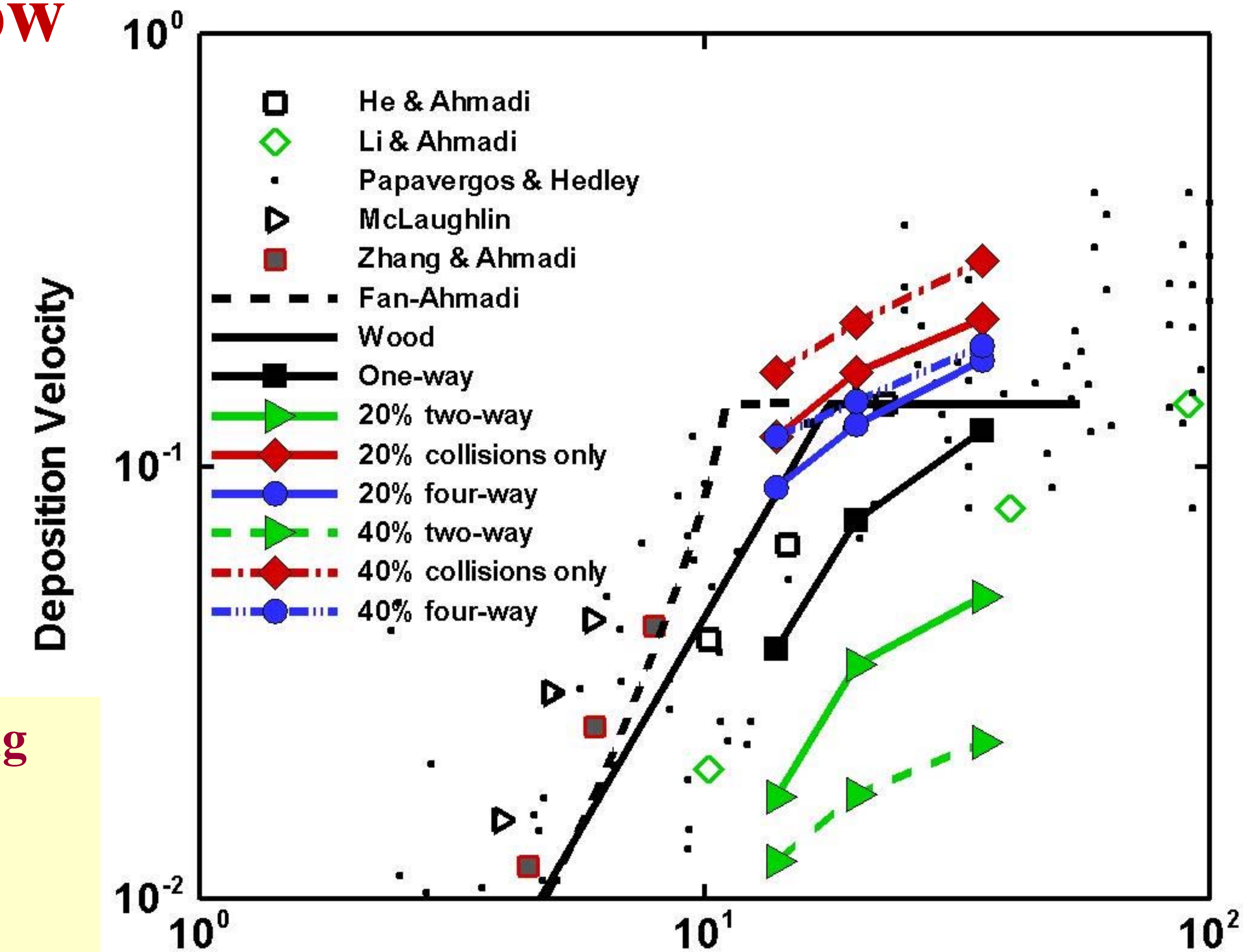
Turbulent Flow

DNS

u_d^+



τ^+

Effect of Mass Loading

Two-Way Coupling

Particle Collisions

Four-Way Coupling

Fiber Equation of Motion

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Translational Motion

$$m^p \frac{d\mathbf{v}}{dt} = \left(m^p - m^f\right)\mathbf{g} + \mathbf{f}^h + \mathbf{f}^L$$

Rotational Motion

$$I_{\hat{x}} \frac{d\omega_{\hat{x}}}{dt} - \omega_{\hat{y}}\omega_{\hat{z}}\left(I_{\hat{y}} - I_{\hat{z}}\right) = T_{\hat{x}}^h$$

$$I_{\hat{y}} \frac{d\omega_{\hat{y}}}{dt} - \omega_{\hat{z}}\omega_{\hat{x}}\left(I_{\hat{z}} - I_{\hat{x}}\right) = T_{\hat{y}}^h$$

$$I_{\hat{z}} \frac{d\omega_{\hat{z}}}{dt} - \omega_{\hat{x}}\omega_{\hat{y}}\left(I_{\hat{x}} - I_{\hat{y}}\right) = T_{\hat{z}}^h$$

Fiber Equation of Motion

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Drag

$$\mathbf{f}^{h} = \mu\pi a\hat{\hat{\mathbf{K}}}\cdot(\mathbf{u}-\mathbf{v})$$

$$\hat{\hat{\mathbf{K}}} = \mathbf{A}^{-1}\hat{\mathbf{K}}\mathbf{A}$$

$$k_{\hat{x}\hat{x}} = k_{\hat{y}\hat{y}} = \frac{16(\beta^2-1)}{\left[(2\beta^2-3)\ln\left(\beta+\sqrt{\beta^2-1}\right)/\sqrt{\beta^2-1}\right]+\beta}$$

$$k_{\hat{z}\hat{z}} = \frac{8(\beta^2-1)}{\left[(2\beta^2-1)\ln\left(\beta+\sqrt{\beta^2-1}\right)/\sqrt{\beta^2-1}\right]-\beta}$$

Equivalent Relaxation Time Shapiro-Goldenberg

$$\tau_{eq}^{+} = \frac{4\beta S a^{+2}}{9}\left(\frac{1}{k_{\hat{x}\hat{x}}}+\frac{1}{k_{\hat{y}\hat{y}}}+\frac{1}{k_{\hat{z}\hat{z}}}\right) = \frac{2\beta S a^{+2}}{9}\,\frac{\ln\left(\beta+\sqrt{\beta^2-1}\right)}{\sqrt{\beta^2-1}}$$

Fiber Equation of Motion

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Equivalent Relaxation Time (Fan-Ahmadi)

$$\tau_{eq}^{+} = \frac{4\beta S a^{+2}}{k_{\hat{x}\hat{x}} + k_{\hat{y}\hat{y}} + k_{\hat{z}\hat{z}}}$$

Hydrodynamic Torque

$$T_{\hat{x}}^{h} = \frac{16\pi\mu a^{3}\beta}{3\left(\beta_0 + \beta^2\gamma_0\right)}\left[\left(1-\beta^2\right)d_{\hat{z}\hat{y}} + \left(1+\beta^2\right)\left(w_{\hat{z}\hat{y}} - \omega_{\hat{x}}\right)\right]$$

$$T_{\hat{y}}^{h} = \frac{16\pi\mu a^{3}\beta}{3\left(\alpha_0 + \beta^2\gamma_0\right)}\left[\left(\beta^2 - 1\right)d_{\hat{x}\hat{z}} + \left(1+\beta^2\right)\left(w_{\hat{x}\hat{z}} - \omega_{\hat{y}}\right)\right]$$

$$T_{\hat{z}}^{h} = \frac{32\pi\mu a^{3}\beta}{3\left(\alpha_0 + \beta_0\right)}\left(w_{\hat{y}\hat{z}} - \omega_{\hat{z}}\right)$$

Fiber Shear Lift

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$$\mathbf{f}^{L} = \frac{\pi^2 \mu a^2}{\nu^{1/2}} \frac{\partial u_x / \partial y}{\left|\partial u_x / \partial y\right|^{1/2}} \left(\hat{\hat{\mathbf{K}}} \cdot \mathbf{L} \cdot \hat{\hat{\mathbf{K}}} \right) \cdot (\mathbf{u} - \mathbf{v})$$

$$\mathbf{L} = \begin{bmatrix} 0.0501 & 0.0329 & 0.00 \\ 0.0182 & 0.0173 & 0.00 \\ 0.00 & 0.00 & 0.0373 \end{bmatrix}$$

Schematics of Ellipsoidal Fiber

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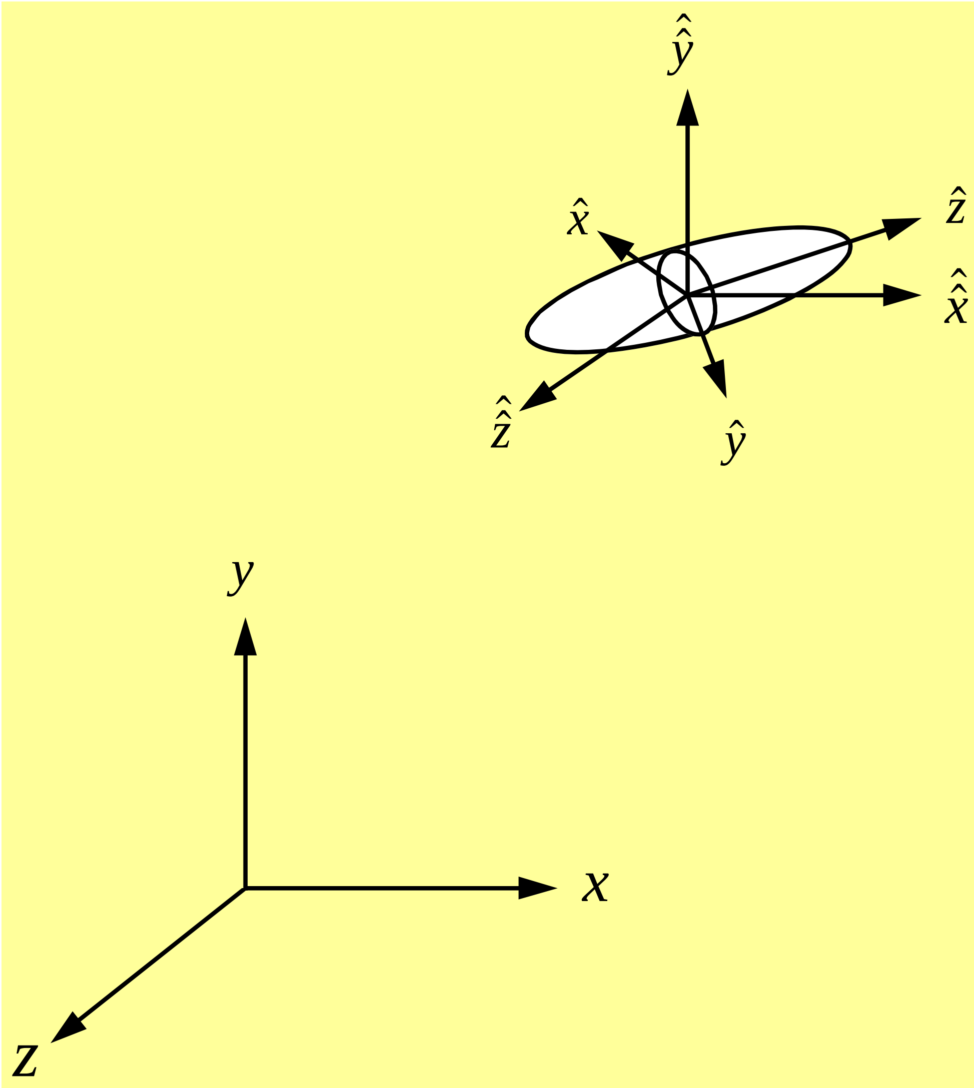

Transformation Matrix and Euler Parameters

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Euler Angles

$$\mathbf{A} = \begin{bmatrix} \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi & \cos\psi\sin\phi + \cos\theta\cos\phi\sin\psi & \sin\psi\sin\theta \\ -\sin\psi\cos\phi - \cos\theta\sin\phi\cos\psi & -\sin\psi\sin\phi + \cos\theta\cos\phi\cos\psi & \cos\psi\sin\theta \\ \sin\theta\sin\phi & -\sin\theta\cos\phi & \cos\theta \end{bmatrix}$$

Euler Parameters

$$\mathbf{A} = \begin{bmatrix} 1-2\left(\varepsilon_2^2+\varepsilon_3^2\right) & 2(\varepsilon_1\varepsilon_2+\varepsilon_3\eta) & 2(\varepsilon_1\varepsilon_3-\varepsilon_2\eta) \\ 2(\varepsilon_2\varepsilon_1-\varepsilon_3\eta) & 1-2\left(\varepsilon_3^2+\varepsilon_1^2\right) & 2(\varepsilon_2\varepsilon_3+\varepsilon_1\eta) \\ 2(\varepsilon_3\varepsilon_1+\varepsilon_2\eta) & 2(\varepsilon_3\varepsilon_2-\varepsilon_1\eta) & 1-2\left(\varepsilon_1^2+\varepsilon_2^2\right) \end{bmatrix}$$

$$\varepsilon_1^2+\varepsilon_2^2+\varepsilon_3^3+\eta^2=1$$

Fiber Deposition Rate

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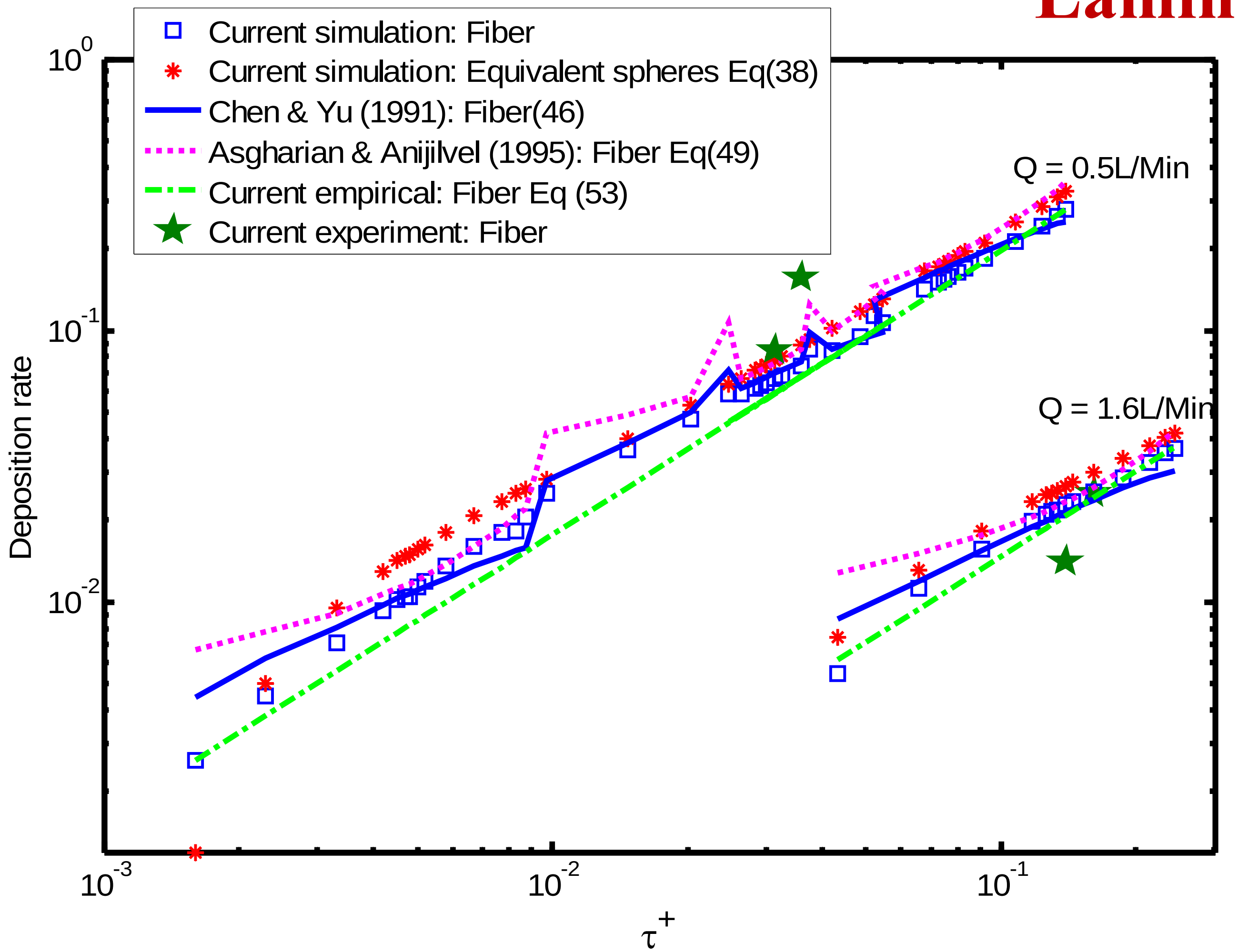
Laminar Flow



Deposition rates of glass fibers and the equivalent spheres.

Fiber Deposition Rate

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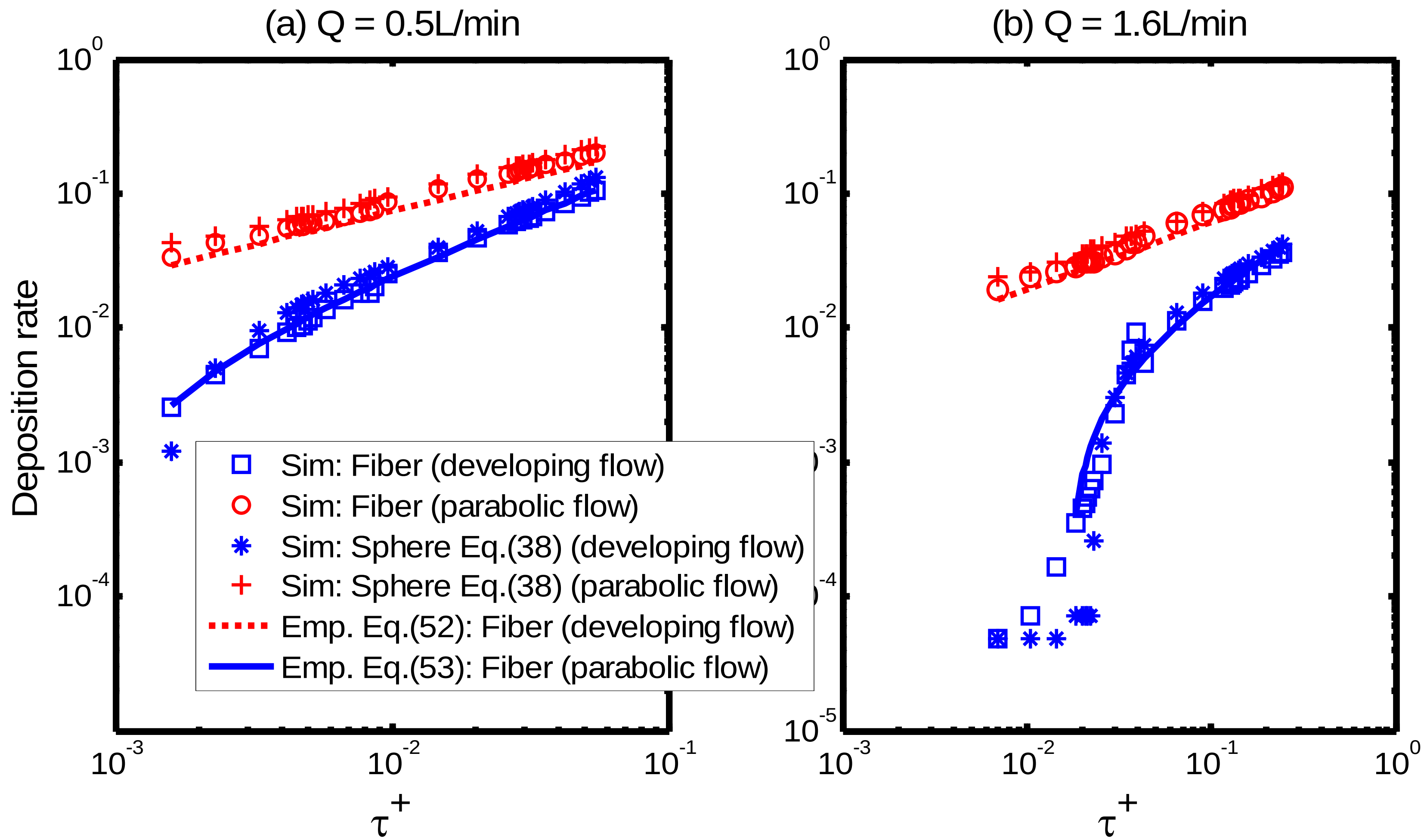

Glass fibers in horizontal pipe

Laminar developing and in parabolic flows

Fiber Deposition Velocity

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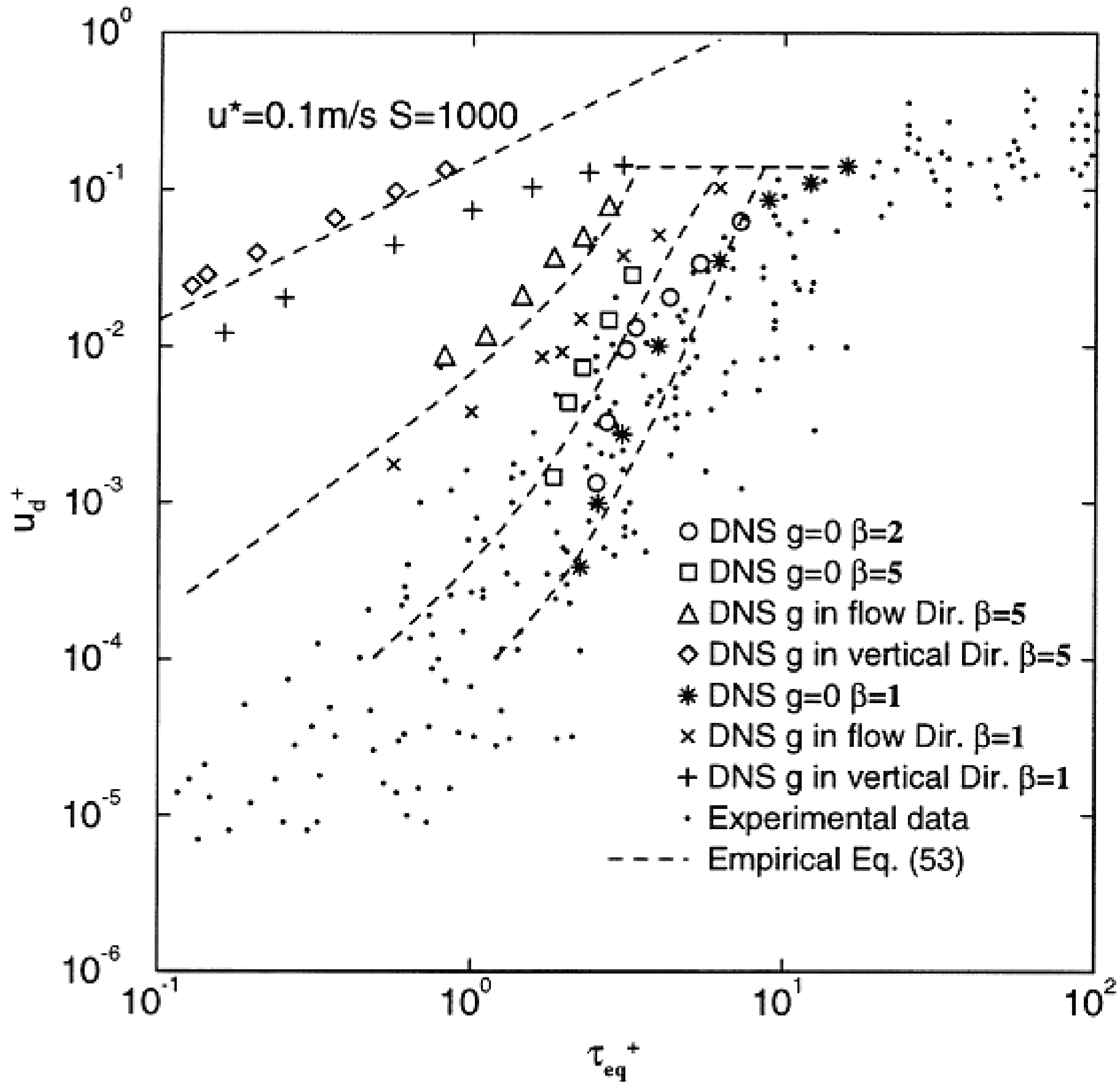
Turbulent Flow

DNS



Fiber Deposition Velocity

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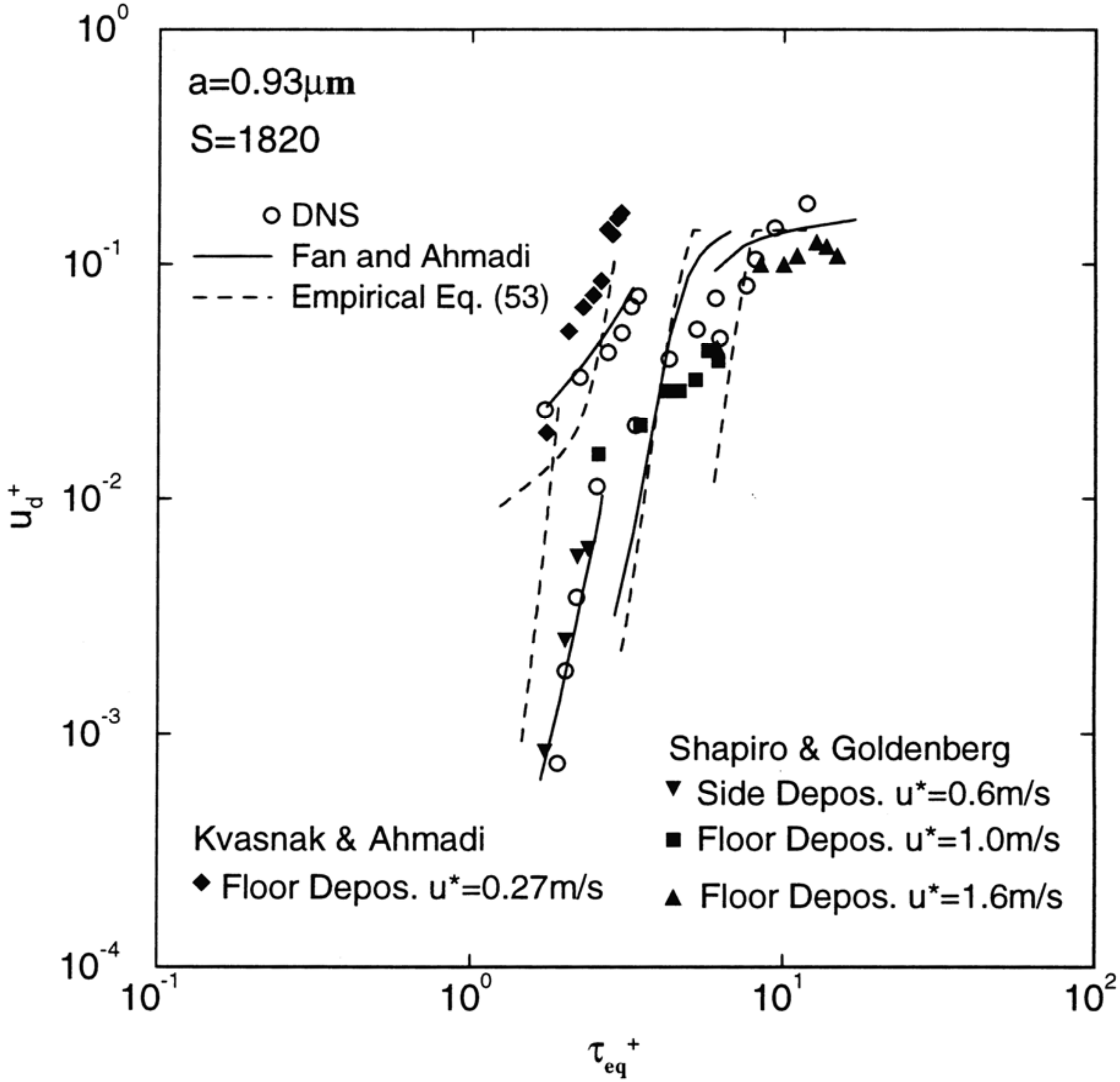
Turbulent Flow

DNS

Comparison with Experimental Data



Empirical Equation for Fiber Deposition Velocity

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$$u_d^+ = \begin{cases} 0.0185 \times \left[\dfrac{\dfrac{\beta L^{+2}}{\beta+3} + \dfrac{4\beta \tau_{eq}^{+2} g^+ L_1^+}{0.01085(\beta+3)(1+\tau_{eq}^{+2} L_1^+)}}{3.42 + \dfrac{\tau_{eq}^{+2} g^+ L_1^+}{0.01085(1+\tau_p^{+2} L_1^+)}} \right] & \\ \times \left[1 + 8e^{-(\tau_{eq}^+ - 10)^2/32}\right] \dfrac{1}{1 - \tau_{eq}^{+2} L_1^+ (1 + \dfrac{g^+}{0.037})} & \text{if } u_d^+ < 0.14 \\ 0.14 & \text{otherwise} \end{cases}$$

$$L_1^+ = \frac{3.08}{S d_{eq}^+} = \frac{0.725}{\sqrt{S \tau_{eq}^+}}$$

$$d_{eq}^+ = \sqrt{\frac{18 \tau_{eq}^+}{S}}$$

Angular Fiber

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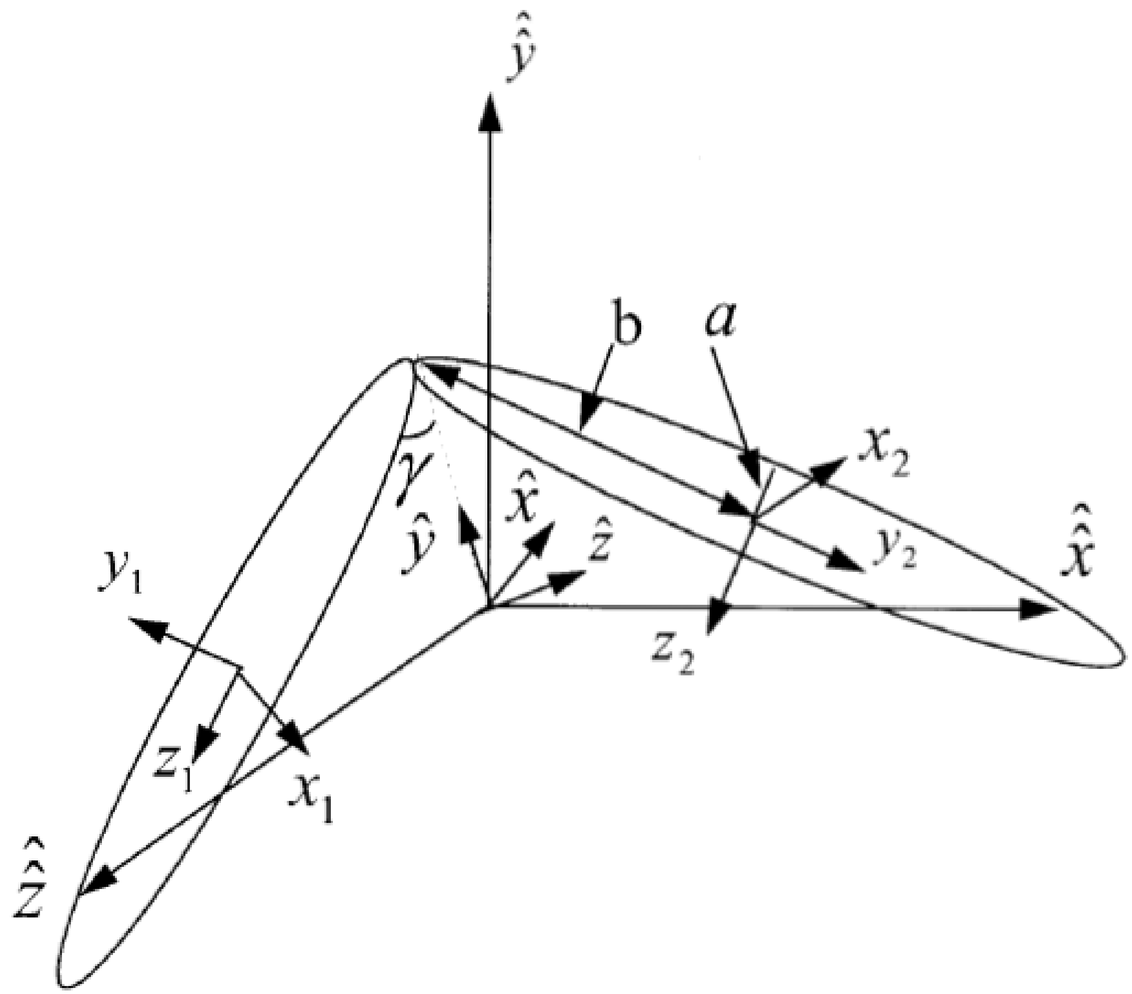

Angular Fiber Sample Trajectories

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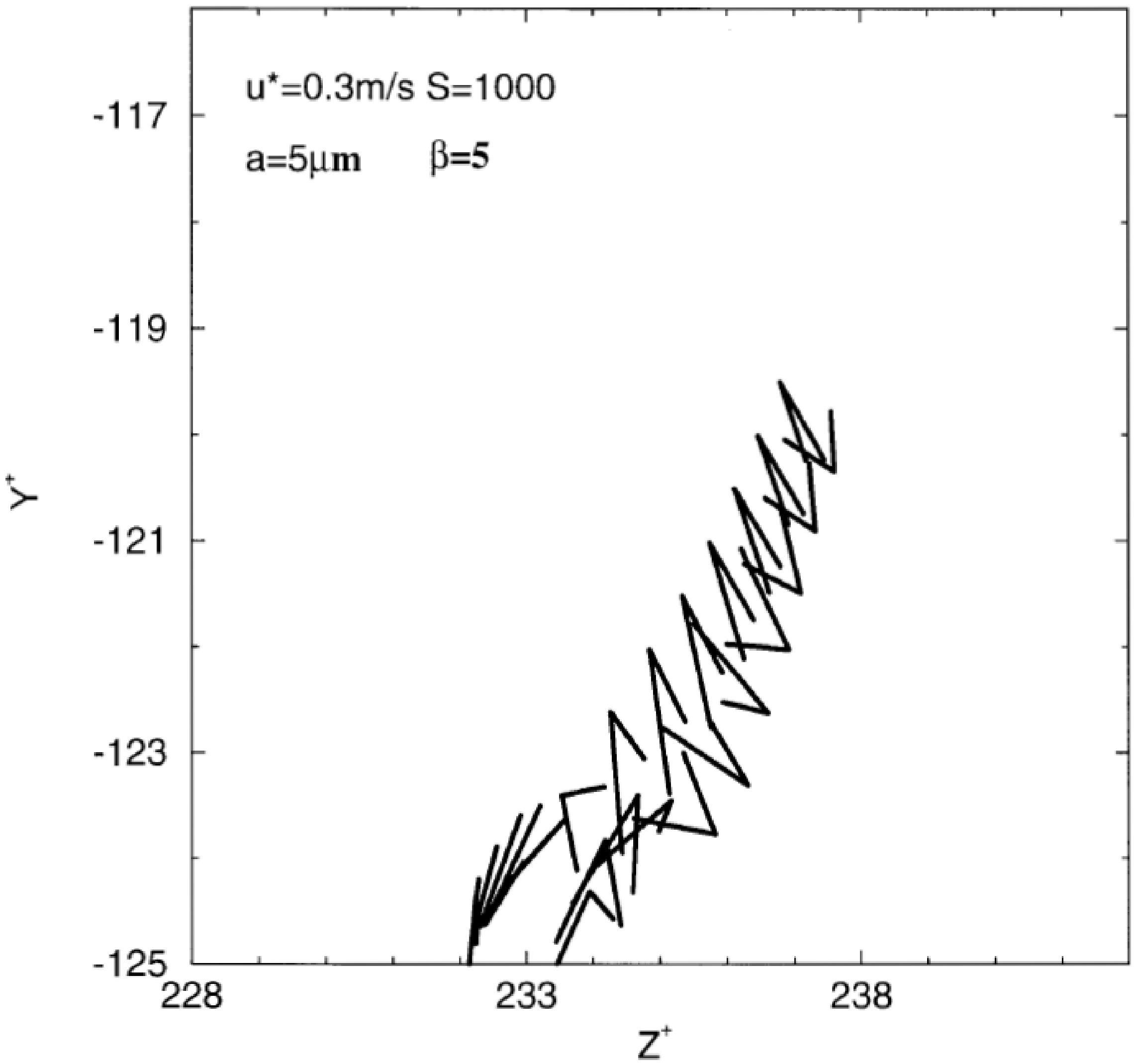

Angular Fiber Deposition Velocity

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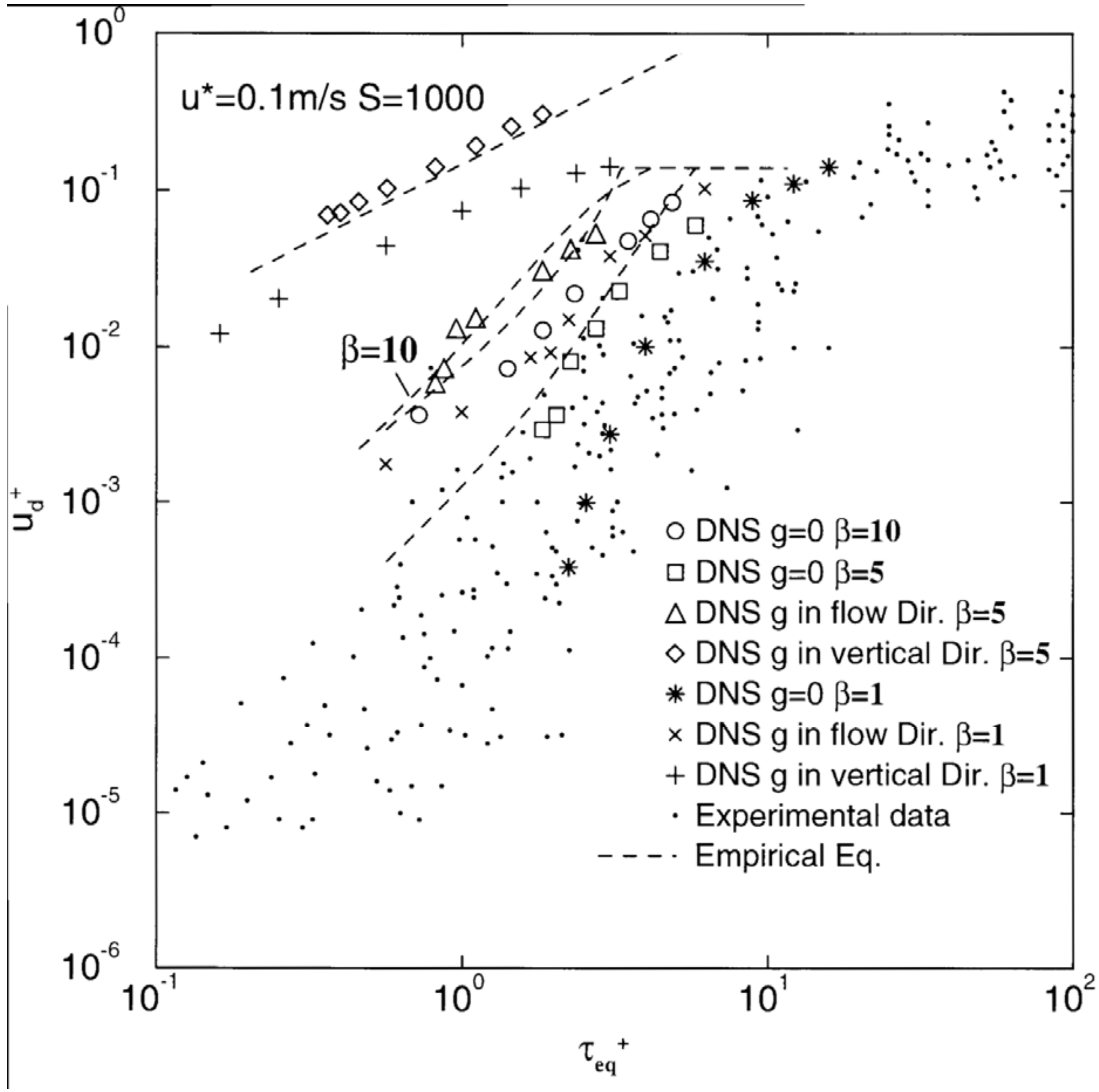
Turbulent Flow

DNS



Angular Fiber Deposition Velocity

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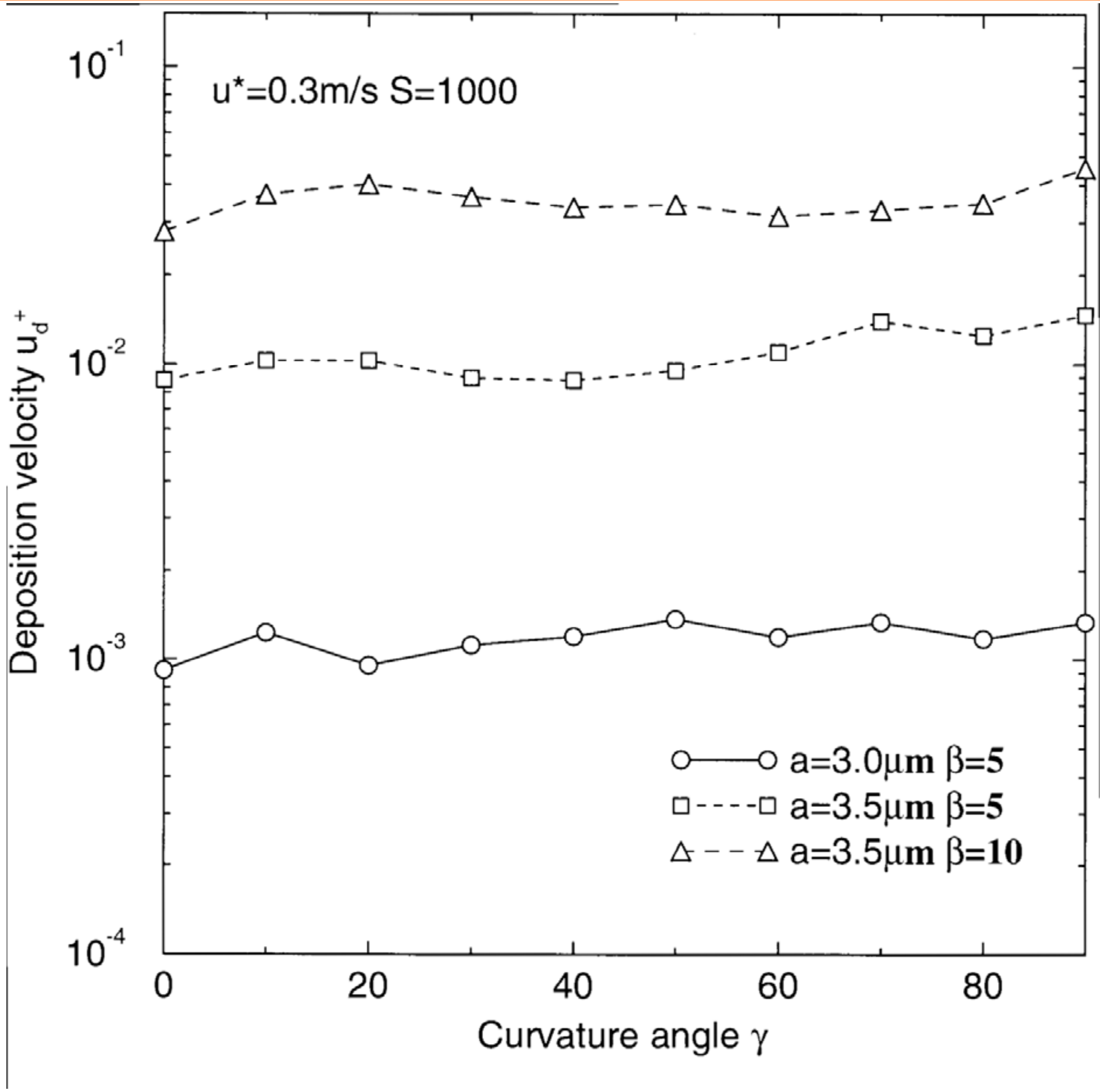
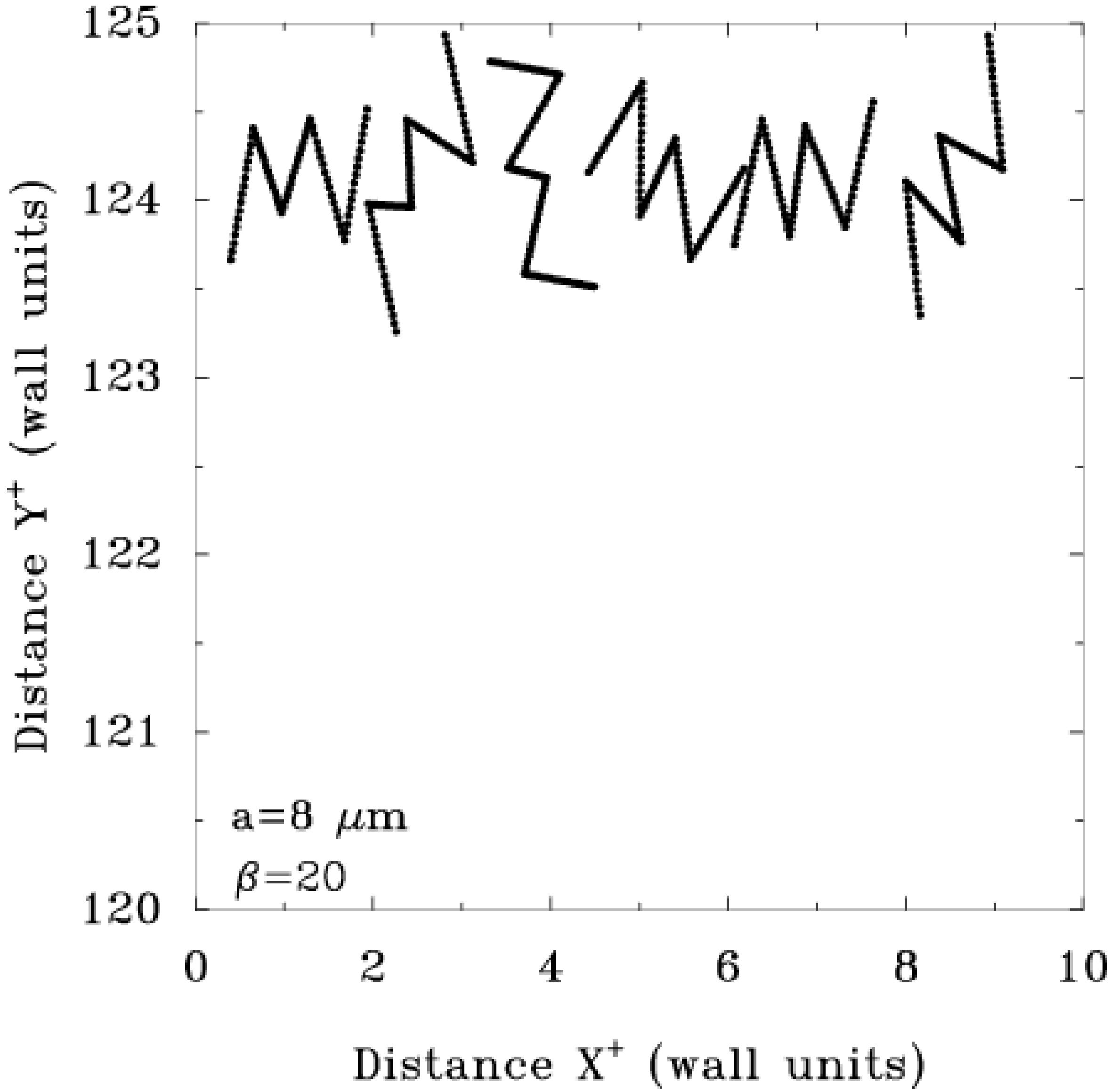


Curly Fiber Motions



Ellipsoidal Particles

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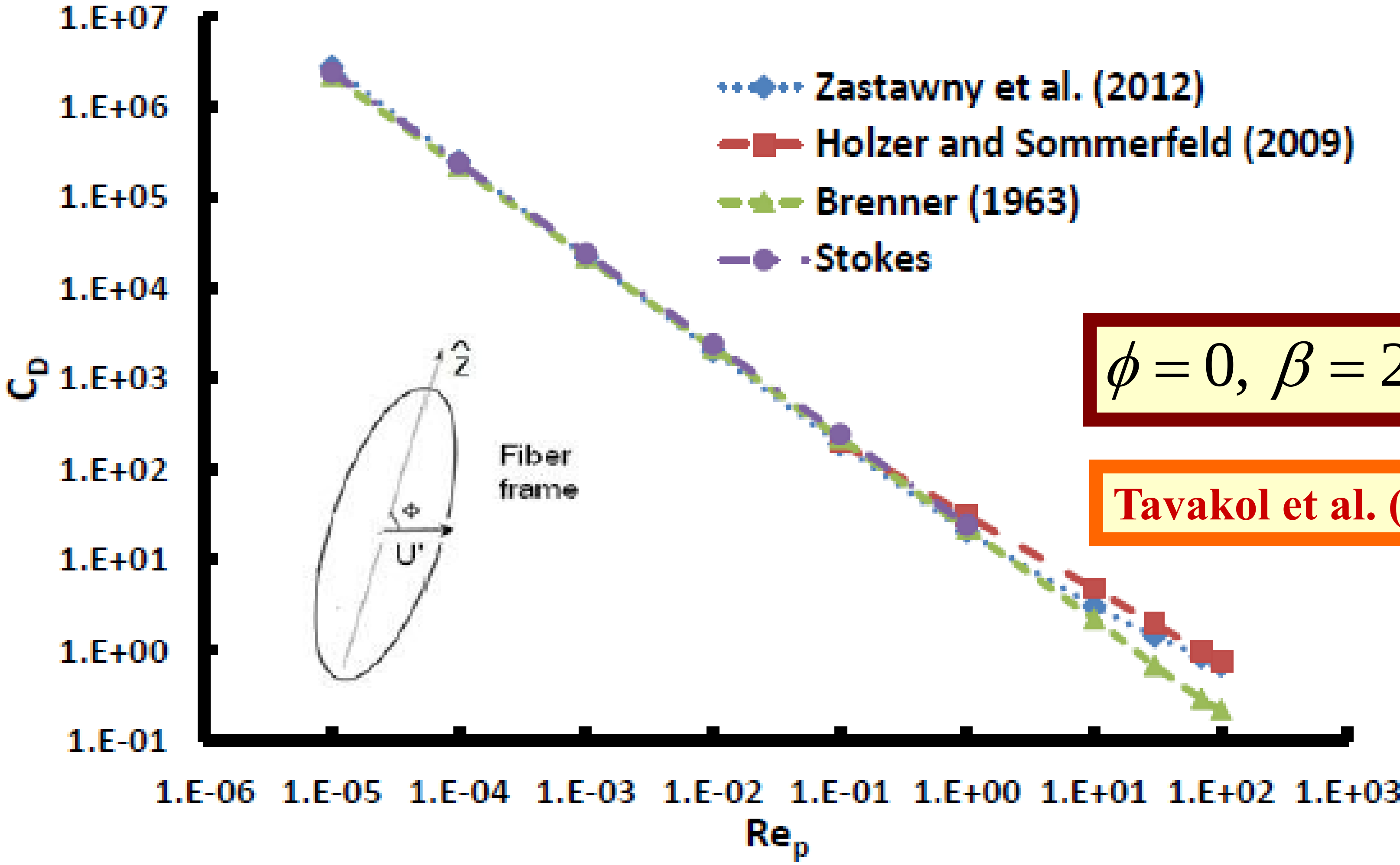
Comparison of creeping and non-creeping drag



$\phi = 0, \ \beta = 2$

Tavakol et al. (2015)

Ellipsoidal Particles

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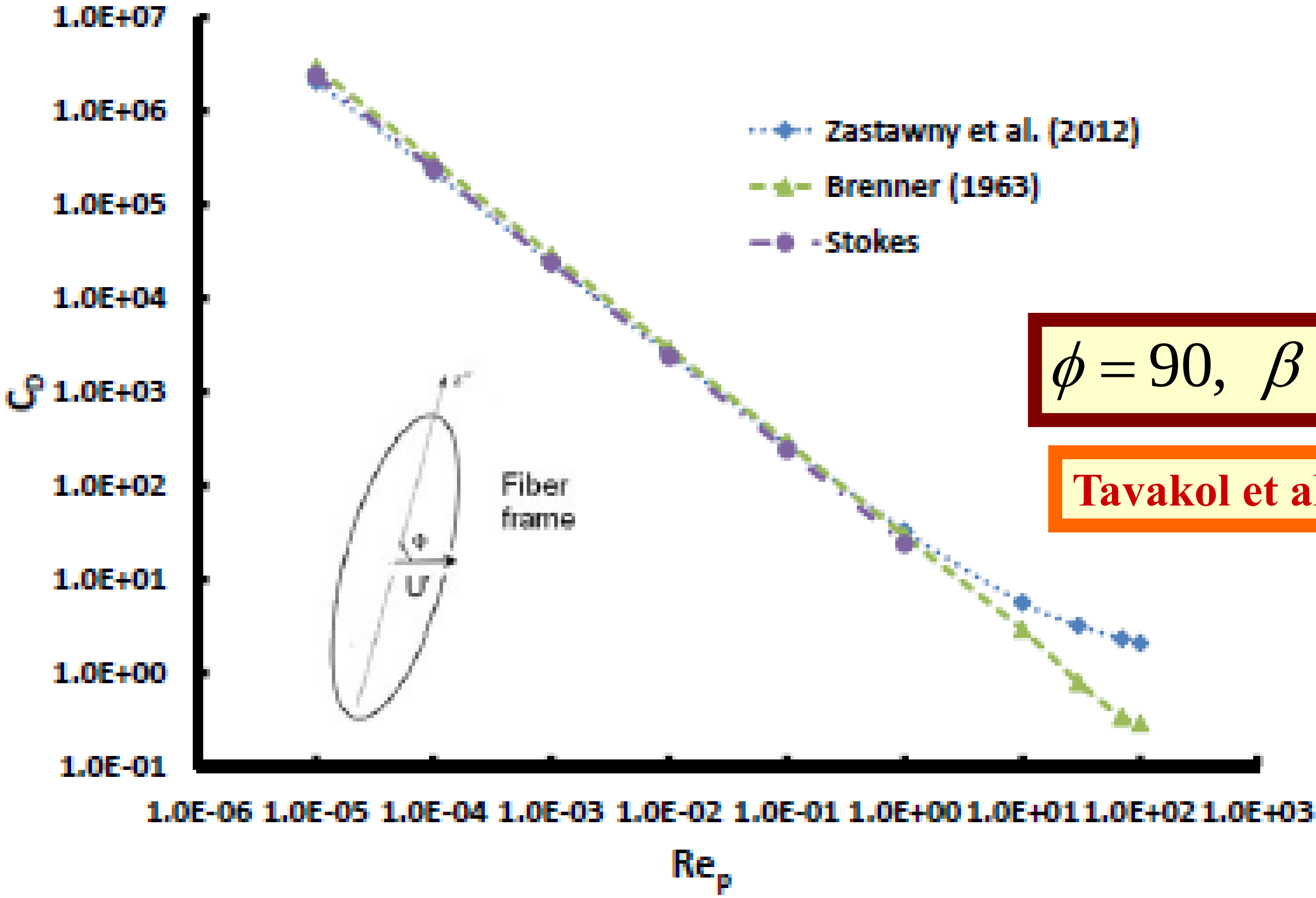
Comparison of creeping and non-creeping drag



$\phi = 90, \ \beta = 5$

Tavakol et al. (2015)

Ellipsoidal Particles

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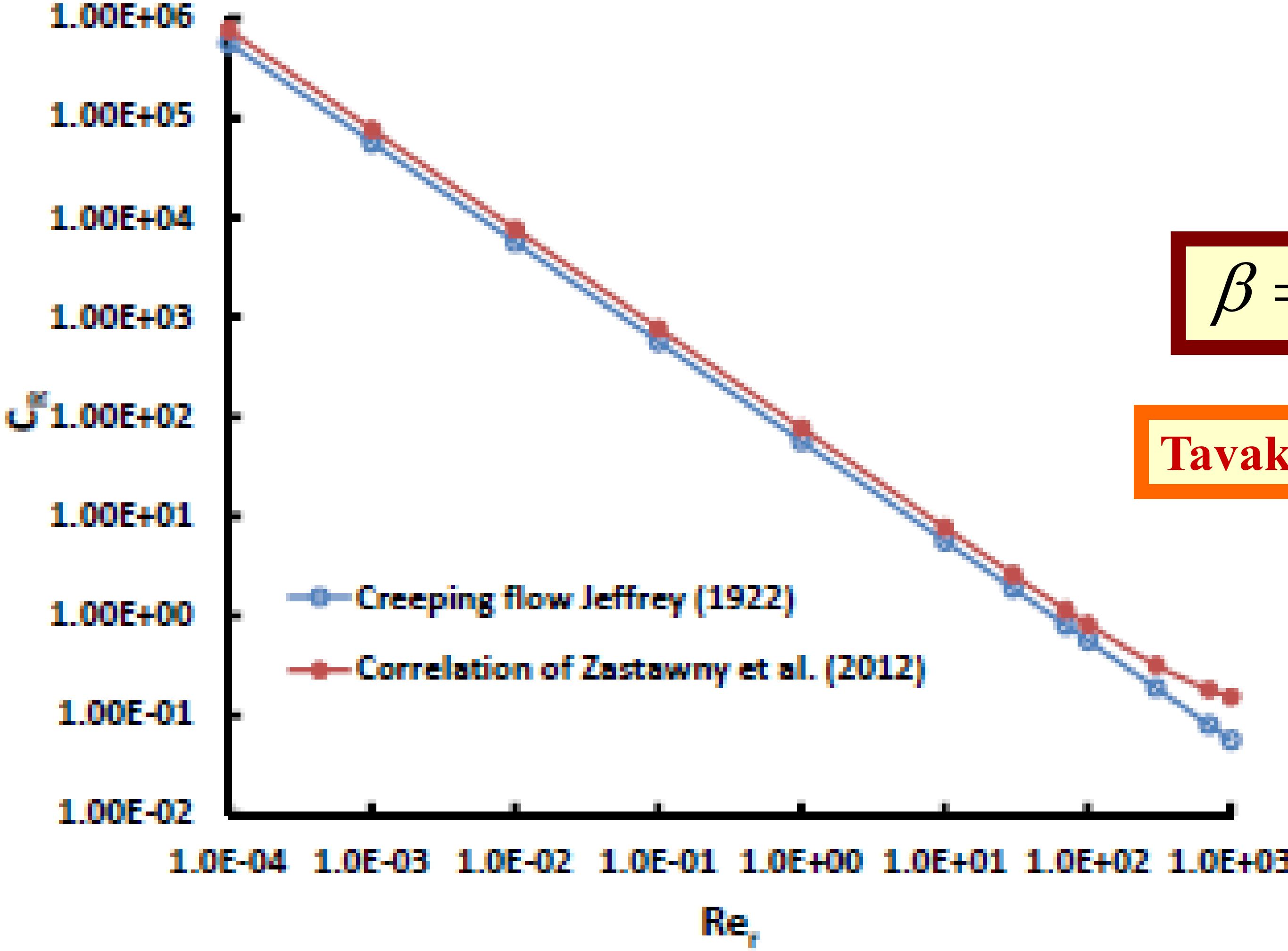
Comparison of creeping and non-creeping torque



$\beta = 5$

Tavakol et al. (2015)

Fiber Transport and Deposition

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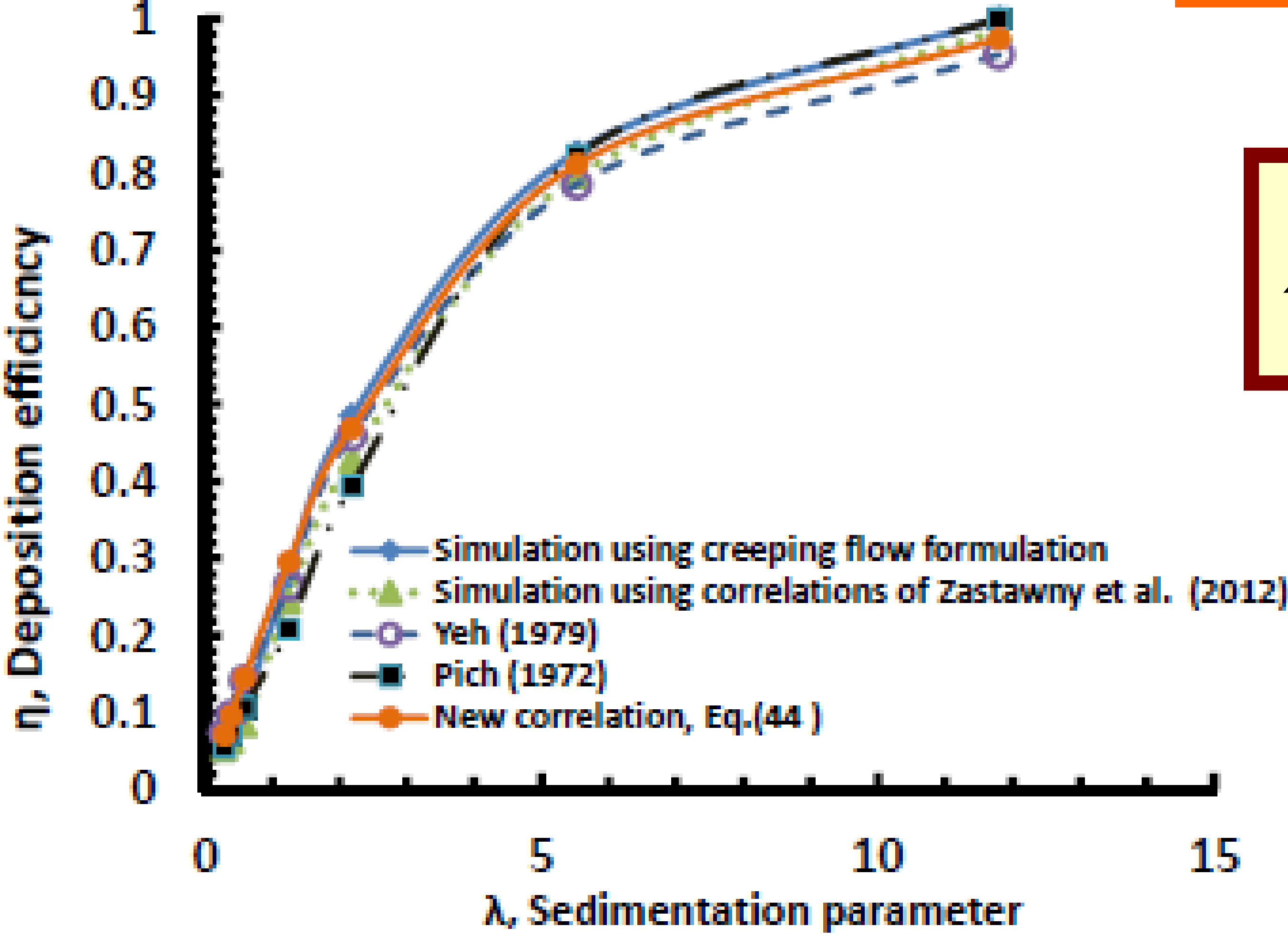
Creeping and Non-creeping Models

Laminar Pipe Flow, a=10µm, β=5

$$\lambda = \frac{\rho_p g \beta d^2}{18 \mu U}$$



Conclusions

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- **Aspect ratio plays an important role on ellipsoidal particle deposition rate.**
- **The simulation results for deposition velocity are in good agreement with the experimental data.**
- **Deposition velocity increases with fiber aspect ratio.**
- **Effect of gravity on particle deposition velocity depends on the magnitude of shear velocity.**
- **The gravitational sedimentation enhances the deposition rate on the lower wall in horizontal duct flows.**

Collaborators

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- Dr. P. Zamankhan
- Dr. L. Tian
- Dr. Kevin Shanley
- Dr. Mazyar Salmanzadeh
- Dr. F-G Fan
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- Dr. M. Shams
- Prof. McLaughlin
- Prof. Saidi
- Dr. A. Li
- Dr. O. Abouali
- Dr. W. Kvasnak
- Dr. X. Zhang

Thank You!

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Questions?