A grayscale visualization of a square cylinder wake, showing complex, swirling flow patterns and vortices. A vertical rod is visible on the left side of the image.

Modification of the modal characteristics of a square cylinder wake obstructed by a multi-scale array of obstacles

J.E. Higham & W. Brevis

Sheffield Environmental Fluid Mechanics Group

The governing equations of a fluid

Convection

Pressure Gradient

Viscous Forces

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho} \nabla \mathbf{p} + \nu \nabla^2 \mathbf{u}$$

∇ - gradient

\mathbf{u} - velocity

\mathbf{p} - pressure

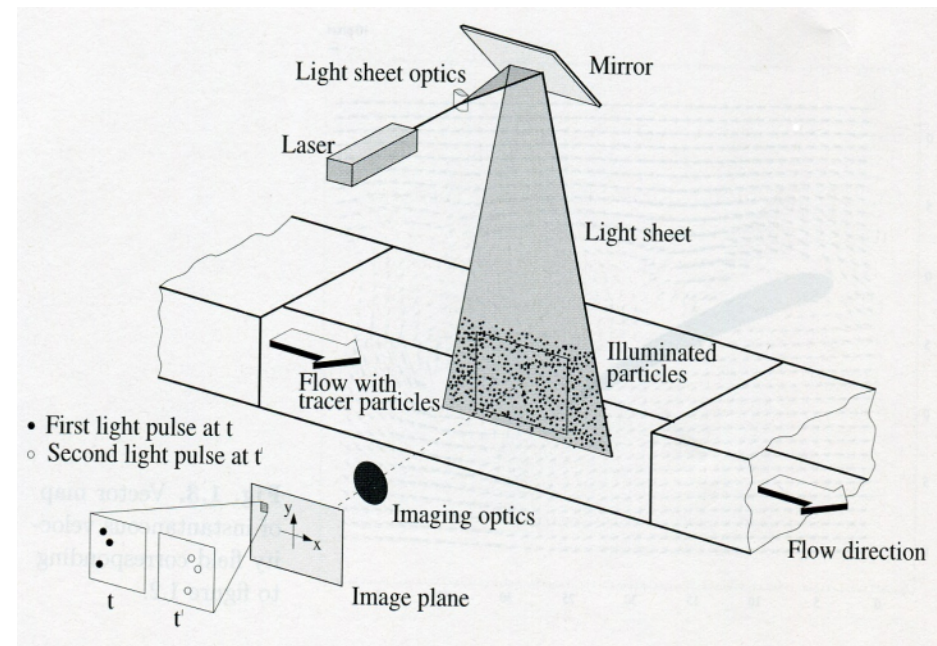
ρ - density

ν - kinematic viscosity

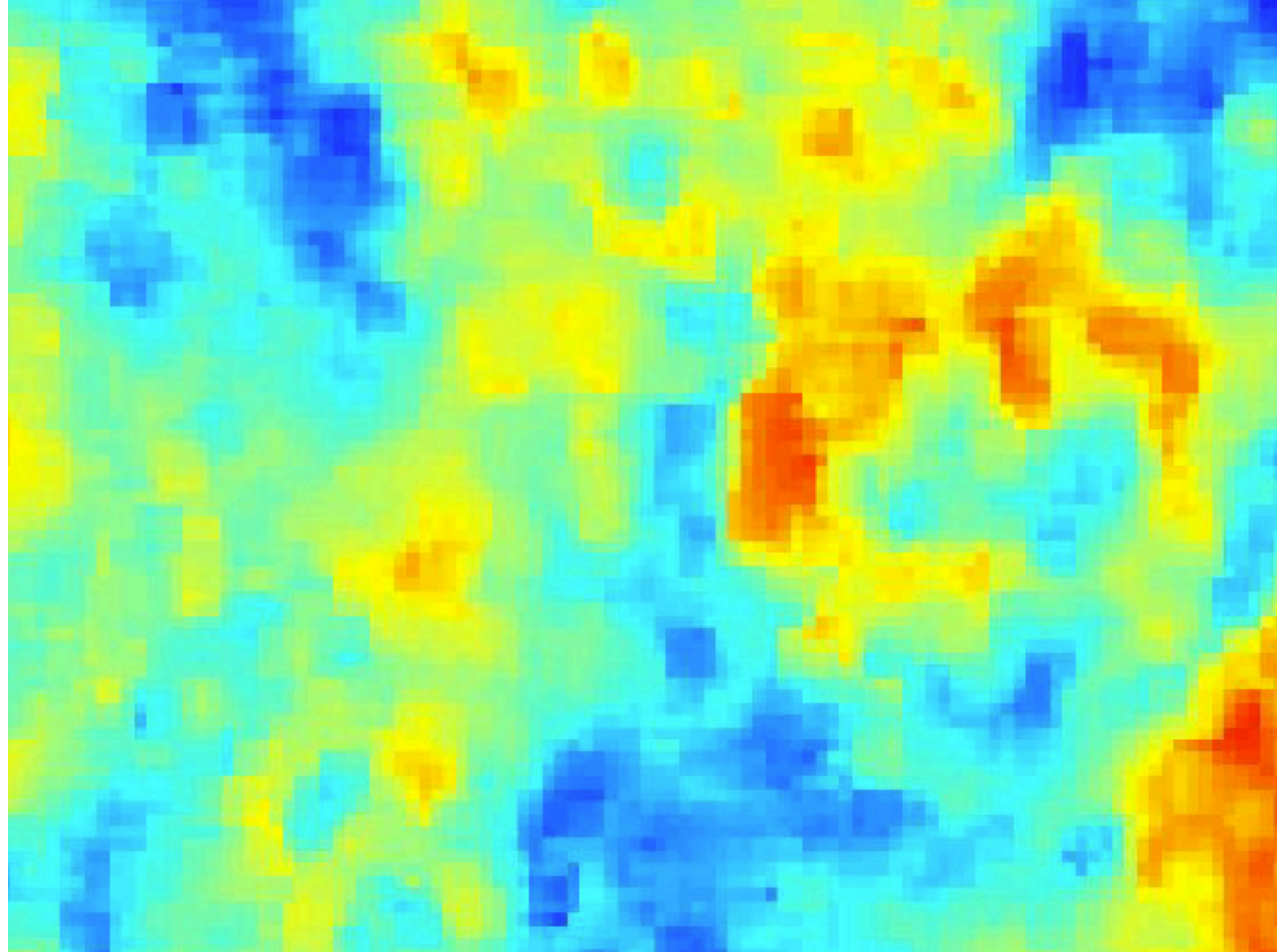
Particle Image Velocimetry

Is an image based method used to determine instantaneous velocity fields by measuring the displacement of a cluster of tracers in a fluid.

- Tracer particles are added to the flow.
- The particles are illuminated.
- Scattered light recorded.
- Displacements are determined from correlations in the Fourier domain.
- This is a fully non-intrusive method.
- PIV is NOT perfect.
- Tomographic PIV etc.



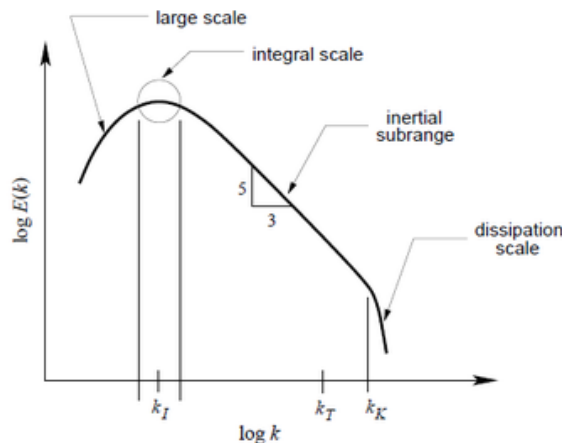
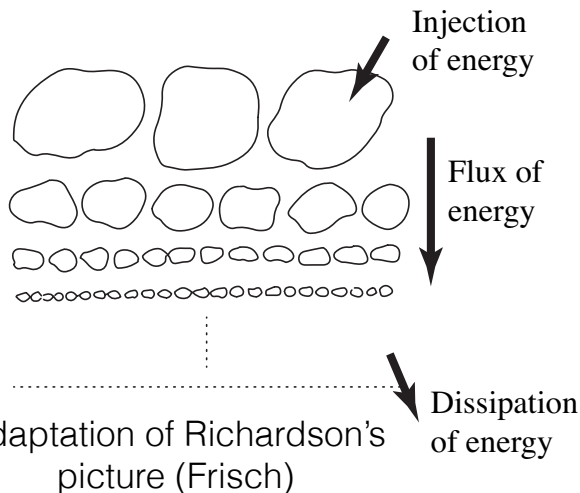
Raffel, Willert, Wereley, Kompenhans.
Particle Image Velocimetry. 2nd Edition,
Springer. 2007



How do we interpret these data?

Osborne Reynolds' Decomposition

$$u_i = \bar{u}_i + u'_i$$



Adaptation of Taylor's spectrum (with Kolmogorov additions).

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla\mathbf{p} + \nu\nabla^2\mathbf{u}$$

$$\rho\bar{u}_j\frac{\partial\bar{u}_i}{\partial x_j} = \bar{f}_i + \frac{\partial}{\partial x_j}\left[-\bar{p}\delta_{ij} + \mu\left(\frac{\partial\bar{u}_i}{\partial x_j} + \frac{\partial\bar{u}_j}{\partial x_i}\right) - \rho\overline{u'_i u'_j}\right]$$

$$\tau_{ij} = -\rho\overline{u'_i u'_j}$$

This is the Reynolds stress i.e. momentum flux.

This show regions which can lead to erosion etc.

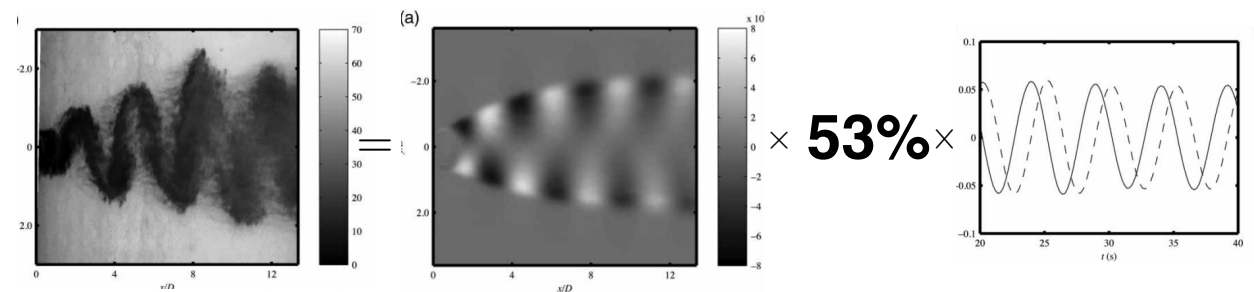
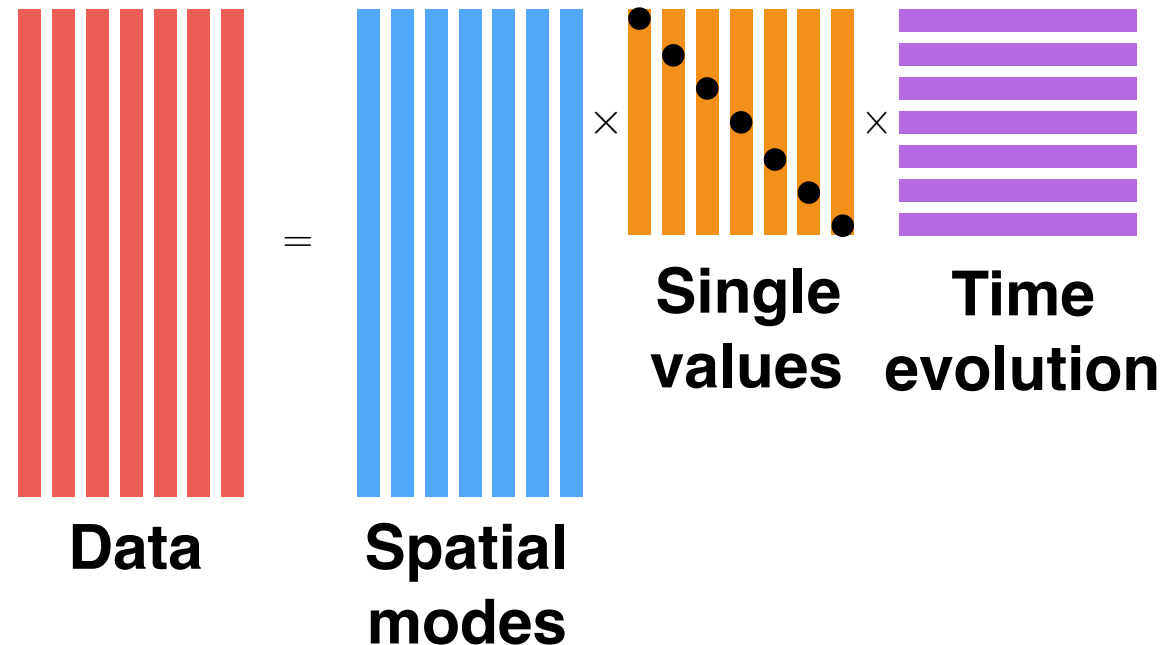
BUT

Does not represent any dynamics, is a very old hat approach.

Modal Decomposition Techniques

(Hussain, A. 1983) Coherent structures: reality of myth?

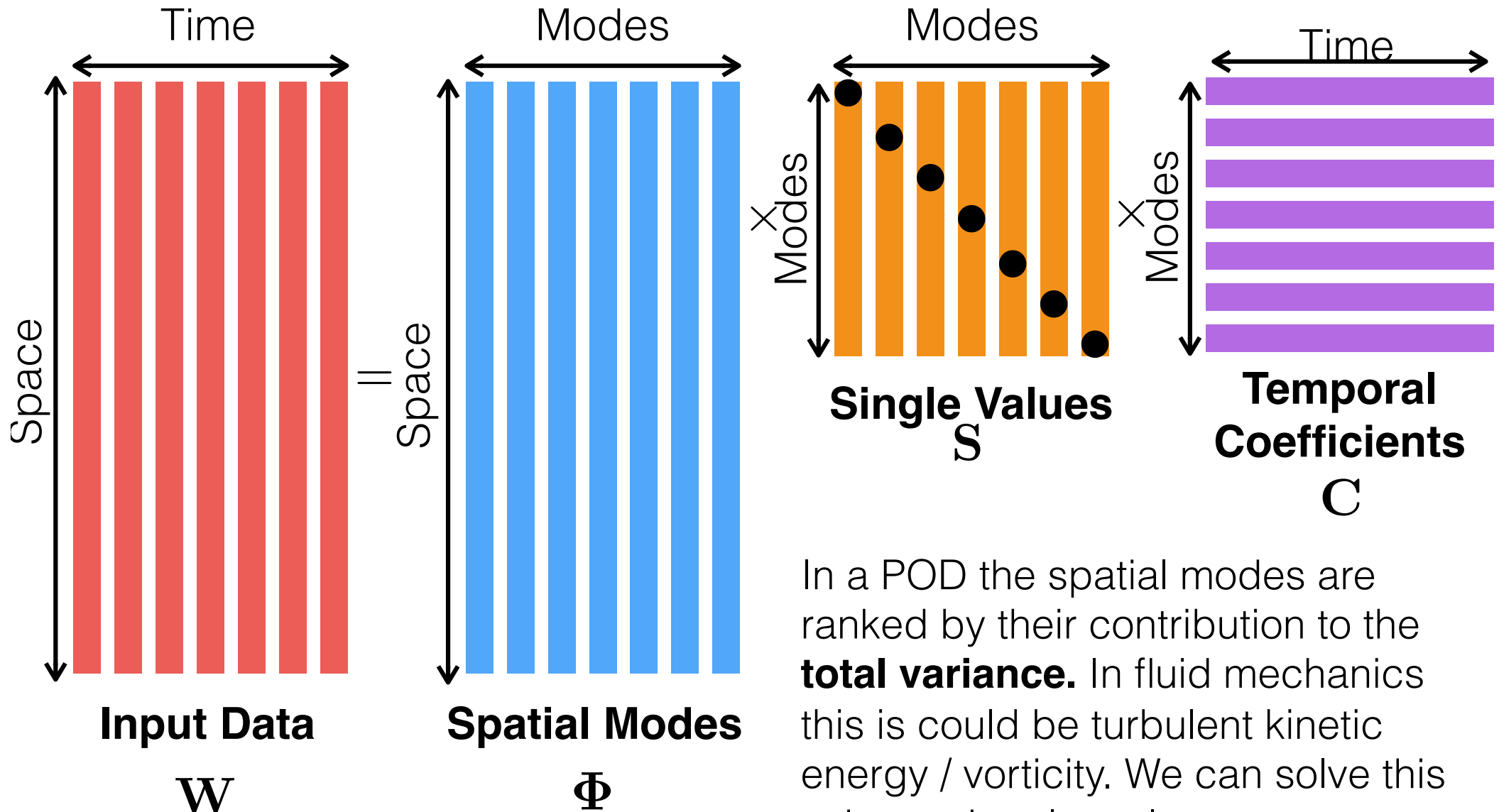
- In fluid mechanics there are two main techniques.
- POD (Proper Orthogonal Decomposition) - based on spatial correlations.
- DMD (Dynamic Mode Decomposition) - based on temporal correlations*.
- The main aim in a modal decomposition technique to extract regions of coherence.
- But what is coherence **



*not explained today, but see Higham et al. (2017) AWR

**not explained today, but see Higham et al. (In Press) JHR

Proper Orthogonal Decomposition

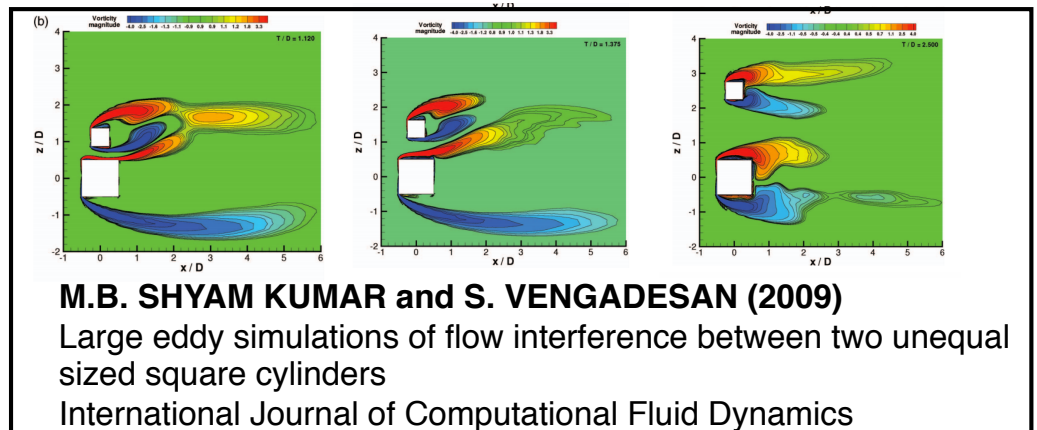
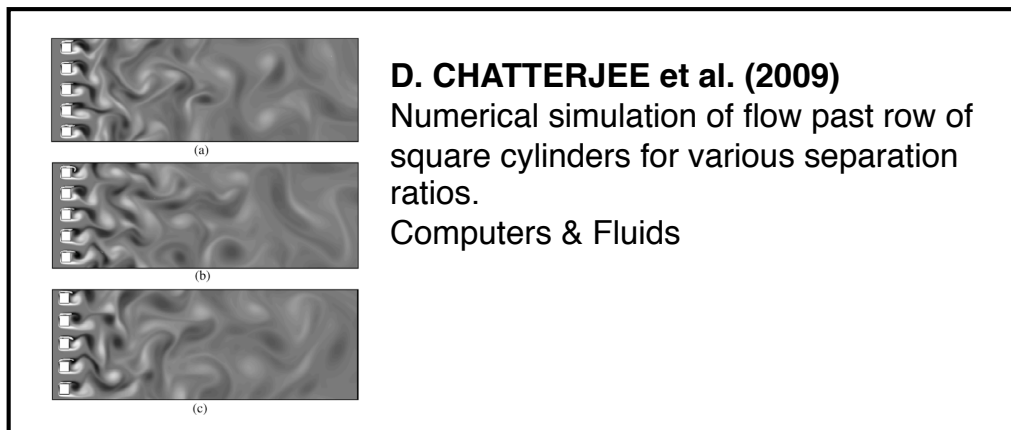
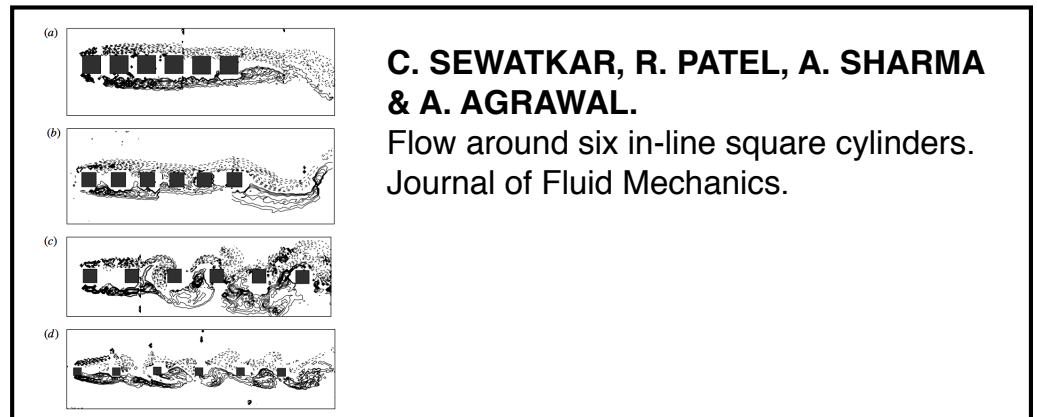
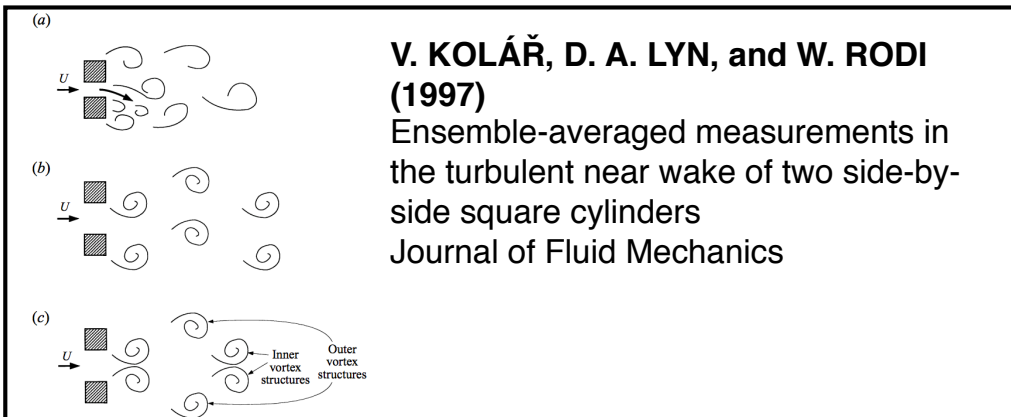
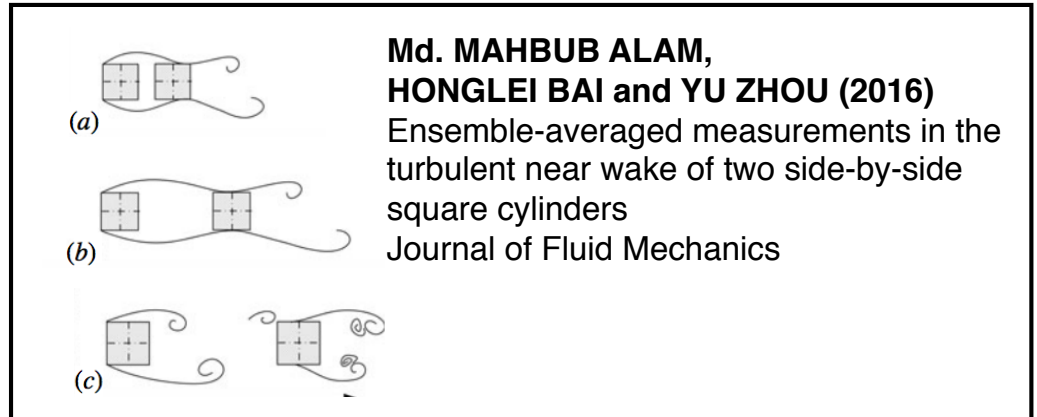
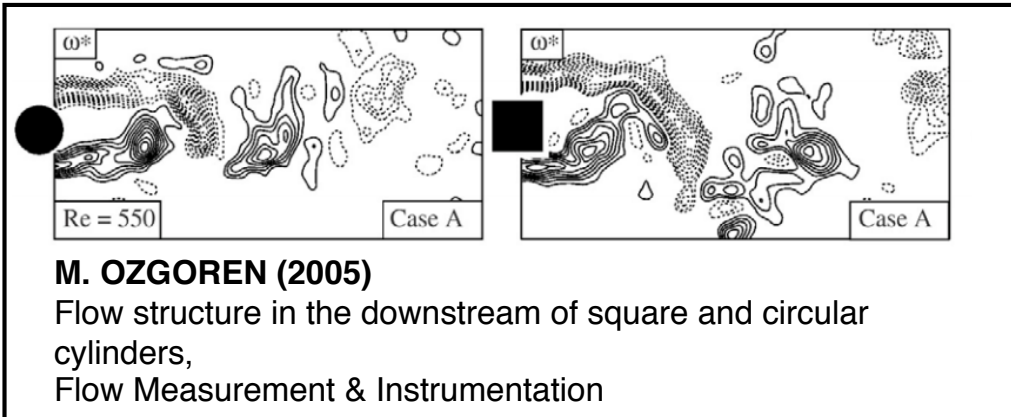


In a POD the spatial modes are ranked by their contribution to the **total variance**. In fluid mechanics this is could be turbulent kinetic energy / vorticity. We can solve this using a singular value decomposition.

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Cylinder wakes

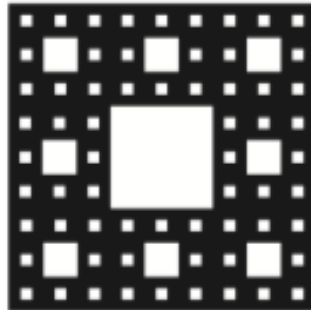


Test cases

Case I



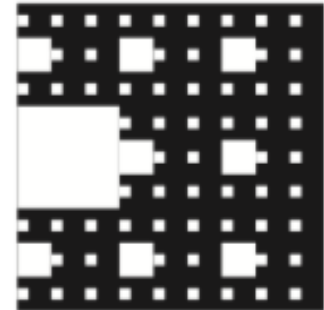
Case II



Case III



Case IV



Experimental Setup

Seeding	Type	Polyamide powder
	Specific gravity	1.016 gcm^{-3}
	Diameter	$100 \mu\text{m}$
Light sheet	Laser type	Double pulsed Nd:YAG
	Maximum energy	200 mJ
	Wave length	532 nm
	Thickness	2mm
Camera	Type	Imager MX 4M
	Resolution	$2048 \times 2048 \text{ px}$
	Pixel size	0.21 mm
	Lens focal length	24 mm
Imaging	Viewing area	$440 \text{ mm} \times 440 \text{ mm}$
PIV Analysis	Interrogation area	final integration window size $16 \text{ px} \times 16 \text{ px}$
	Overlap	75%
	Approximate resolution	$3.5 \text{ mm} \times 3.5 \text{ mm} \times 3.5 \text{ mm}$

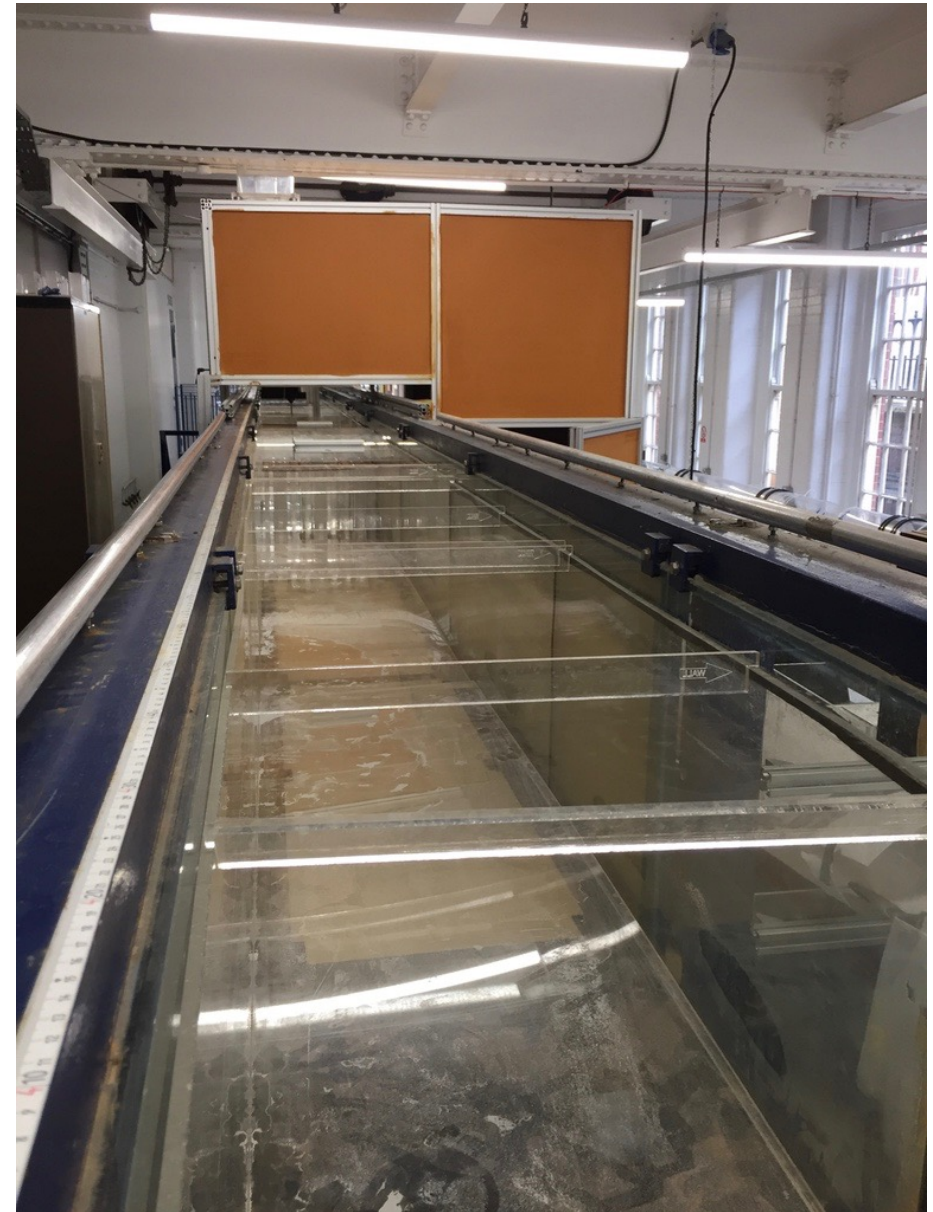
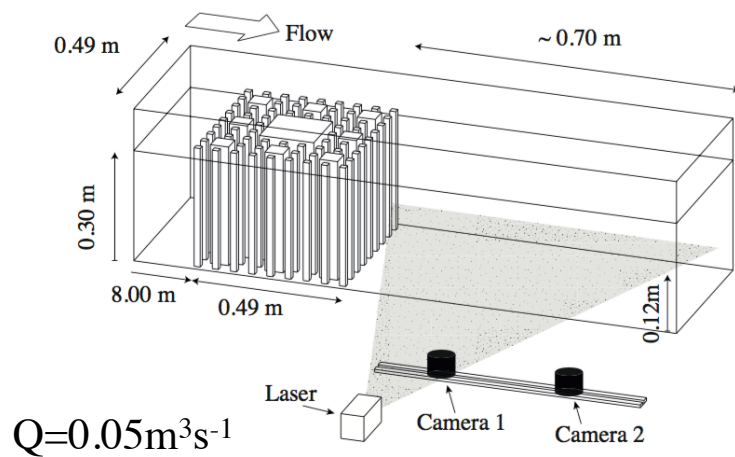


Figure 2: View of the measurement section and experimental setup (not to scale)

Streamwise mean flow

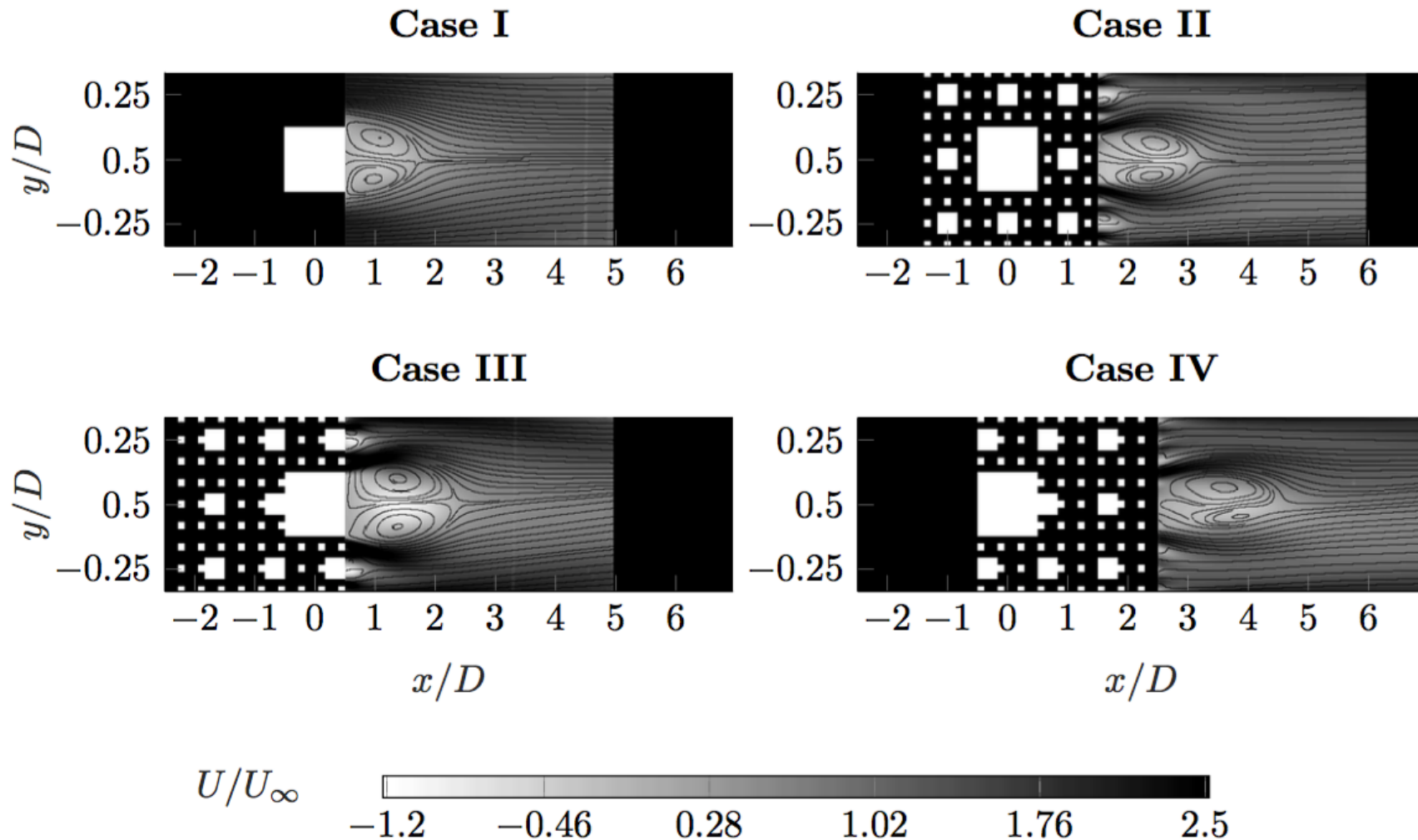


Figure 3: Contour plots of the streamwise mean velocity, U , normalised by the bulk velocity, U_∞ , with streamlines overlaid

(Higham J.E. & Brevis W. (In Press) - Experimental Thermal and Fluid Science.)

Streamwise mean flow (centre line)

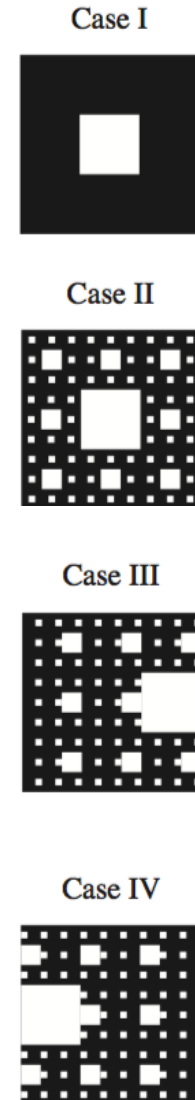
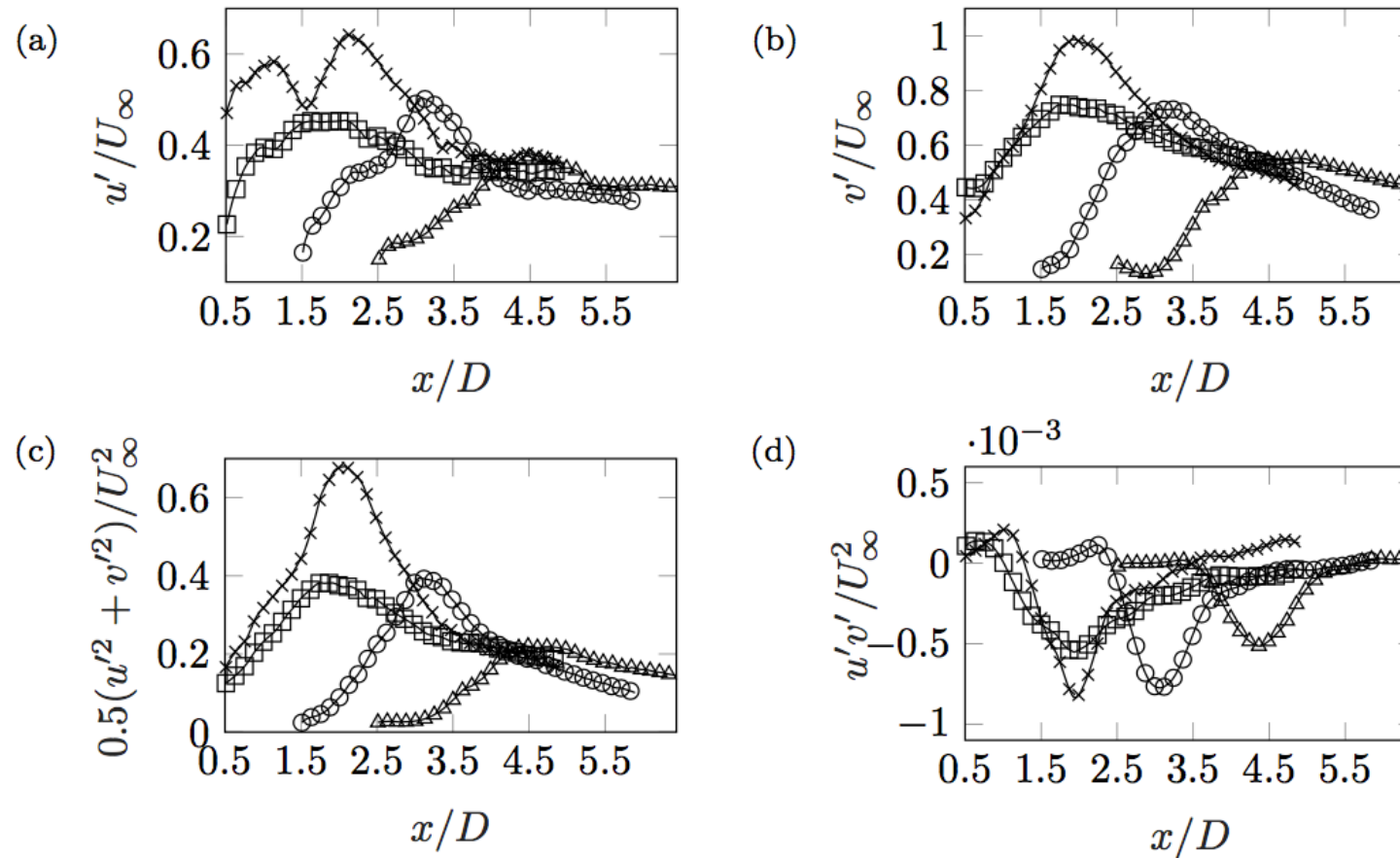


Figure 4: Centre line time averaged statistics. Case I – \square , case II – \bigcirc , case III – \times & case IV – \triangle . (a) u'/U_∞ , (b) v'/U_∞ , (c) $0.5(u'^2 + v'^2)/U_\infty^2$ & (d) $u'v'/U_\infty^2$.

POD (Φ_1 & 3)

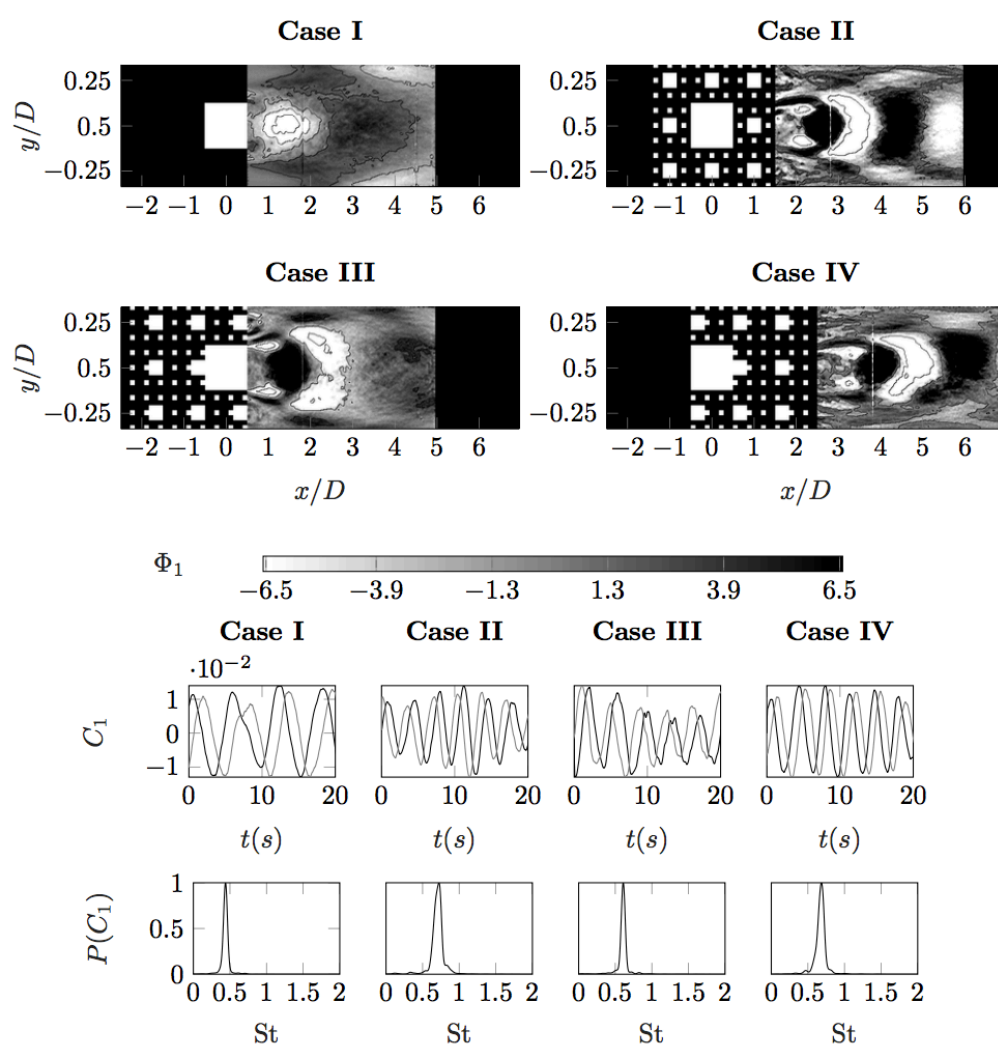


Figure 6: Top: POD modes Φ_1 . Middle: POD temporal coefficients C_1 & C_2 (grey). Bottom: Fourier power spectrum of C_1 . (Φ_2 is not plotted as it is a conjugate pair of Φ_1 .)

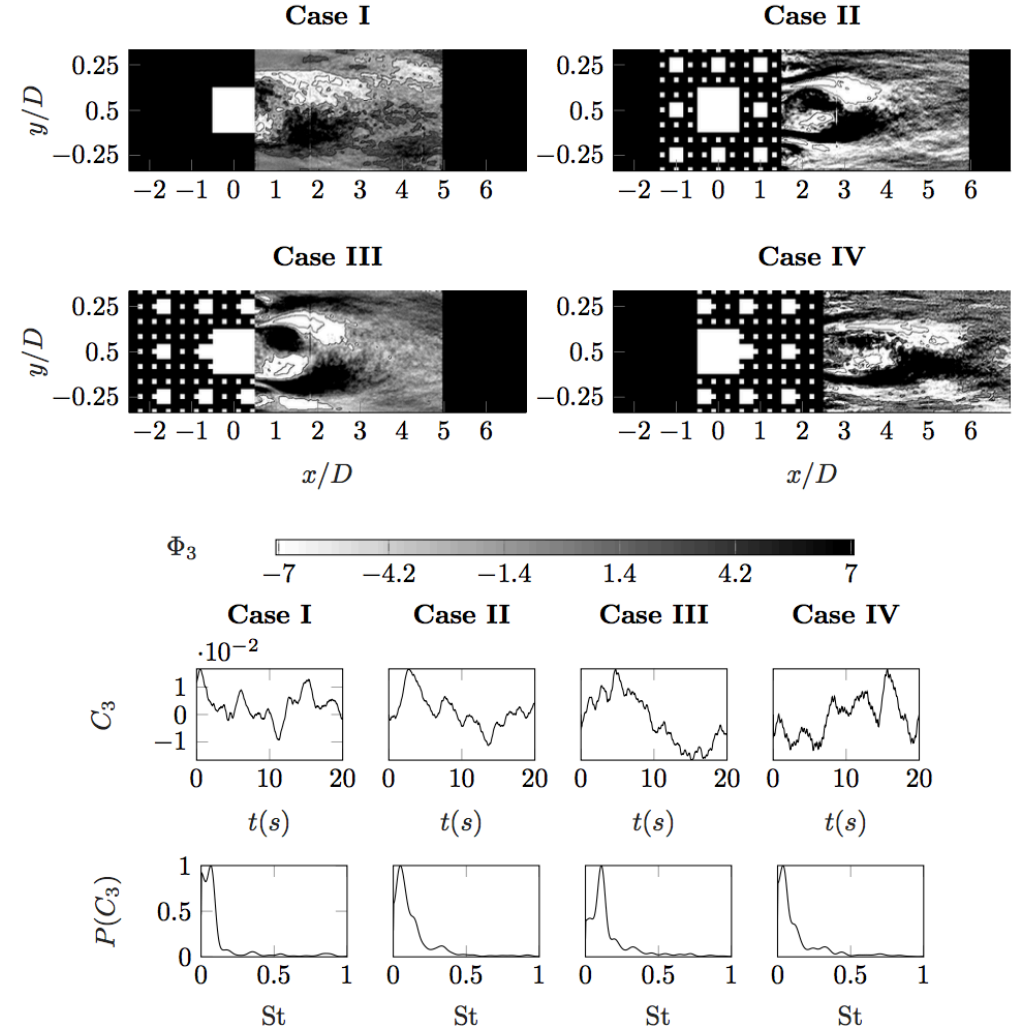


Figure 7: Top: POD modes Φ_3 . Middle: POD temporal coefficients C_3 . Bottom: Fourier power spectrum of C_3 .

POD (Φ_4)

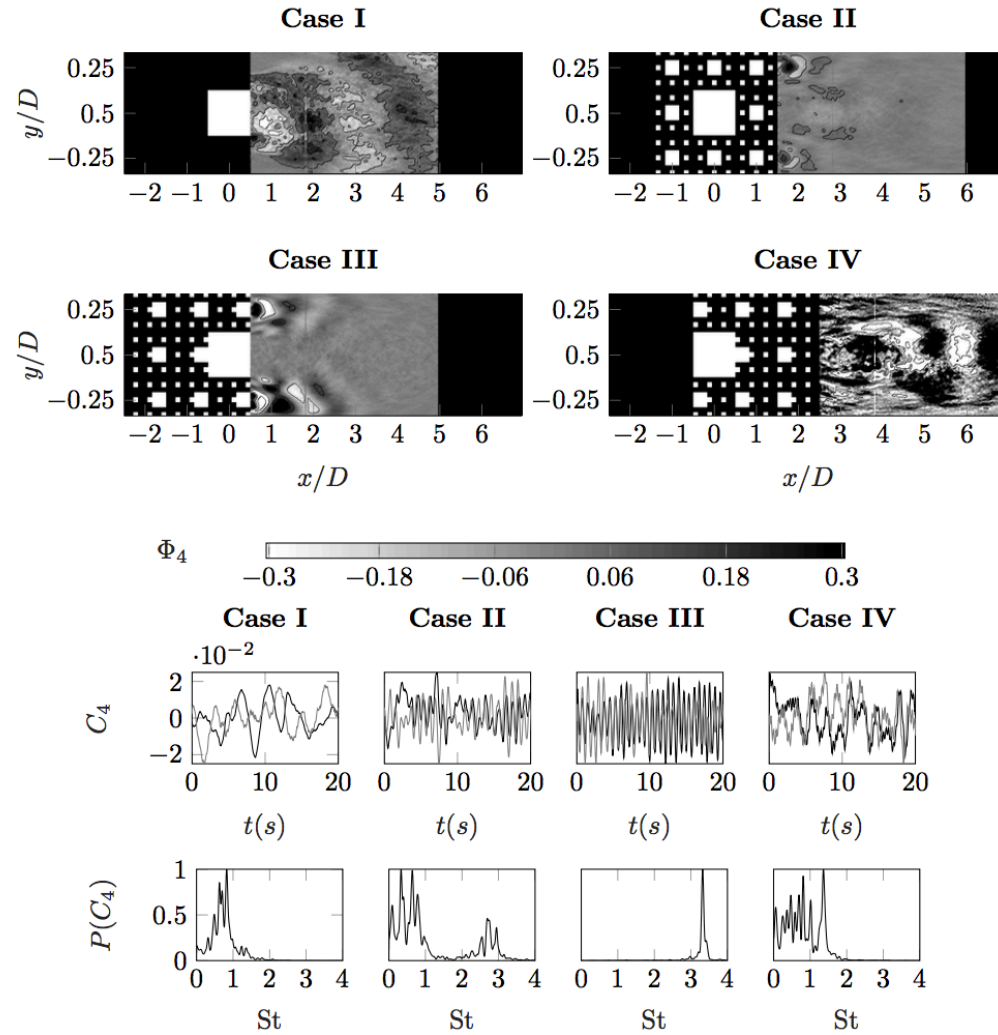


Figure 8: Top: POD modes Φ_4 . Middle: POD temporal coefficients C_4 & C_5 (grey). Bottom: Fourier power spectrum of C_4 . (Φ_5 is not plotted as it is a conjugate pair of Φ_4 .)

POD Spectra

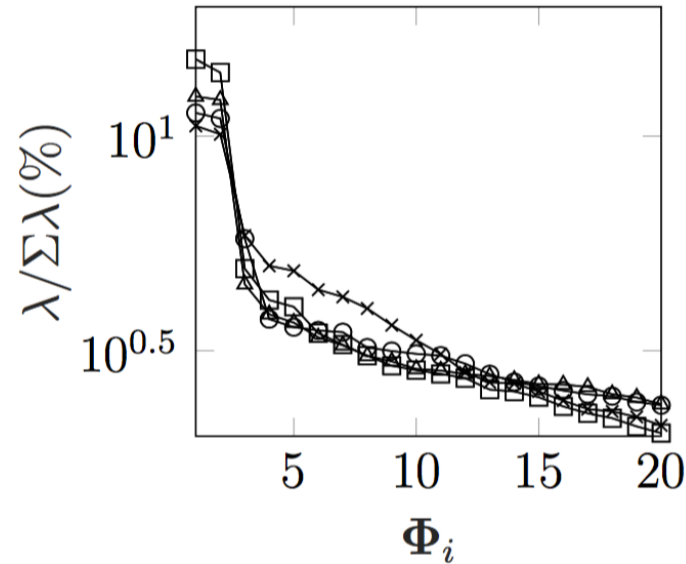
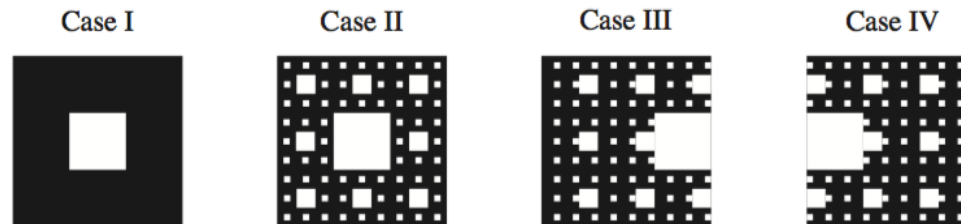


FIGURE 6.5: POD spectra where $\lambda = \text{diag}(\mathbf{S})$, these values represent the contribution of each the spatial modes Φ_i to the total variance. Case I – \square , case II – \circ , case III – \times & case IV – \triangle .



Conclusions

- The POD offers a wealth of information not typically attained from time-averaging.
- From these results we can see that the structures of the turbulence has changed.
- Therefore, we suggest that the arrangement of the multi-scales can be used to manipulate the turbulent flows.
- From further work we also show from time resolve data the ratio between the Taylor and Kolmogorov micro scales changes.

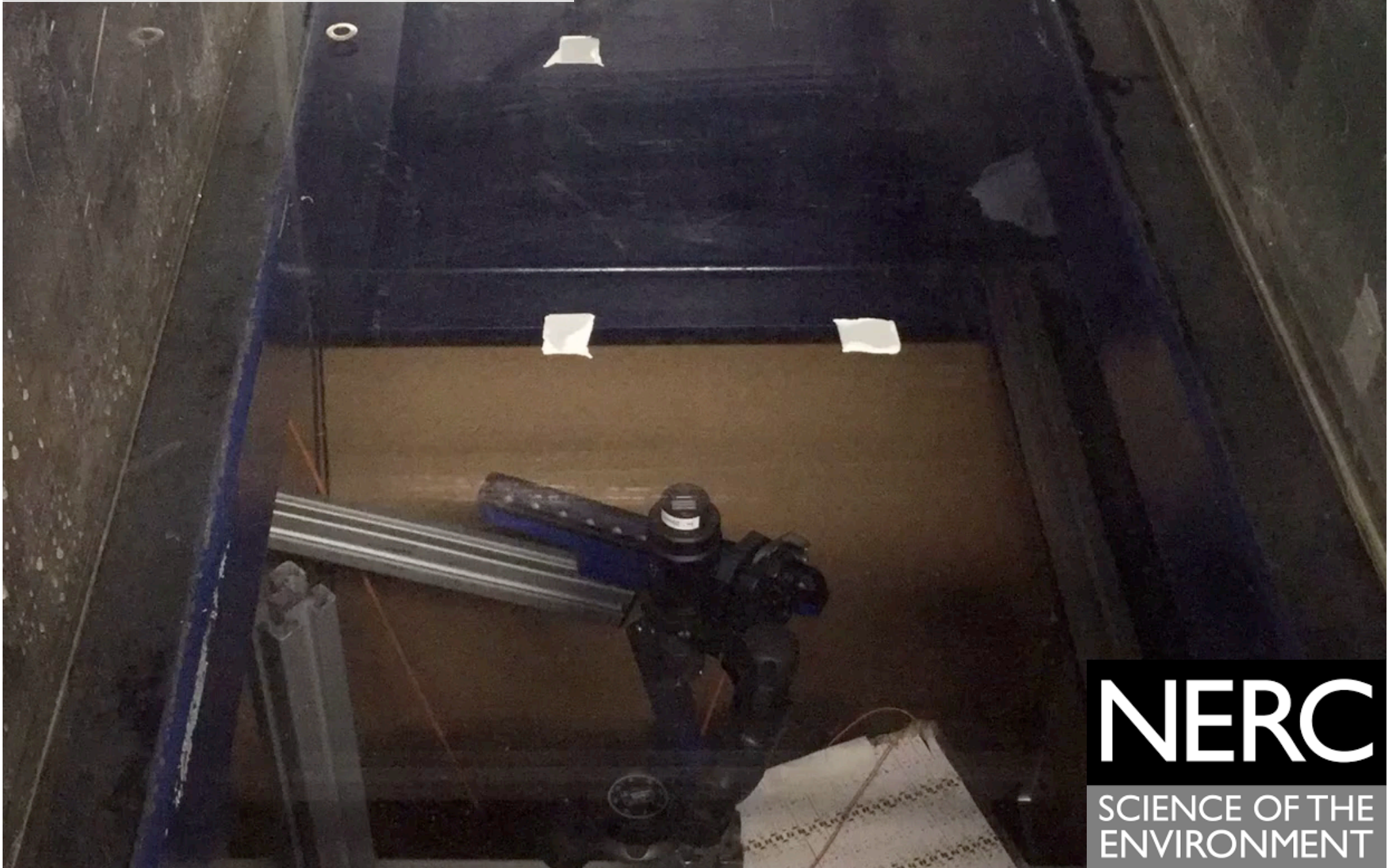
Future...

- So we know the Navier-Stokes doesn't apply - can we apply these modal decomposition techniques to the fluidised beds and the risers...

Questions?



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