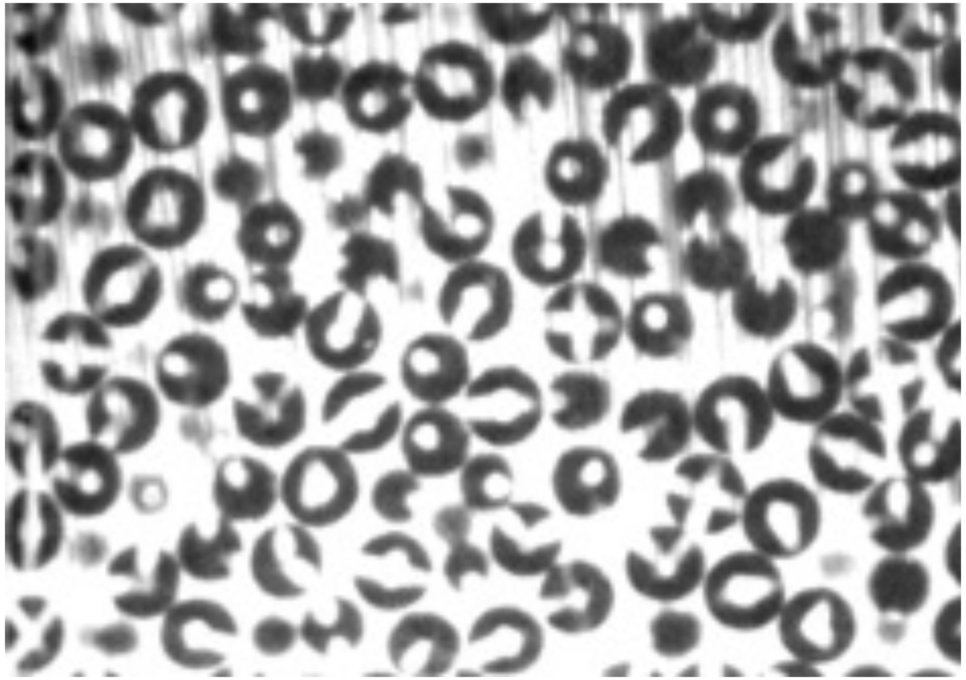
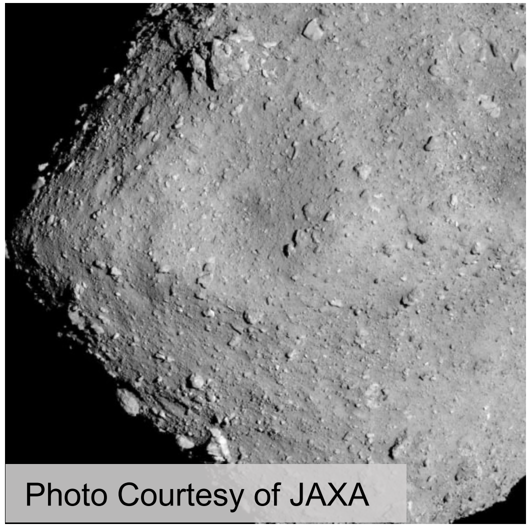
Experimentally measuring three-dimensional granular rotations.

Zackery A. Benson



Examples of granular materials

Asteroids and other ET objects can be modeled as granular material



- Collection of discrete particles that interact with a wide range of forces
- Inherently far from equilibrium
- Bulk properties depend heavily on material history

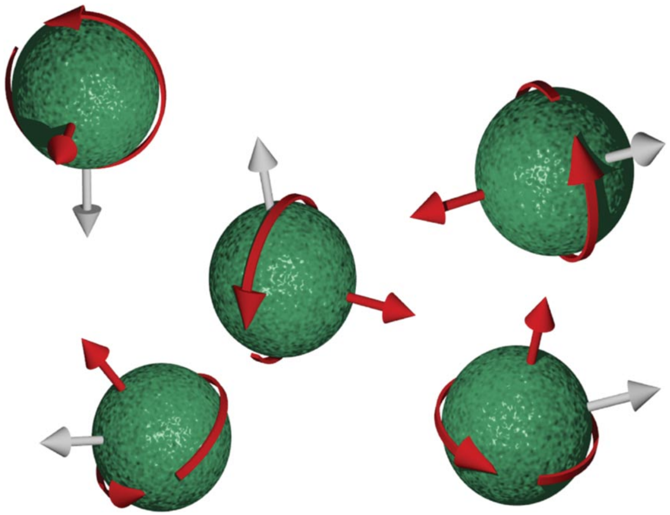
Ballast (bed of rocks) provides structural stability to railroad tracks

Rotations are needed for the complete study of particle dynamics

Rotational motion accounts for ½ the total degrees of freedom

For dense systems, energy dissipation is dominated by frictional contacts instead of collisions

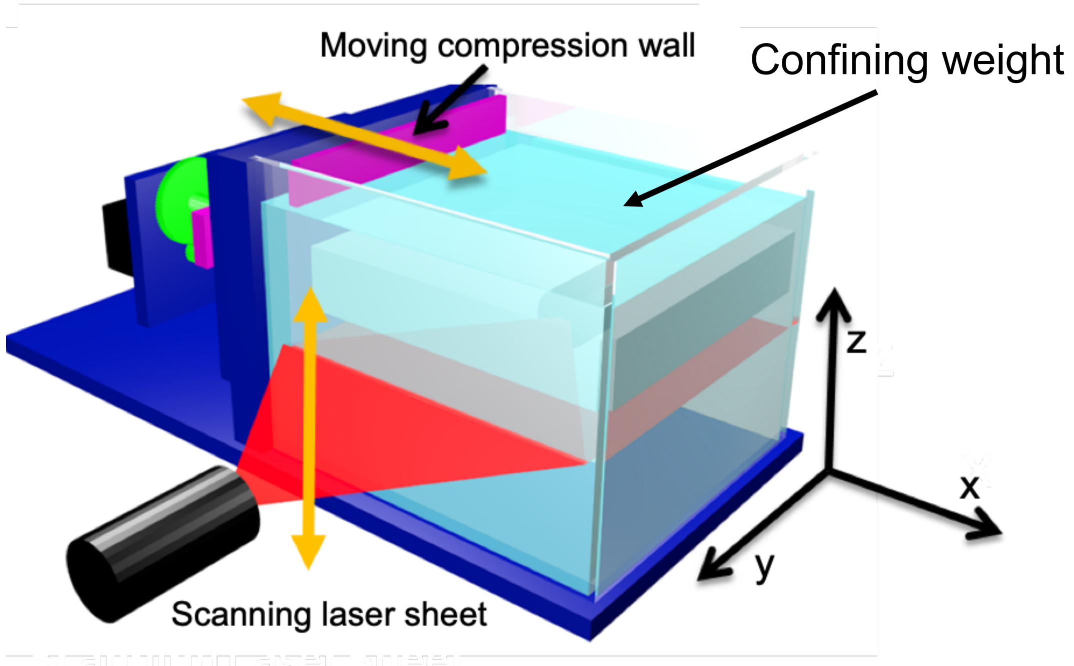
Collective rotations can emerge on multiple scales



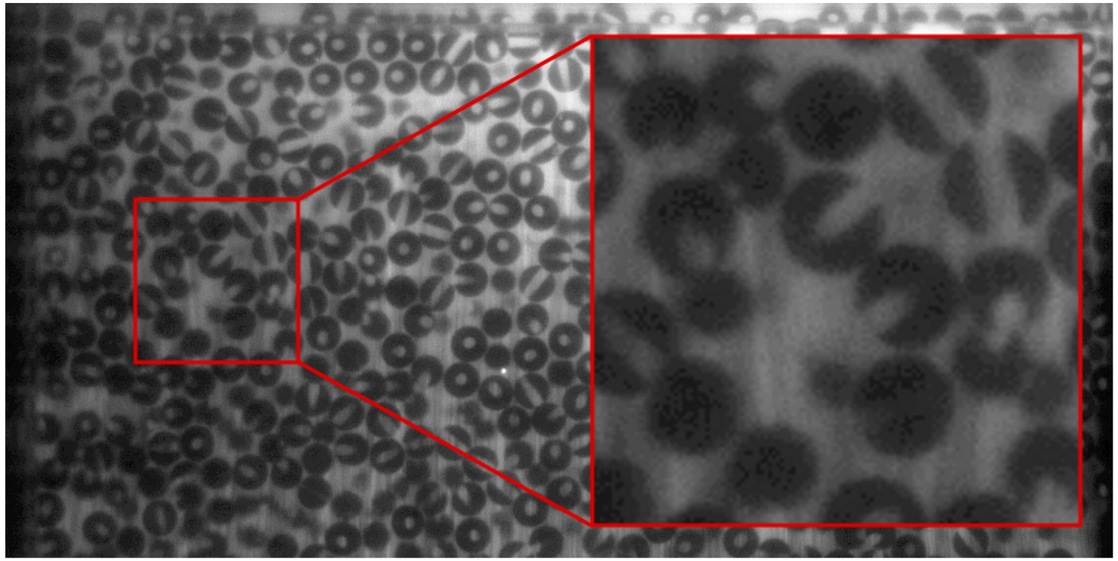
N. V. Brilliantov et. al. *PRL*, **98** (2007) 128001

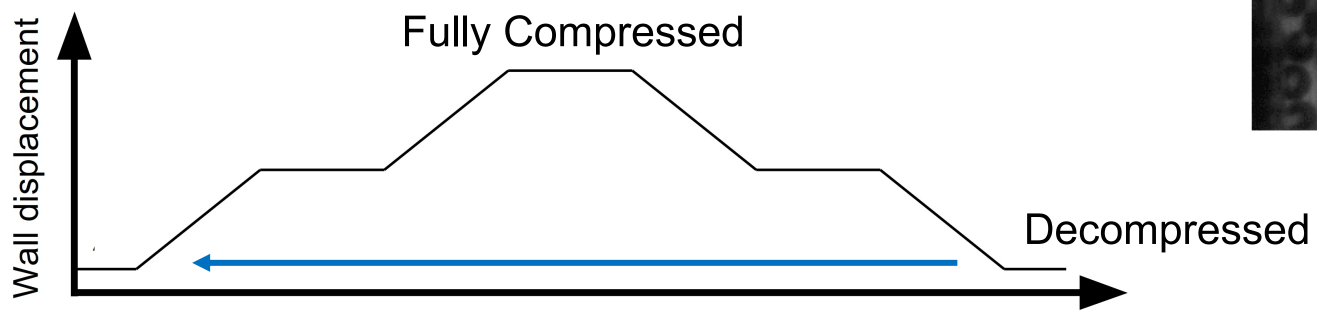
D. V. Stager et. al. *PRL*, **116** (2016) 254301

Our experimental system



Material: 20,000 acrylic beads (n=1.49)
Radius = 0.25 cm
Fluid: Triton X100
Compression amplitude: 1% (~0.15cm)
Packing fraction ≈ 0.6

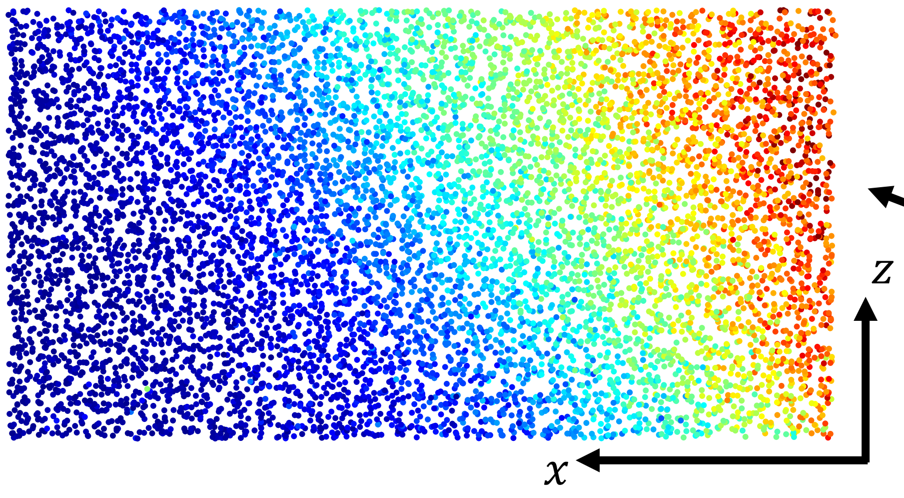




A. Peshkov et. al. *PRE* **100** (2019)

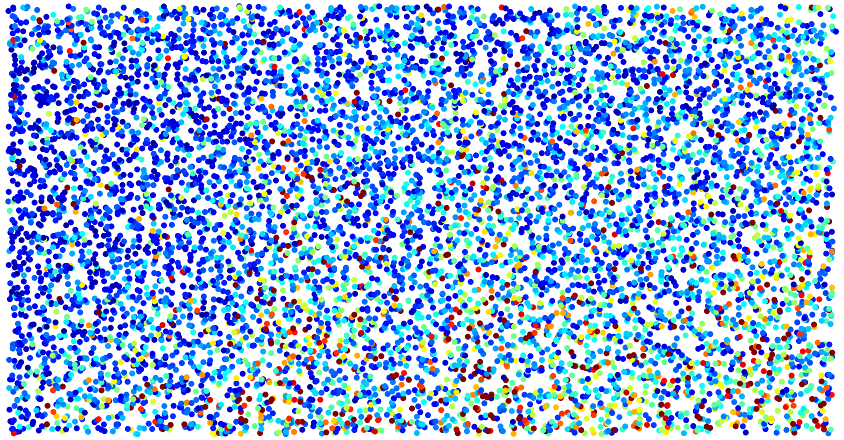
Capturing 3D rotational motion during cyclic compression

Mid Cycle Motion



Track positions

Mid Cycle Rotations



Track orientations

Material: 20,000 acrylic beads (n=1.49)
Radius = 0.25 cm
Fluid: Triton X100
Compression amplitude: 1% (~0.15cm)
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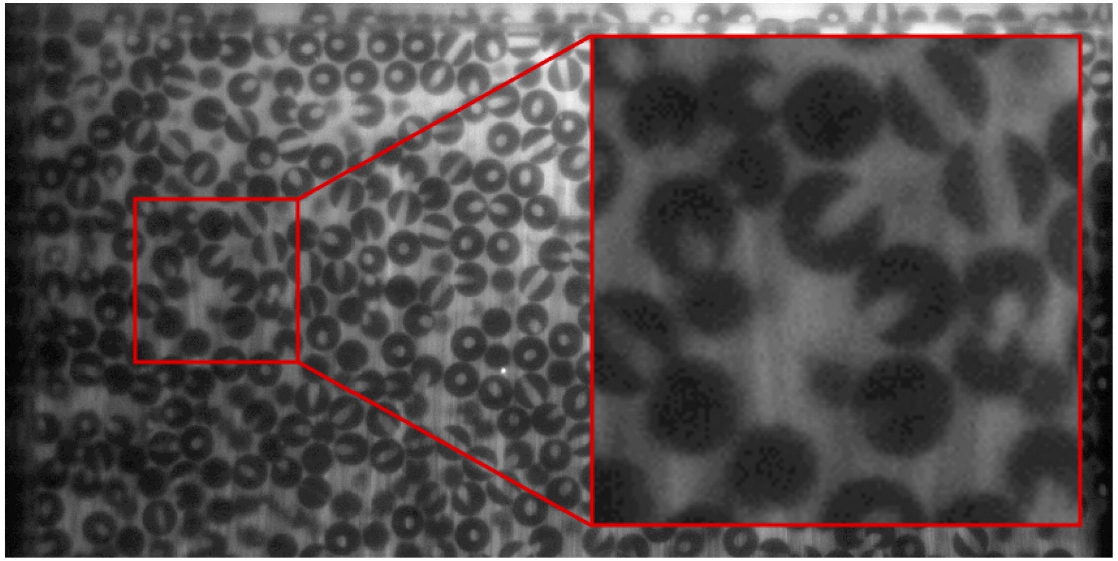

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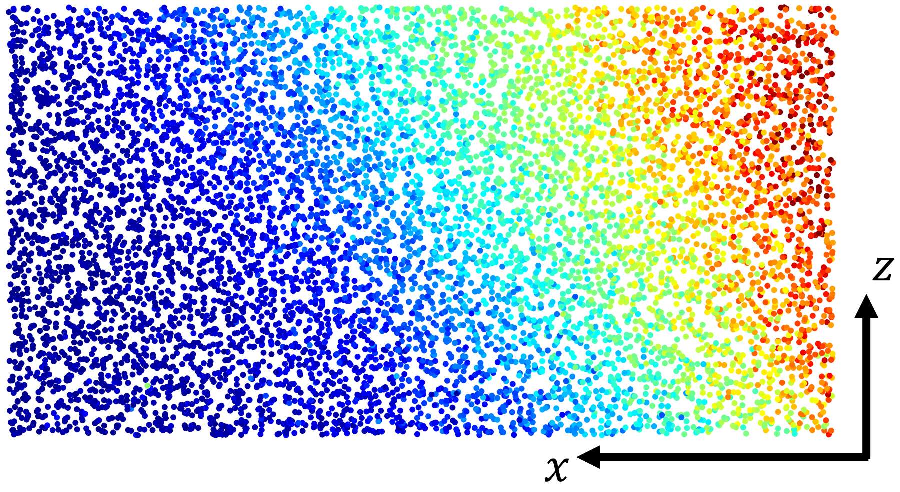
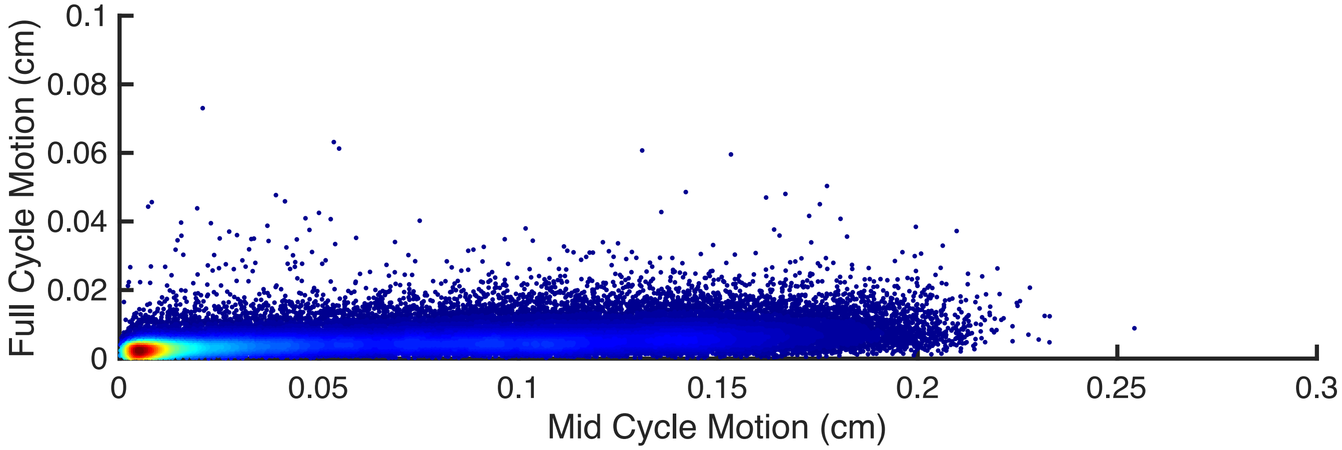
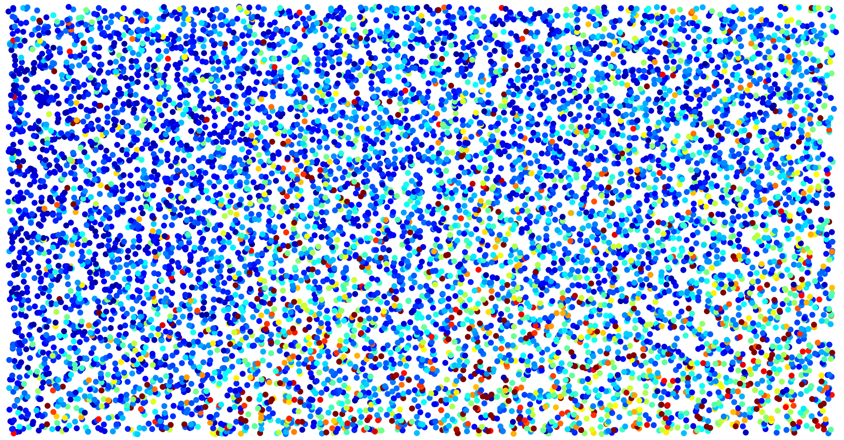
Tracking individual grains positions and orientations

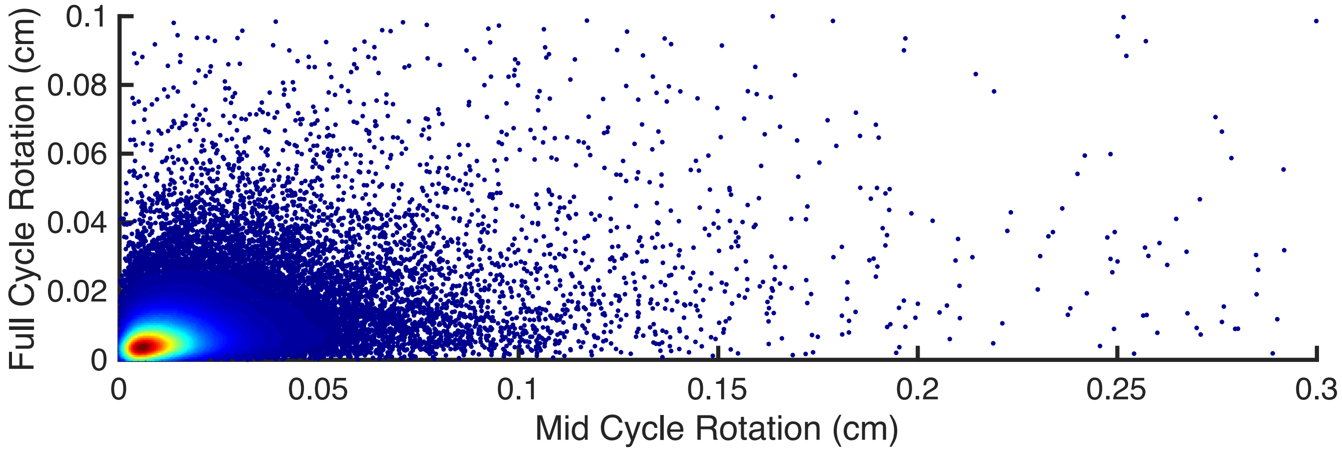
Mid Cycle Motion





Mid Cycle Rotations





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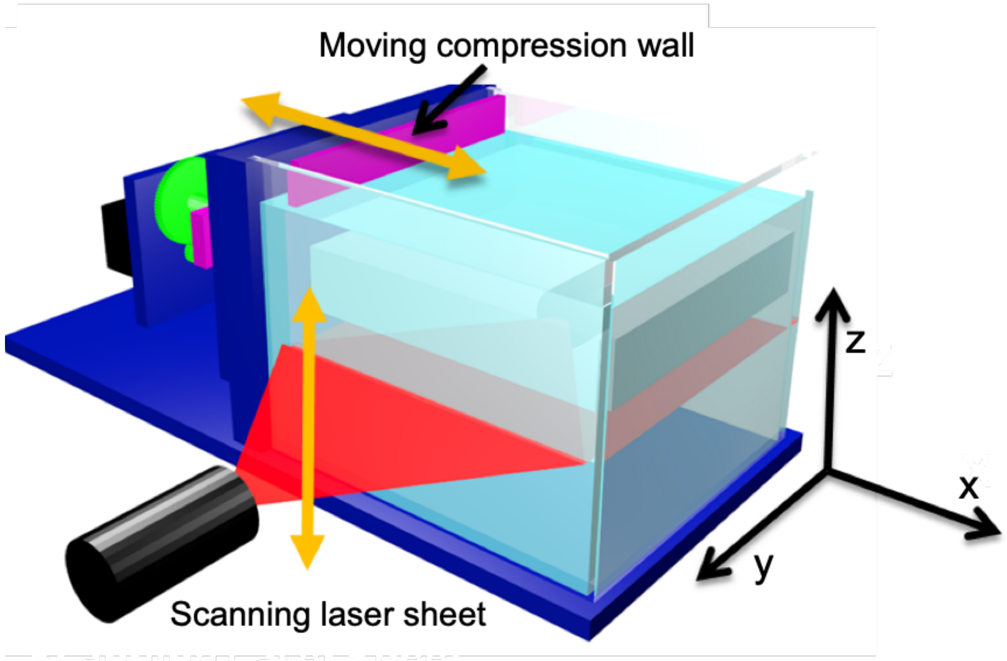
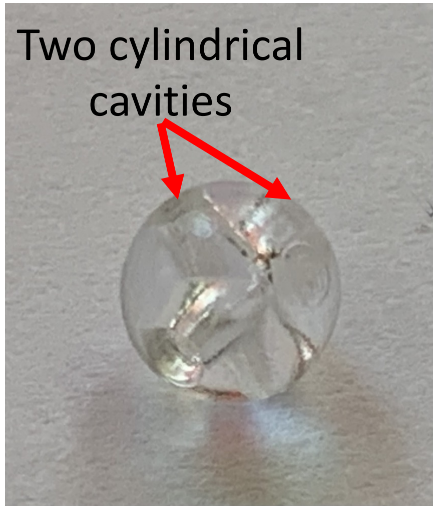
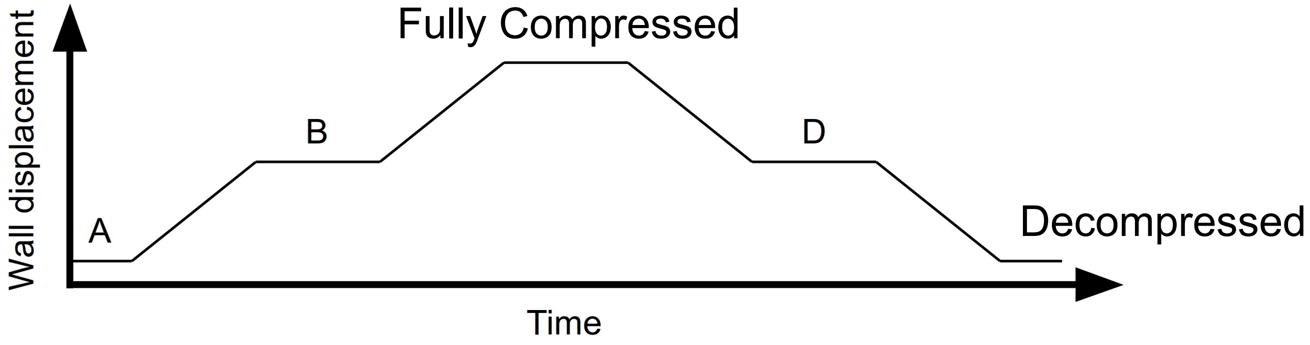
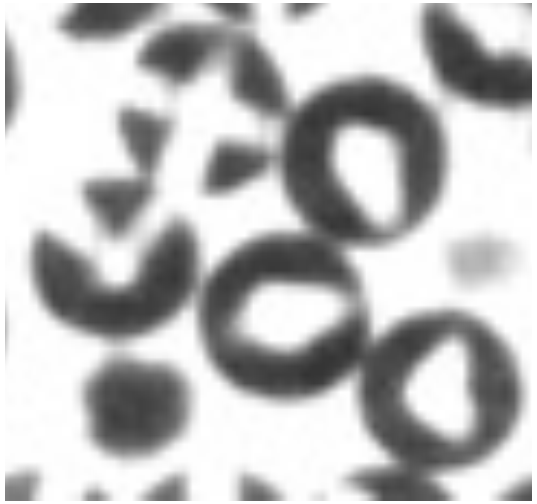
Two holes quantify all rotational degrees of freedom

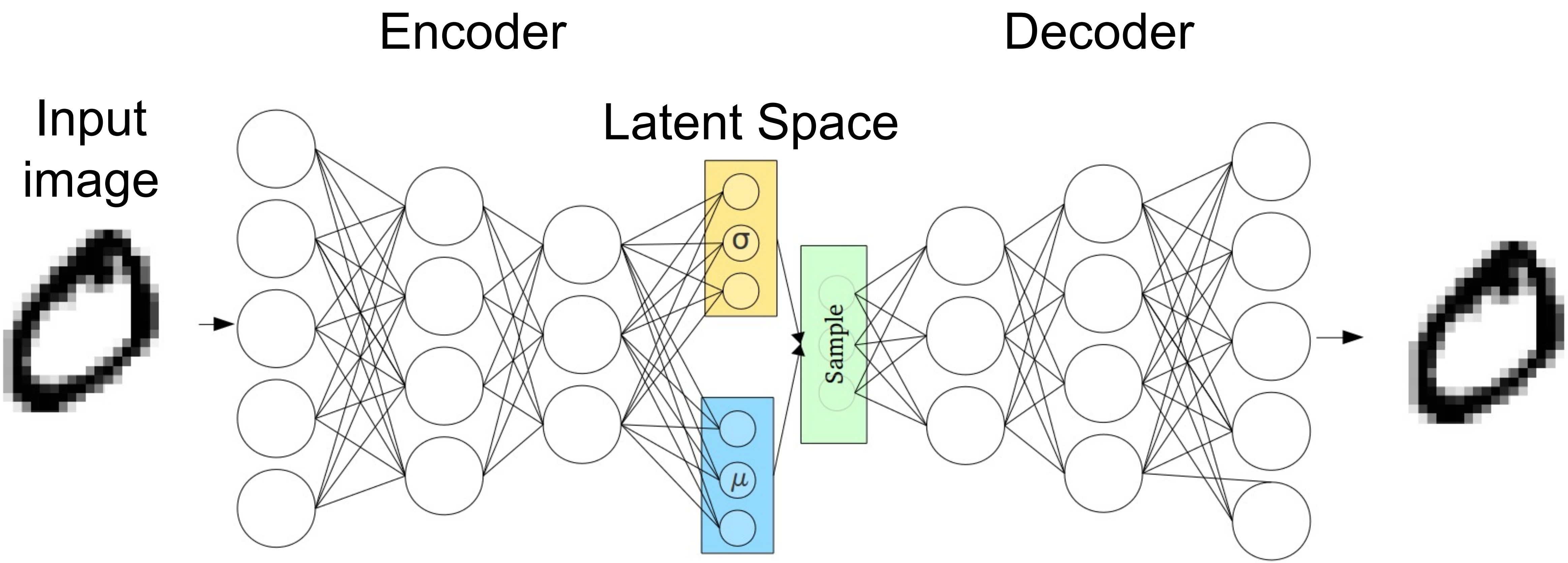








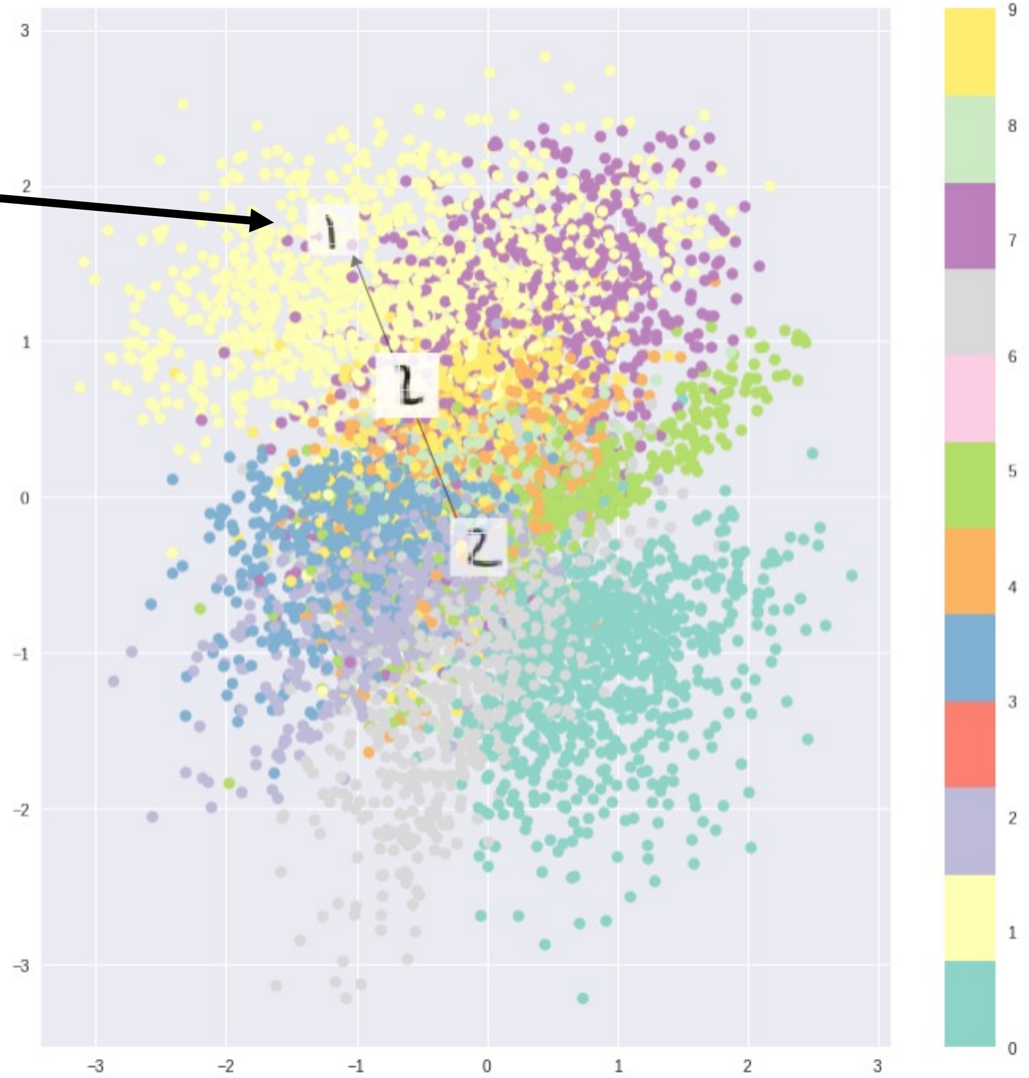
Application of variational auto encoders for image analysis



https://towardsdatascience.com/intuitively-understanding-variational-autoencoders-1bfe67eb5daf

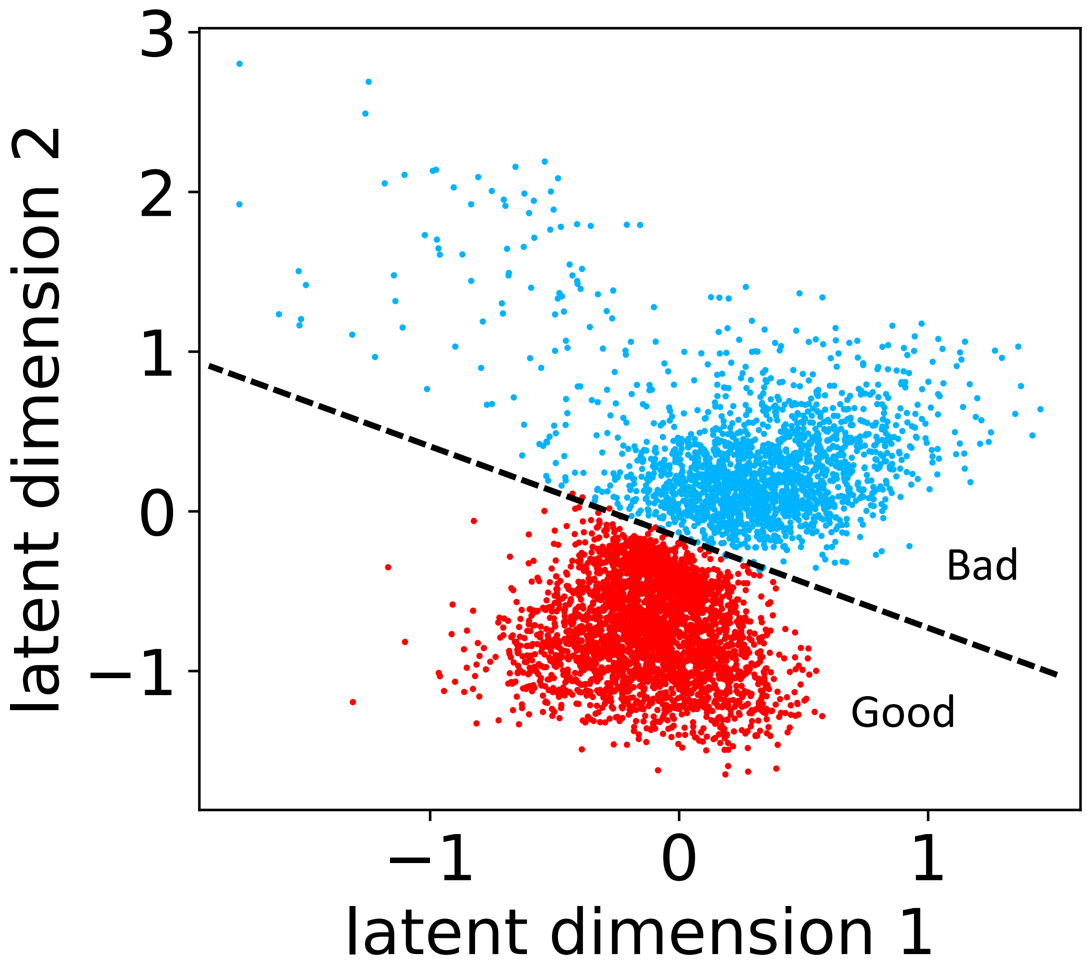
VAE encoding handwritten digits in a 2D latent space

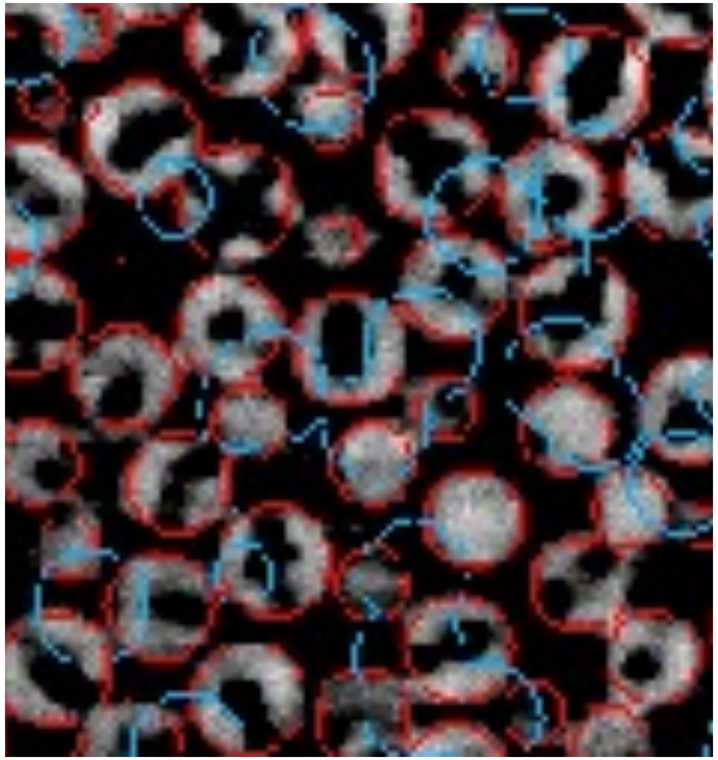
This region corresponds to the digit 1



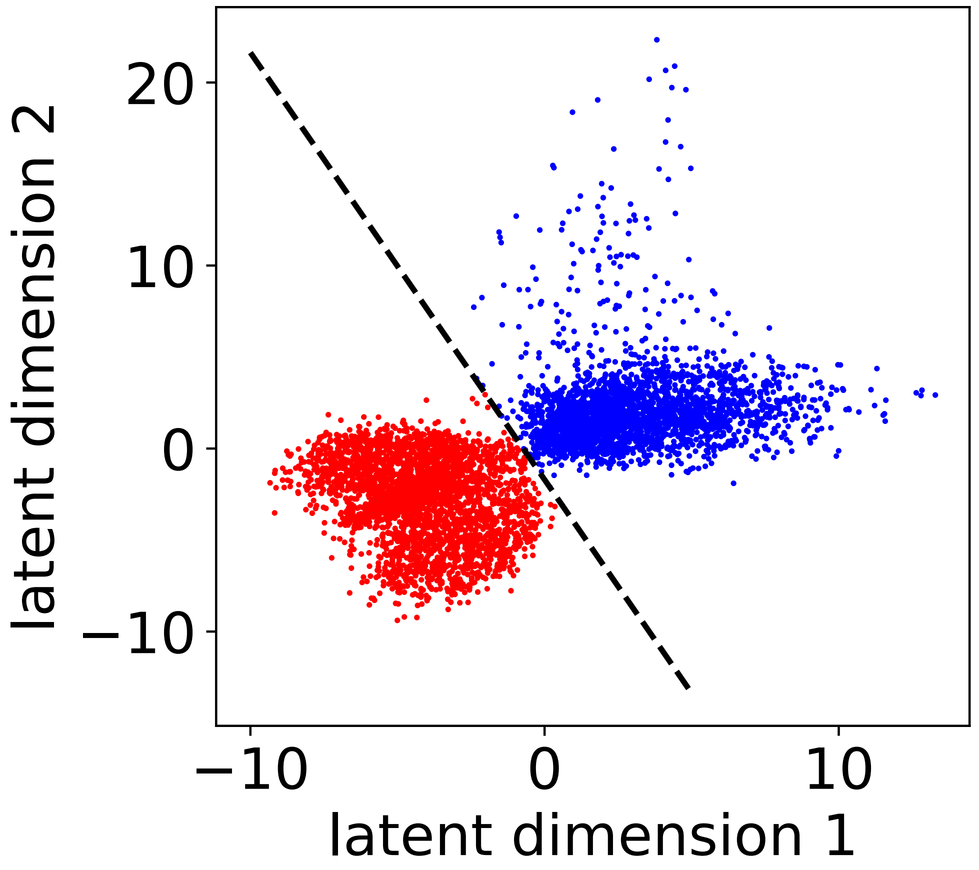
https://towardsdatascience.com/intuitively-understanding-variational-autoencoders-1bfe67eb5daf

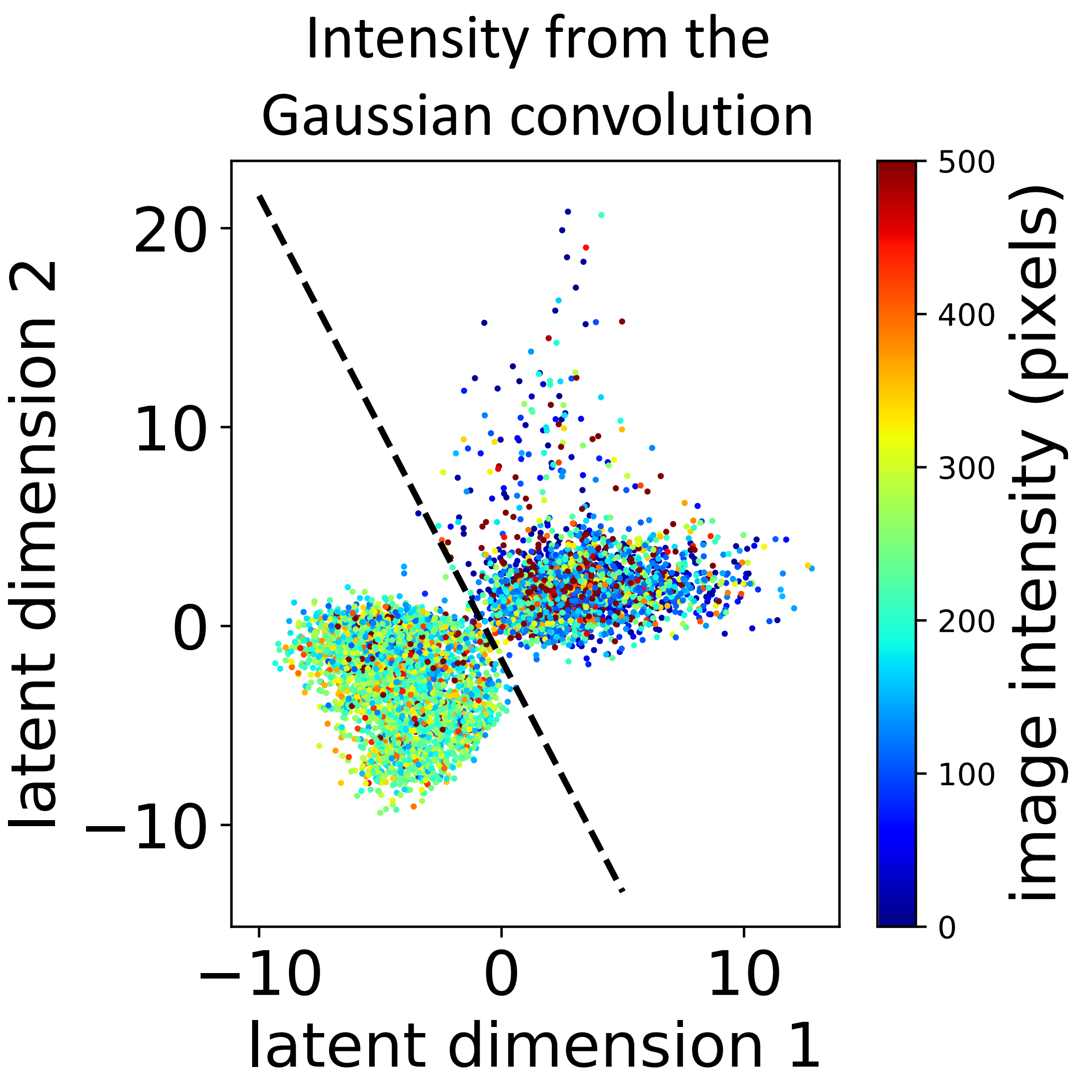
Position identification with VAE





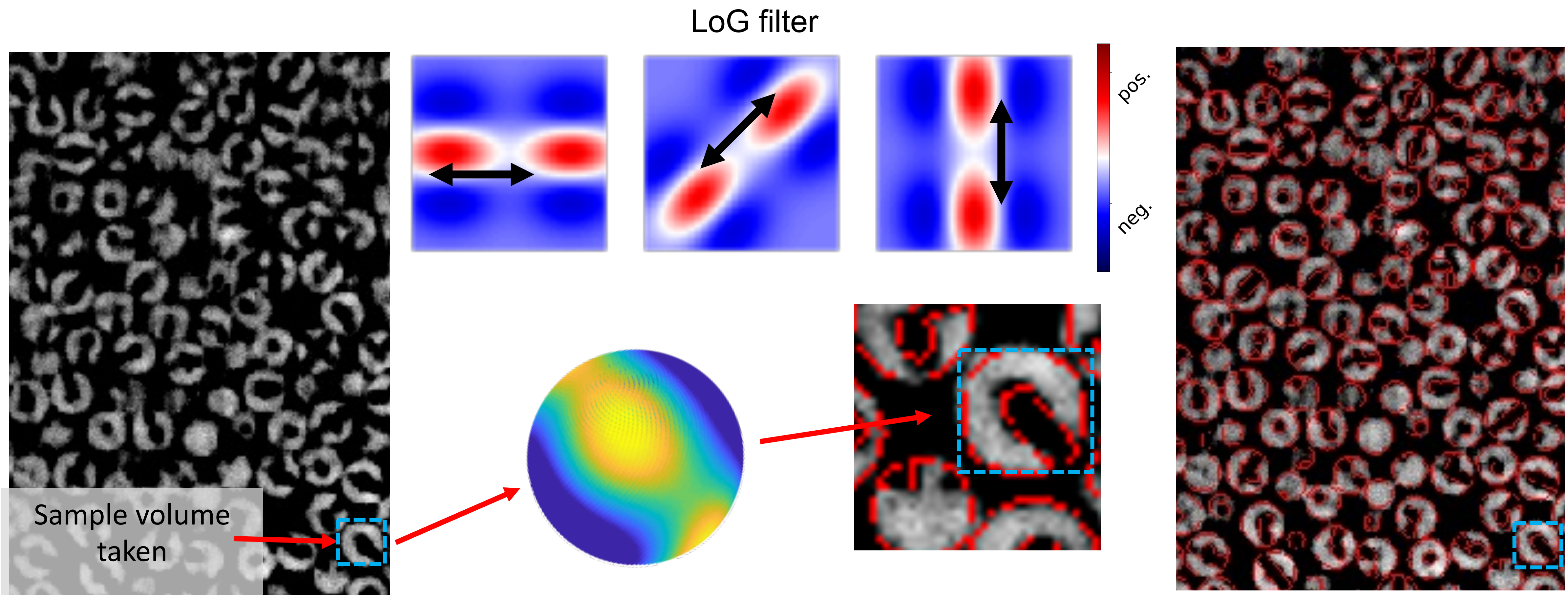
Application of VAE colored by pixel intensity





Latent space rotates during different training runs, but the feature separation remains the same

Orientation extraction

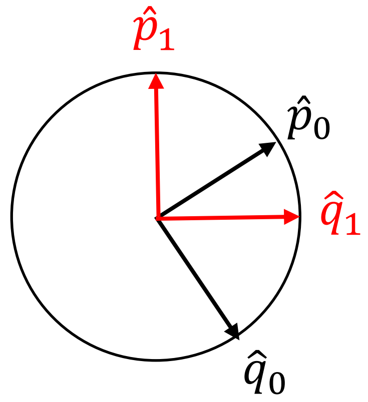


Computing rotations

The rotations for the experiment are calculated by the Kabsch algorithm.

$$C \rightarrow (\hat{p}_0 \quad \hat{q}_0) \cdot \begin{pmatrix} \hat{p}_1 \\ \hat{q}_1 \end{pmatrix}$$

2D projection of rotating grain



Singular value decomposition

$$C = U\Sigma V^T$$

$$d = sign\{\det(VU^T)\}$$

$$R = V \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & d \end{pmatrix} U^T$$

$U \; and \; V^T$ → left and right singular vectors
Σ → contains the singular values
R → Rotation matrix

Computing contact point rotations

Relative deformation of the contact point

$$\Delta\vec{U} = (\vec{v}_1 - \vec{v}_2) + (\vec{\omega}_1 \times \vec{d}_{12} - \vec{\omega}_2 \times \vec{d}_{21})$$

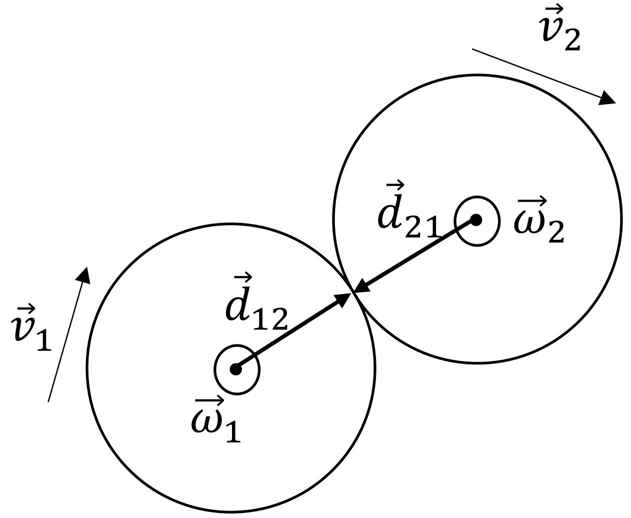
Take the tangential component

$$\Delta\vec{U}_{sld} = \Delta\vec{S} - (\Delta\vec{S} \cdot \hat{d}_{12})\hat{\mathrm{d}}_{12}$$

Rolling displacement

$$\Delta\vec{\omega} = \vec{\omega}_1 - \vec{\omega}_2$$

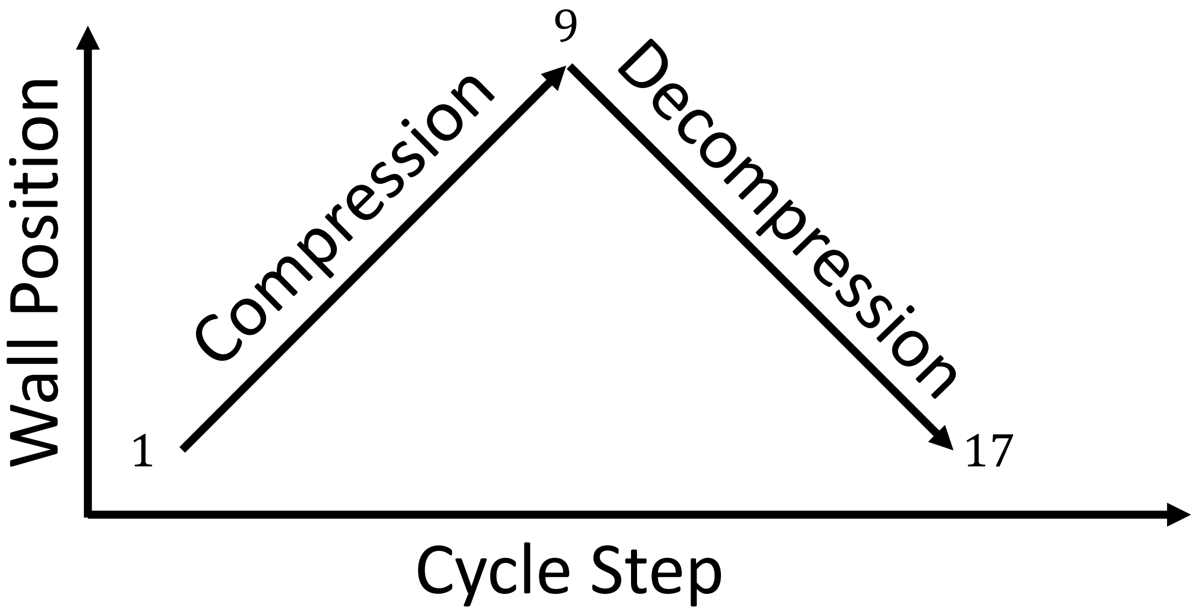
$$\Delta\vec{\mathrm{U}}_{\mathrm{roll}} = \Delta\vec{\omega} \times \vec{d}_{12}$$

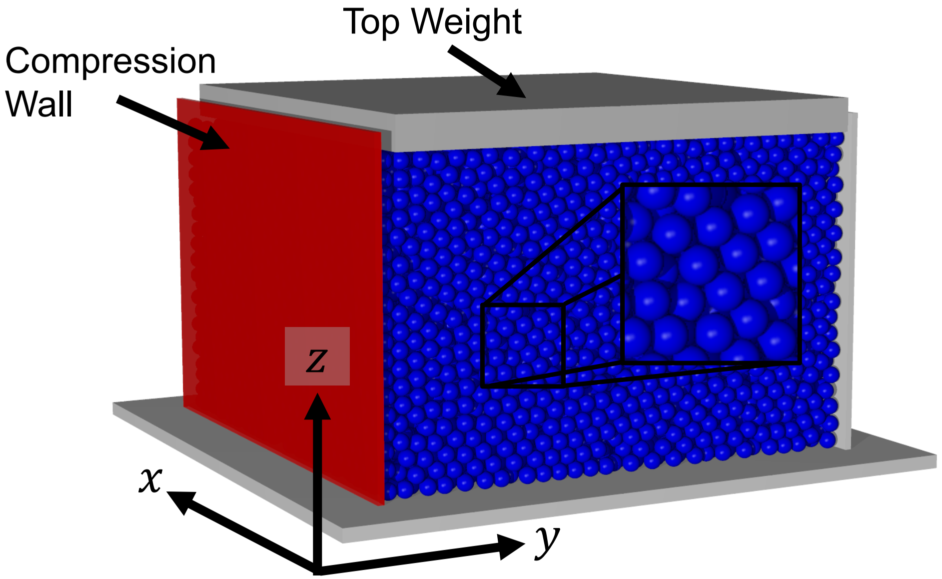


$\omega_i \rightarrow$ angular velocity vector
$\vec{v}_i \rightarrow$ displacement vector
$\vec{d}_{ij} \rightarrow$ vector from center of i to the contact point

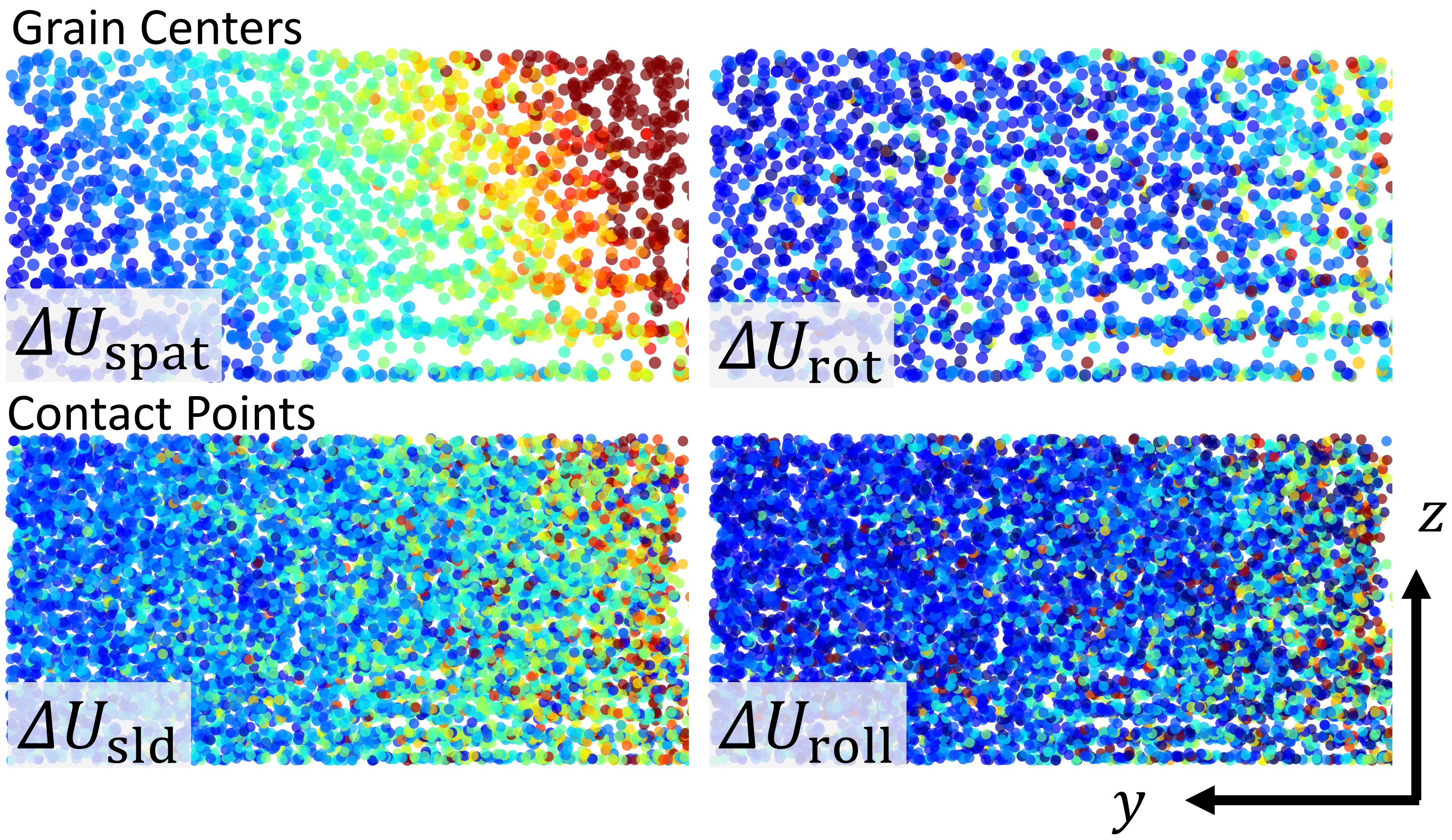
Compression protocol

17 images per full cycle
taken at equal intervals

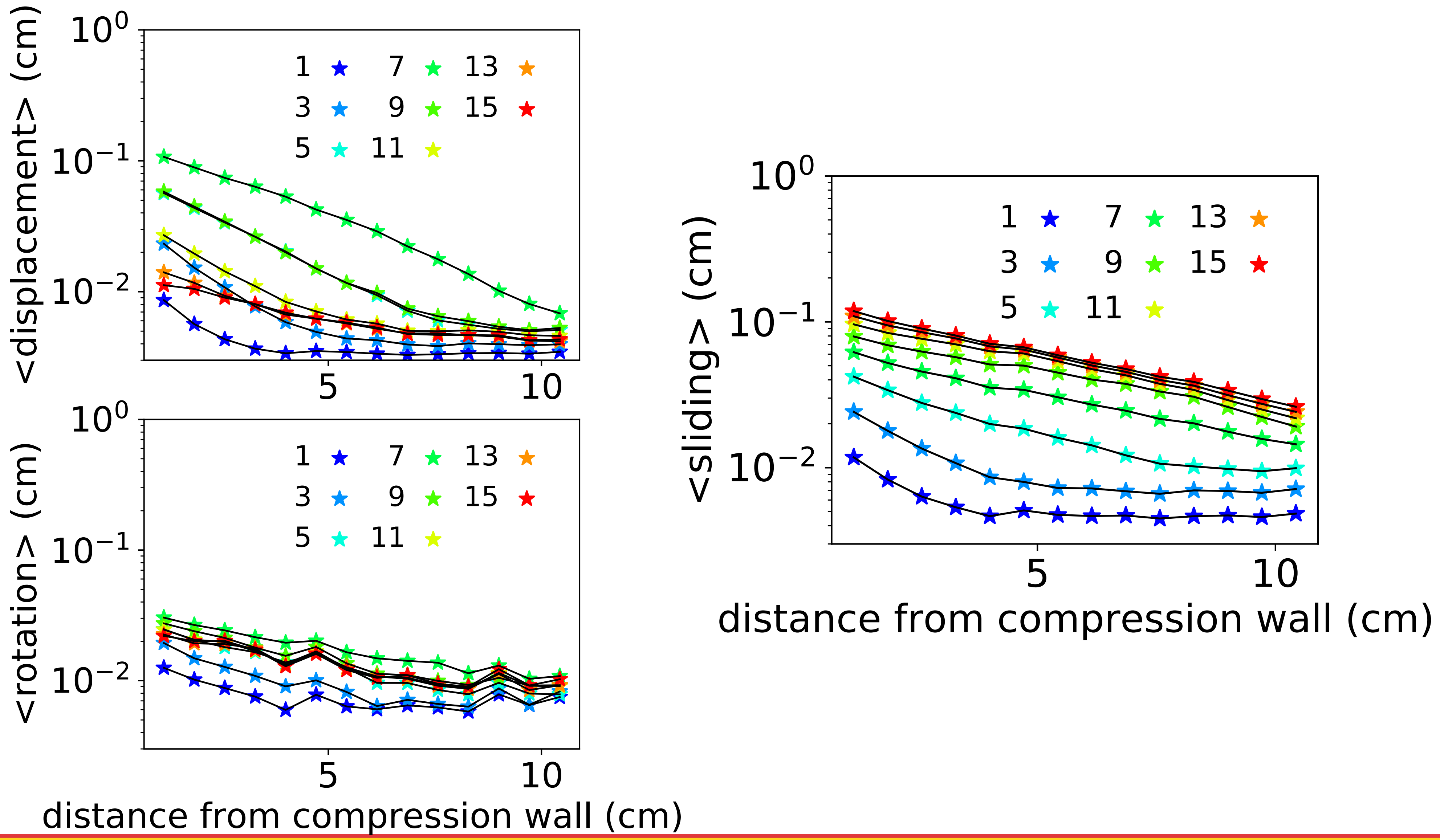




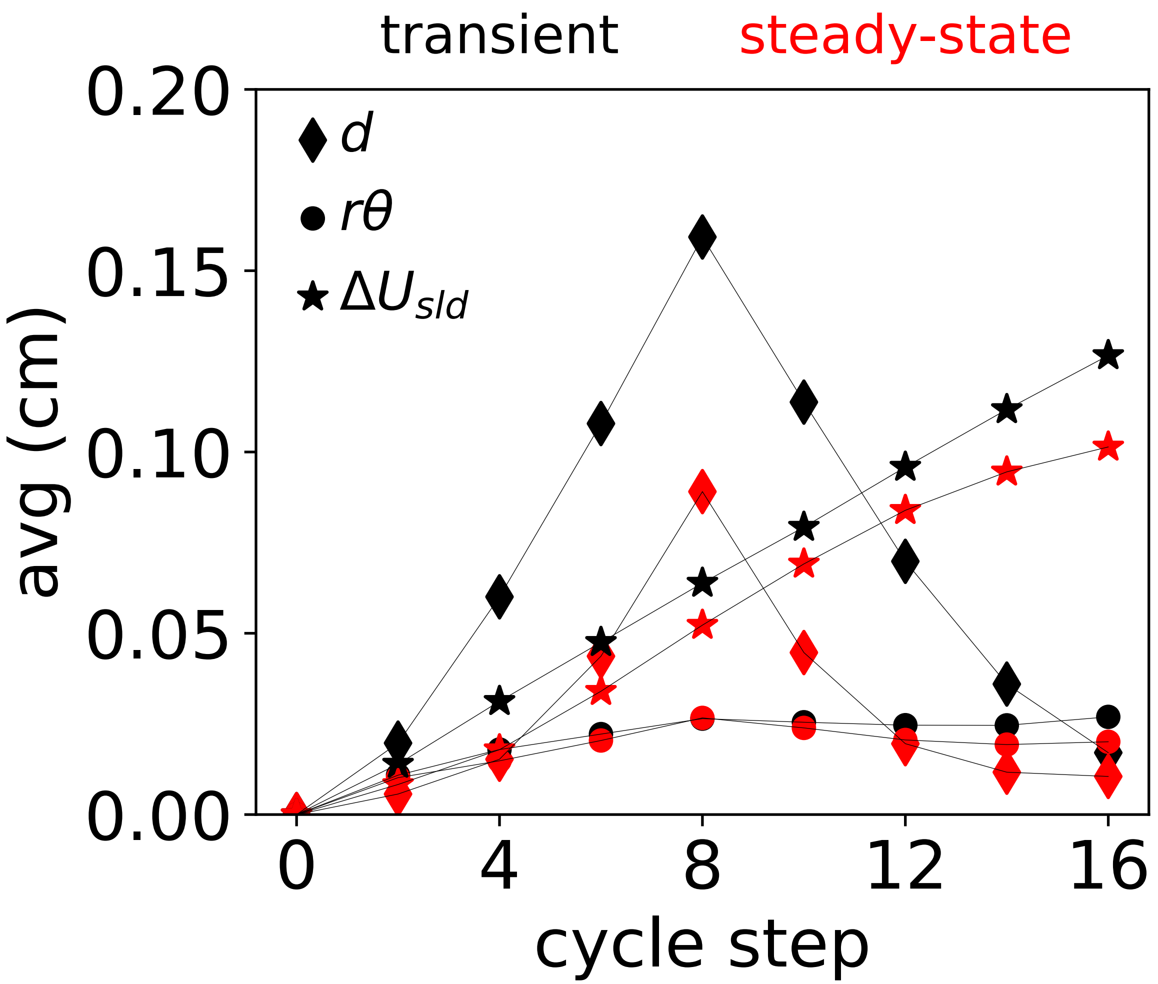
Spatial distribution of displacements at full compression

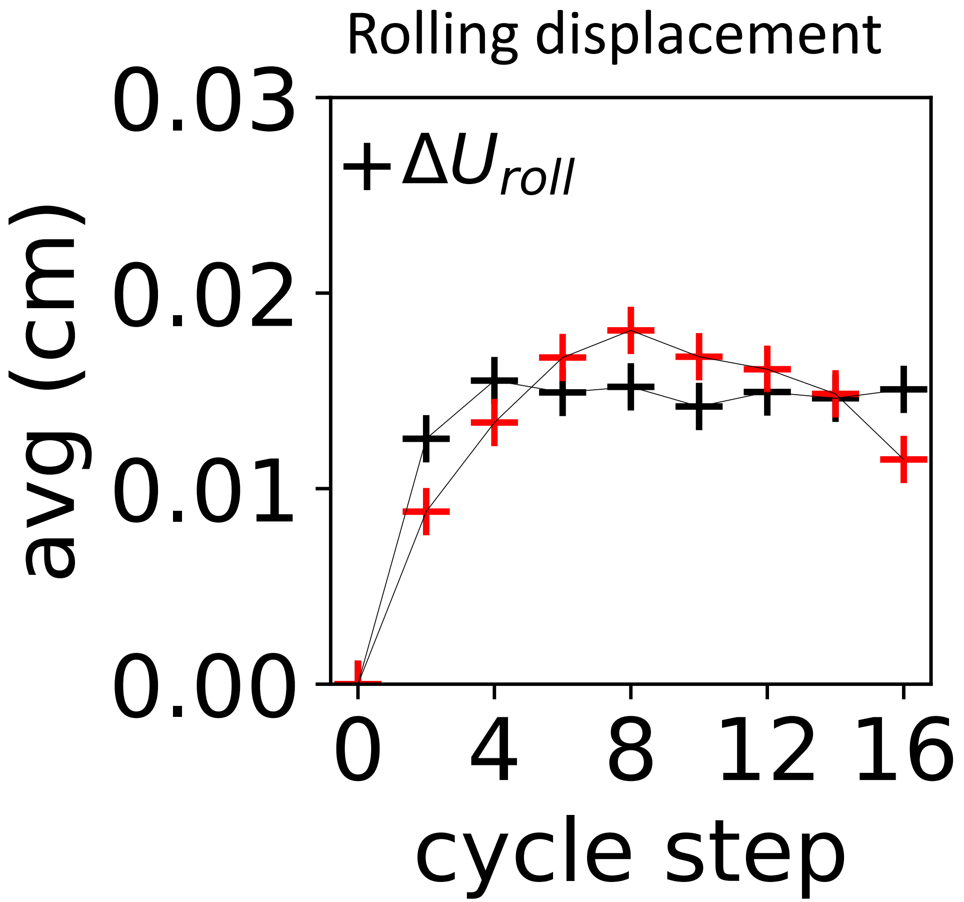


Types of motion penetrates at different lengths in the sample

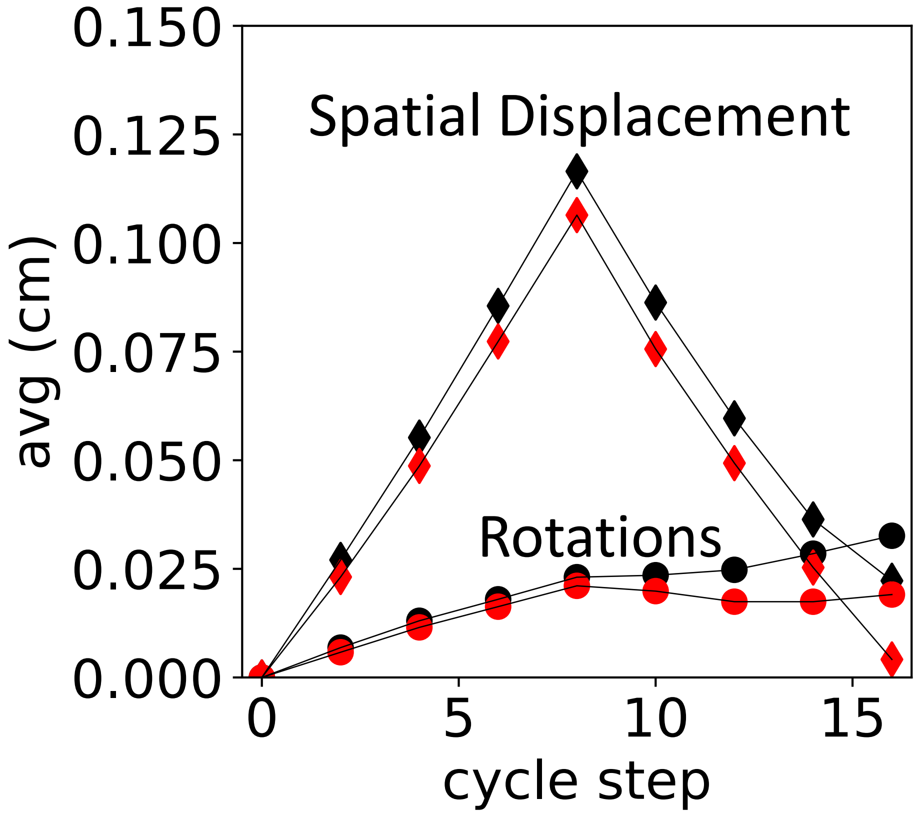


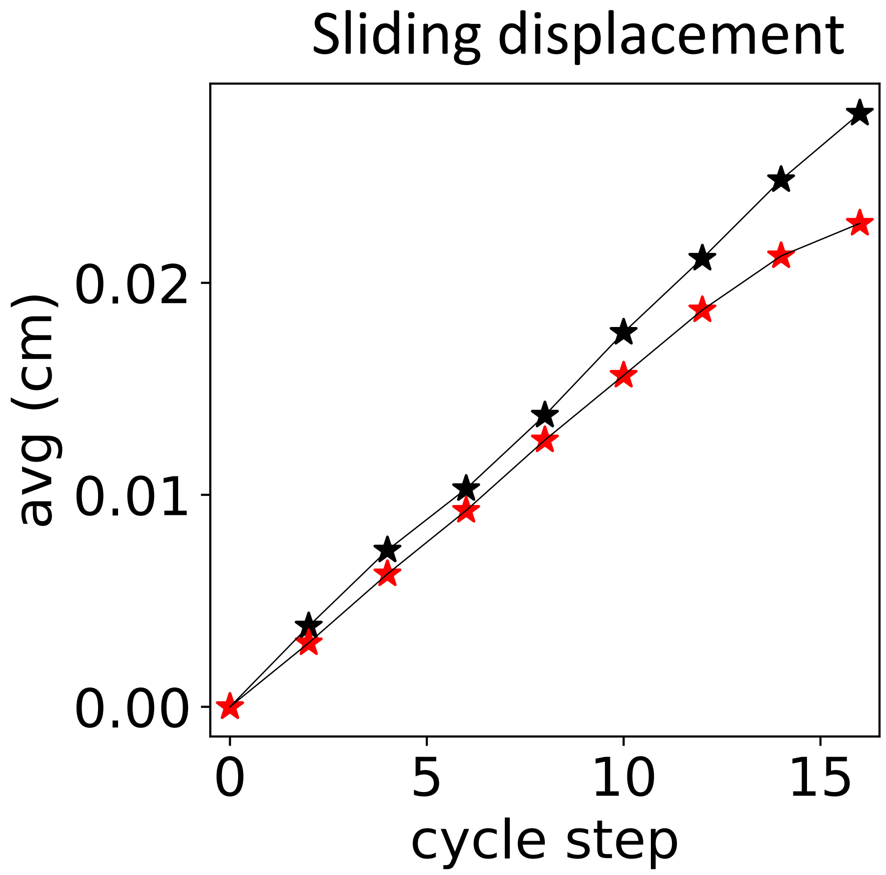
Mean displacements within a cycle





Comparison with DEM simulations





Conclusion and Acknowledgements

Measured 3D rotations of granular spheres.

Implemented VAE to aid in grain identification.

Quantified sliding displacements during cyclic compression.

Found agreement between simulations and experiments in rotational displacement.

Acknowledgements

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