Interface Retaining Coarsening for Gas-Liquid Multiphase Flows

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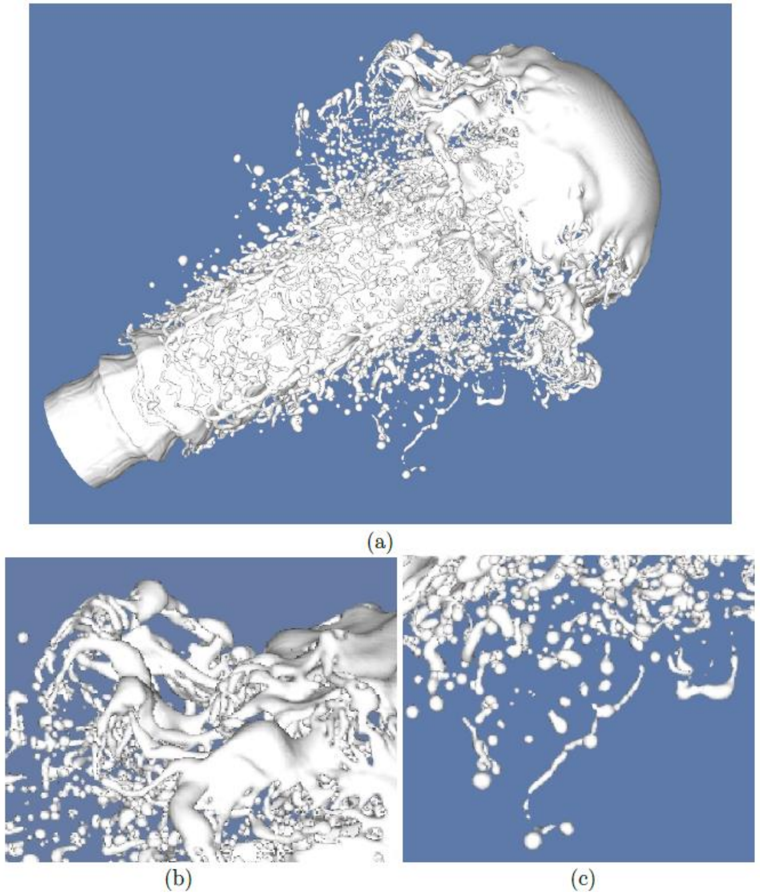
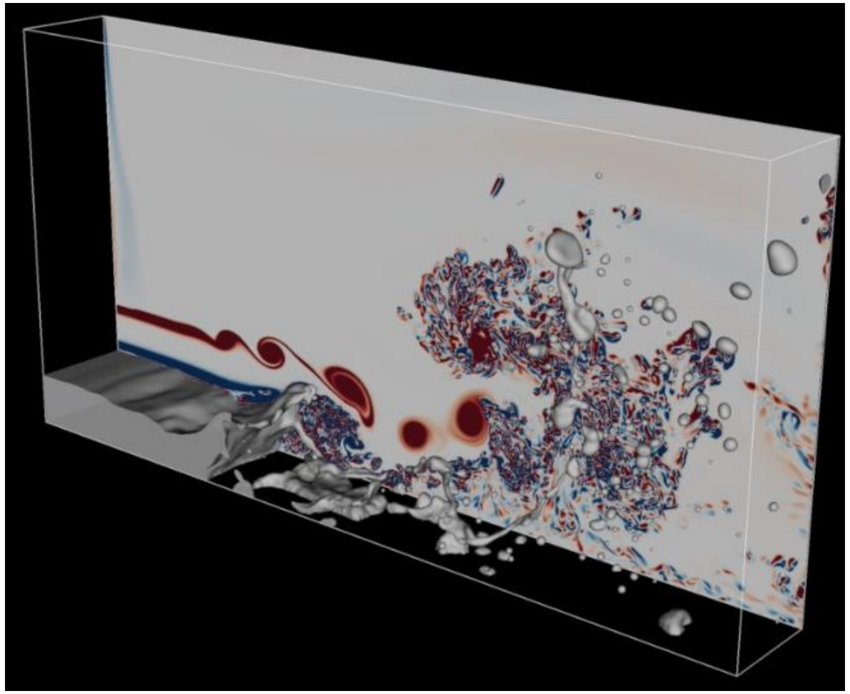
Motivation

- Multi-scale problems are ubiquitous in multiphase-flows such as liquid jets and breaking waves.
- Reduced order model is needed to characterize and predict the complicated flows.



Tryggvason, G., Scardovelli, R., & Zaleski, S. (2011). *Direct numerical simulations of gas–liquid multiphase flows*. Cambridge University Press.

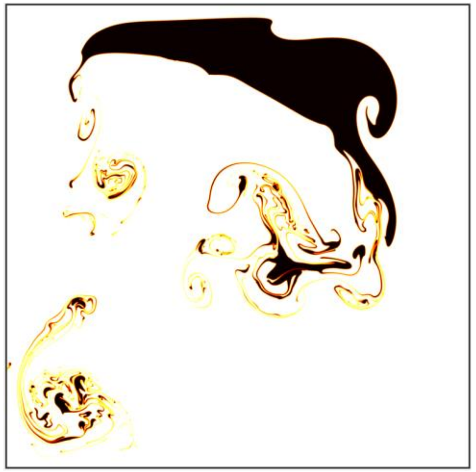


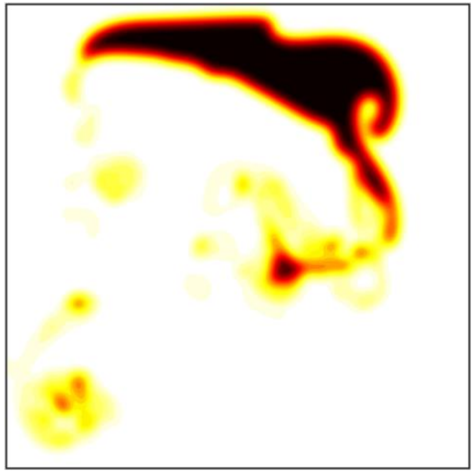
Y. Ling, D. Fuster, G. Tryggvason & S. Zaleski. A two-phase mixing layer between parallel gas and liquid streams: multiphase turbulence statistics and influence of interfacial instability. J. Fluid Mech. 859 (2019), 268–307.

Large Eddy Simulations (LES) in Multiphase flows

- Separates the large and small scales by a low-pass filter
- Simulate the unsteady motions of the large scales and model the effects of the small scales on the large scales motion

Is it appropriate to smooth out the whole field?

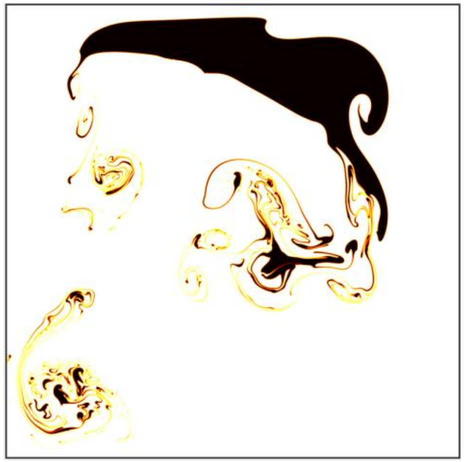


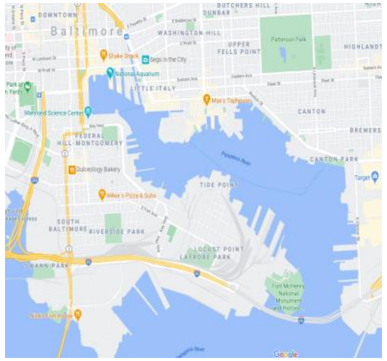


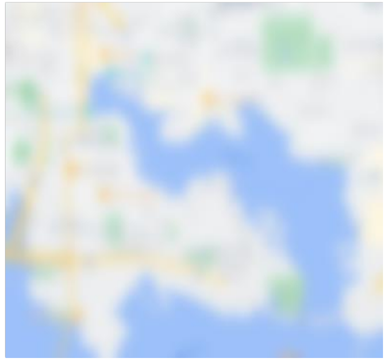
- M. Hermann and M. Gorokhovski. An outline of a LES subgrid model for liquid/gas phase interface dynamics. Proceedings of the 2008 CTR summer program, center for turbulence research, pages 171-181, 2008.
- E. Labourasse, D. Lacanette, A. Toutant, P. Lubin, S. Vincent, O. Lebaigue, J.-P. Caltagirone, and P. Sagaut. Towards large eddy simulation of isothermal two-phase flows: Governing equations and a priori tests. International Journal of Multiphase Flow, 33:1-39, 2007.
- D. Lakehal. Status and future developments of large-eddy simulation of turbulent multi-fluid flows (LEIS and LESS). International Journal of Multiphase Flow, 104:322{� A337,2018.

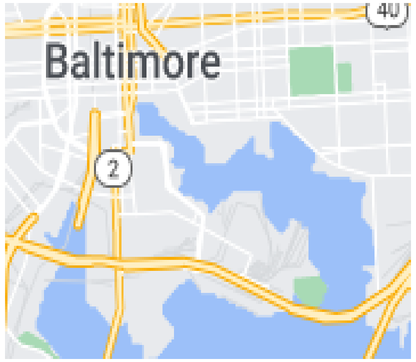
Retain **sharp** interface in the reduced order model

- For multiphase flows where sharp moving phase boundaries separate different fluids or phases, the dynamics of the interface often determines the behavior of the flow.
- In a coarse, or reduced order model, it may therefore be important to retain a sharp interface for the resolved scales. Somewhat like modeling of disperse flows often retain bubbles or drops as point particles.









better

Coarse field model
(or LES for Multiphase flow)

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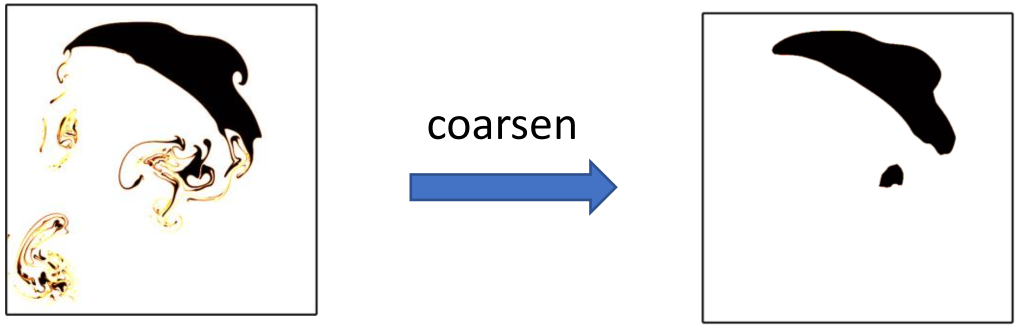
1. coarsening $\tilde{f}(t,\boldsymbol{x})$



2. Evolve the coarsened field and make sure the time evolution $\partial\tilde{f}(t,\boldsymbol{x})/\partial t$ is correct, by adding subgrid terms

$$\frac{\partial\tilde{\rho}\tilde{u}}{\partial t}+\nabla\cdot\tilde{\rho}\tilde{u}\tilde{u}=-\nabla\tilde{p}+\nabla\cdot\mu(\nabla\tilde{u}+\nabla\tilde{u}^T)+\sigma\tilde{\kappa}\tilde{n}\tilde{\delta}+\boldsymbol{\nabla}\cdot\boldsymbol{F}$$

$$\frac{\partial\widetilde{H}}{\partial t}+u_I\cdot\nabla\widetilde{H}=0$$

$$\frac{\partial\alpha'}{\partial t}+\tilde{u}\cdot\nabla\alpha'=\nabla\cdot D_e\nabla\alpha'+S_I$$

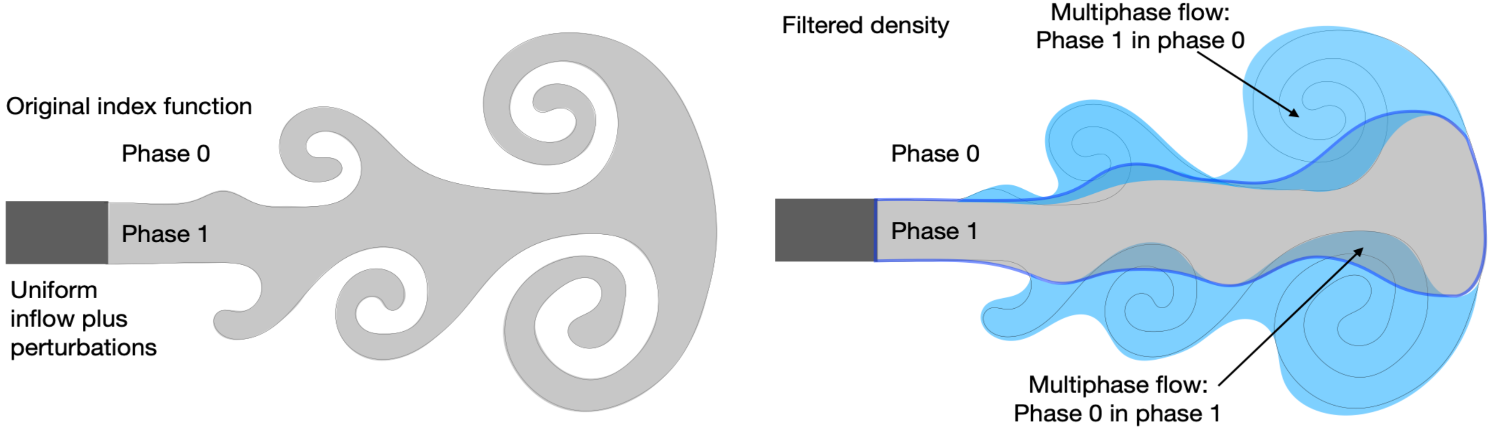
3. Model/Correlate the closure terms with coarsen fields

Correlate F, u_I, D_e with filtered field $f(\tilde{u},\widetilde{H},\nabla\tilde{u},\cdots)$

Coarsening Strategy

The goal is to separate the flow into large scale flow which are evolved deterministically and small scale flow which is modeled.

- The large- and the small-scale phase distribution are separated by coarsening the index function, keeping the interface sharp.
- The interface and the flow are simplified in a consistent way, maintaining incompressibility of the coarsened field.
- The simplified interface retains jump conditions for the flow variables, such as momentum and mixture fractions.
- As the coarsening is reduced, the flow field approaches the results given by direct numerical simulations (DNS).



Filtering the index function

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Filtering and diffusion are closely linked. The solution of the time dependent constant coefficient diffusion equation in 2D unbounded domain

$$\frac{\partial g(t,\mathbf{x})}{\partial t} = D\nabla^2 g(t,\mathbf{x})$$ is given by $$g(\mathbf{x},\tau) = \frac{1}{4\pi D\tau}\int_{Area} e^{-\frac{||\mathbf{x}-\mathbf{x}'||^2}{4D\tau}} g_o(\mathbf{x}')da'$$

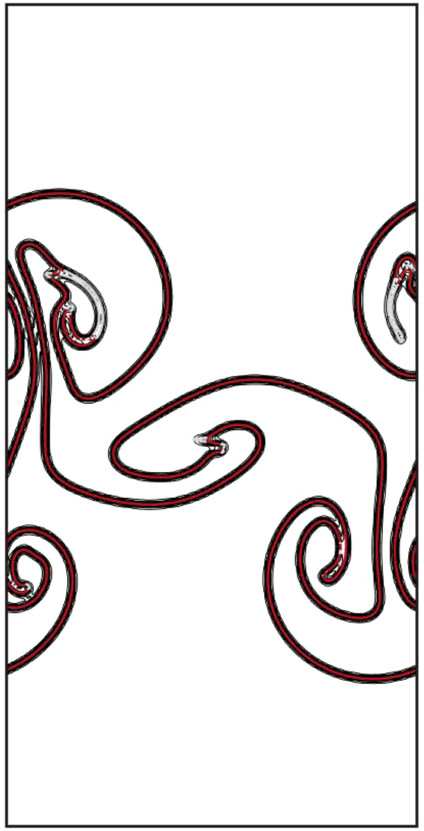
If we take $4D\tau = \Delta^2/6$ filtering and diffusion are interchangeable. Same in 3D

For a use of the same idea to singularize smooth fields, see: X. Chen, J. Lu and G. Tryggvason. "Condensing Smooth Layers into Singular Sheets by Weighted Coordinate Smoothing." Journal of Computational Physics. 431, 110140

We prefer to work with the diffusion equations, since we can generalize the coarsening by modifying it and making the diffusion coefficient variable, such as by preventing diffusion or making it directionally dependent, and work close to boundaries

We also have the freedom to coarsen different variables in slightly different ways

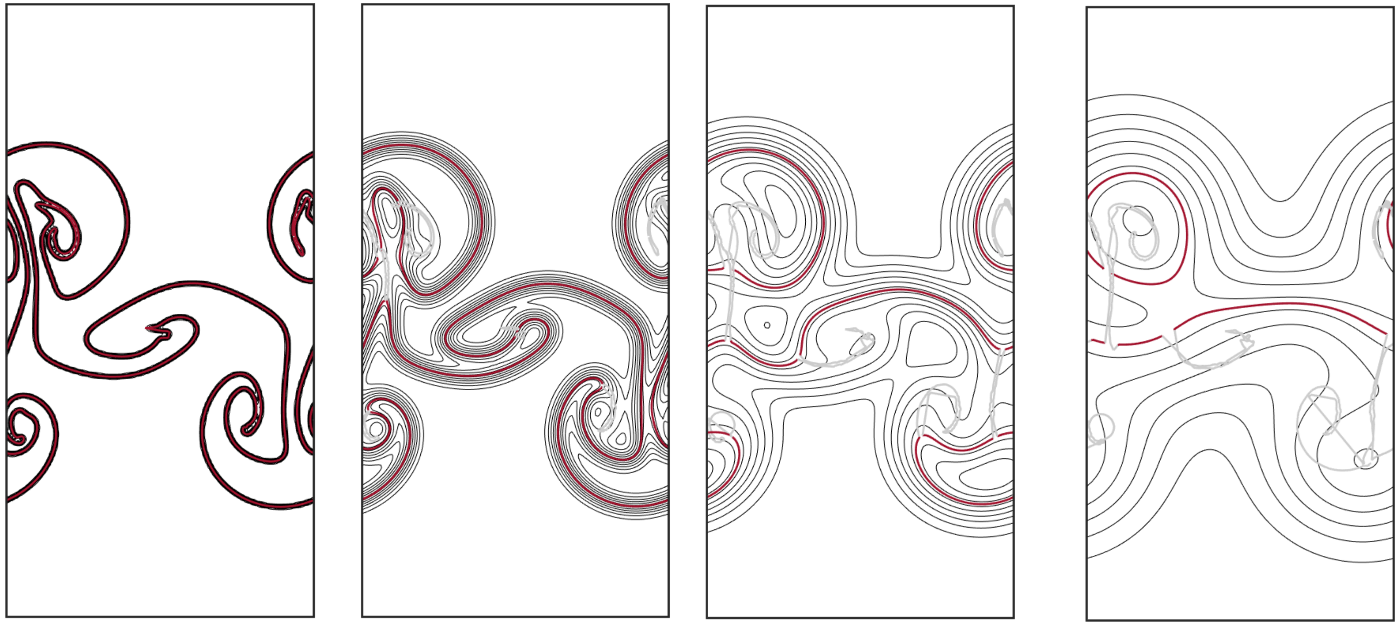
Coarsening the interface



Evolution in pseudo-time

Evolve the index field in pseudo time by

$$\frac{\partial \chi}{\partial \tau} = D\nabla^2\chi.$$

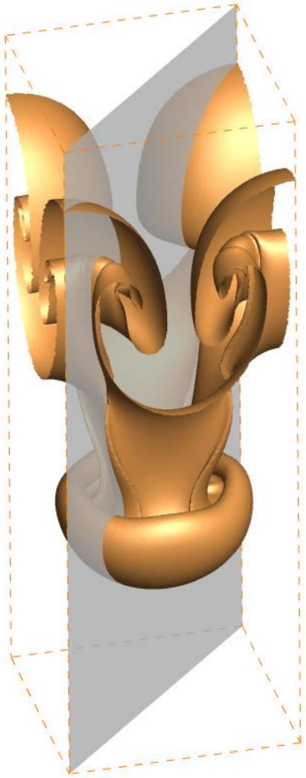


$\tau = 0$ | $\tau = 4e-4$ | $\tau = 2.6e-3$ | $\tau = 1e-2$

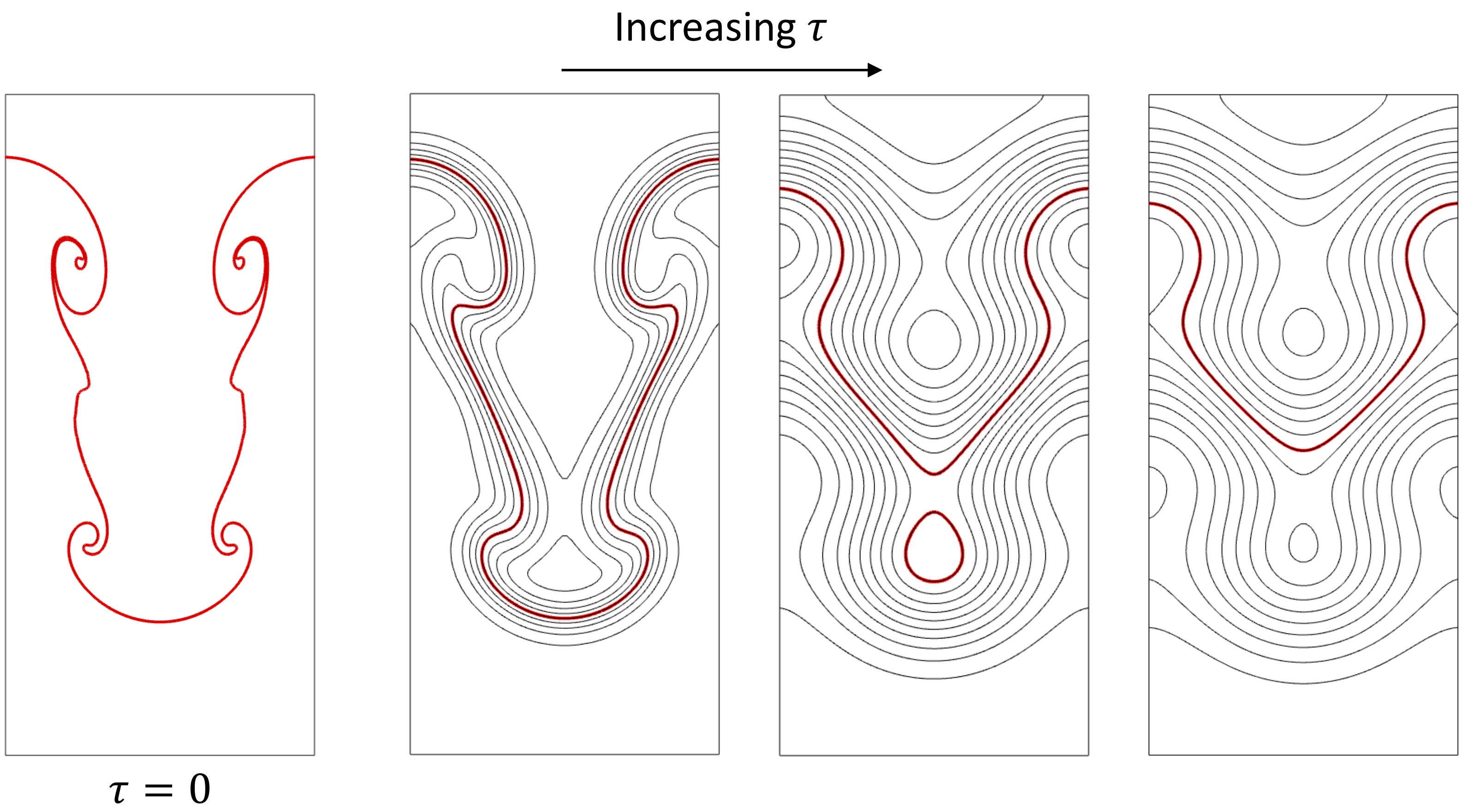
Interface velocity is determined from $\frac{D\chi}{D\tau} = \frac{\partial \chi}{\partial \tau} + \boldsymbol{u} \cdot \nabla\chi = 0.$

$$\boldsymbol{u}_I = u_n \boldsymbol{n} = -\frac{(\chi_I - \chi)}{|\nabla\chi|^2 \Delta\tau}\nabla\chi \quad \text{and} \quad \frac{d\boldsymbol{x}_I}{d\tau} = \boldsymbol{u}_I$$

Coarsening the interface

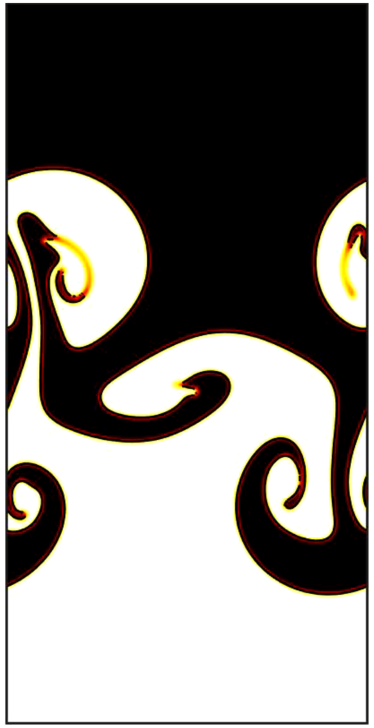


Original interface



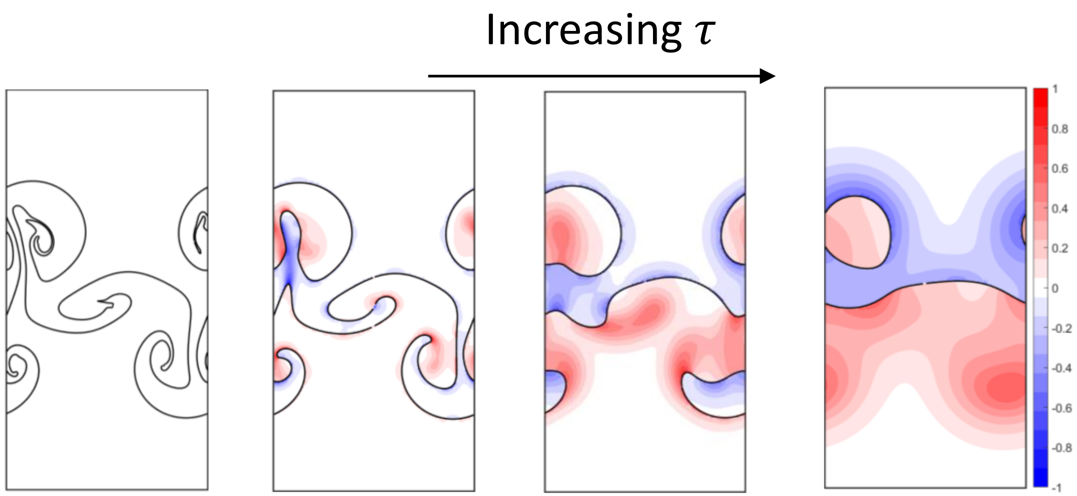
Interface plot in red with diffused index function contour on the grey plane

Coarsening the volume fraction field



Evolution of volume fraction in pseudo time

Evolution of volume fraction perturbation in pseudo time
$\alpha' = \alpha - \tilde{\chi}$



As the interface is coarsened by diffusing the index field, the small-scale field is determined by simultaneously solving another diffusion equation

$$\widetilde{D}(x,\tau) = D \text{ modified by setting } D(x_f) = 0;$$
$$\frac{\partial \alpha}{\partial \tau} = \nabla \cdot \widetilde{D} \nabla \alpha$$

This mixes the phase left behind as the interface is moved with the other phase and prevents diffusion across the interface

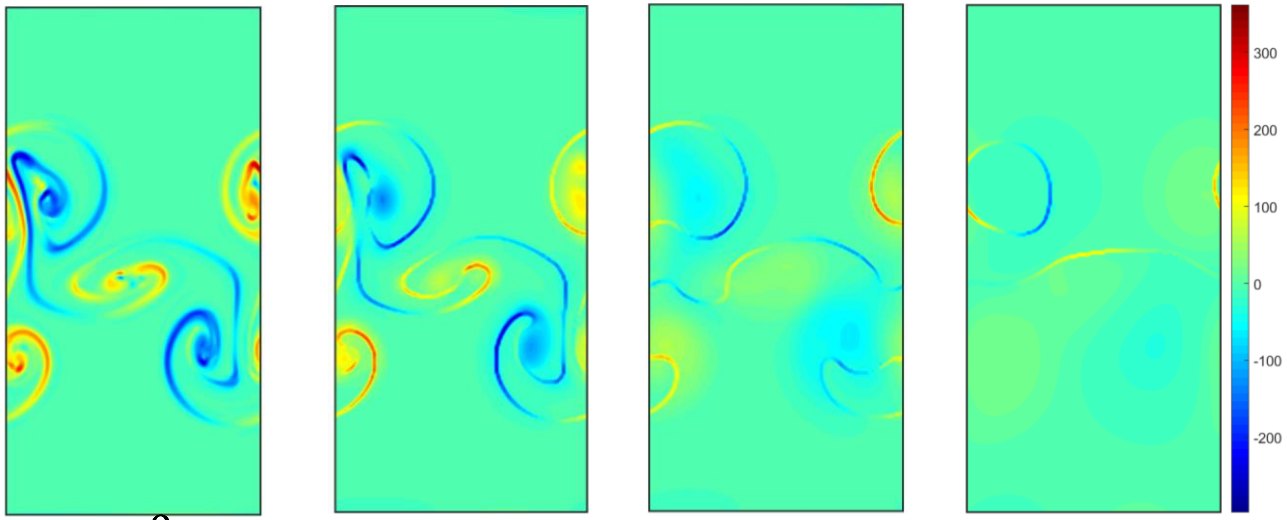
Coarsening the momentum

$$\frac{\partial \widetilde{\rho u}}{\partial \tau} = \nabla \cdot \tilde{D} \nabla \widetilde{\rho u} + \nabla p^*$$

$$\tilde{u} = \frac{\widetilde{\rho u}}{\tilde{\rho}} \text{ and } \nabla \cdot \tilde{u} = 0$$

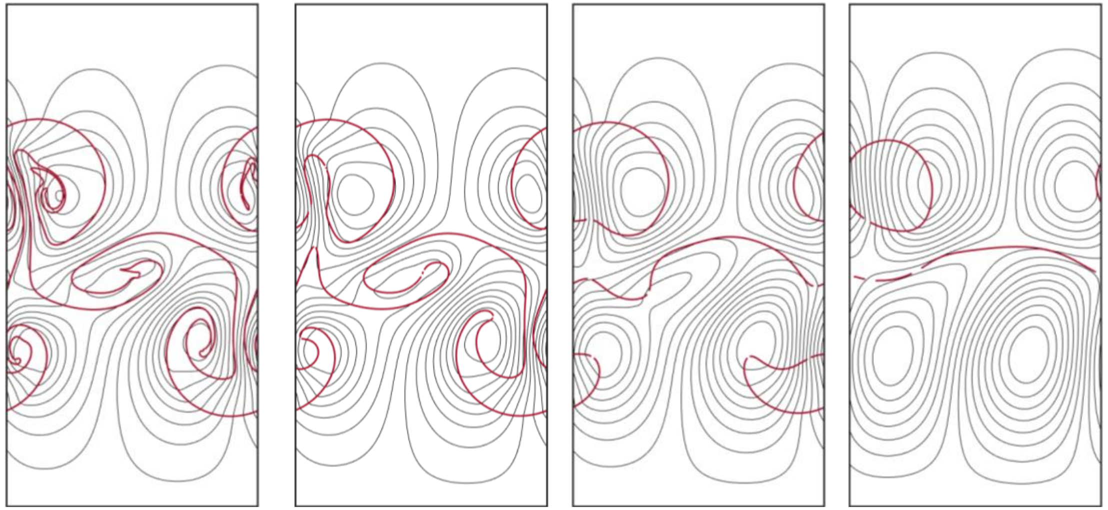
Diffusing the momentum separately on either side of the interface generally does not preserve incompressibility, so we evolve it in pseudo time using a pressure like term to keep the divergence of the volumetric velocity equal to zero.

Increasing τ



$\tau = 0$

The evolution of the vorticity in pseudo time



Increasing τ

The evolution of the stream function in a stationary frame of reference in pseudo time

Evolve the coarsened field

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As a start we are using a very simple homogeneous mixture model where we need models for the interface velocity, dispersion of the small scales and stresses due to the small scales

$$\Delta u = u_f - u_I = f(\kappa, \tilde{\chi} \ldots)$$

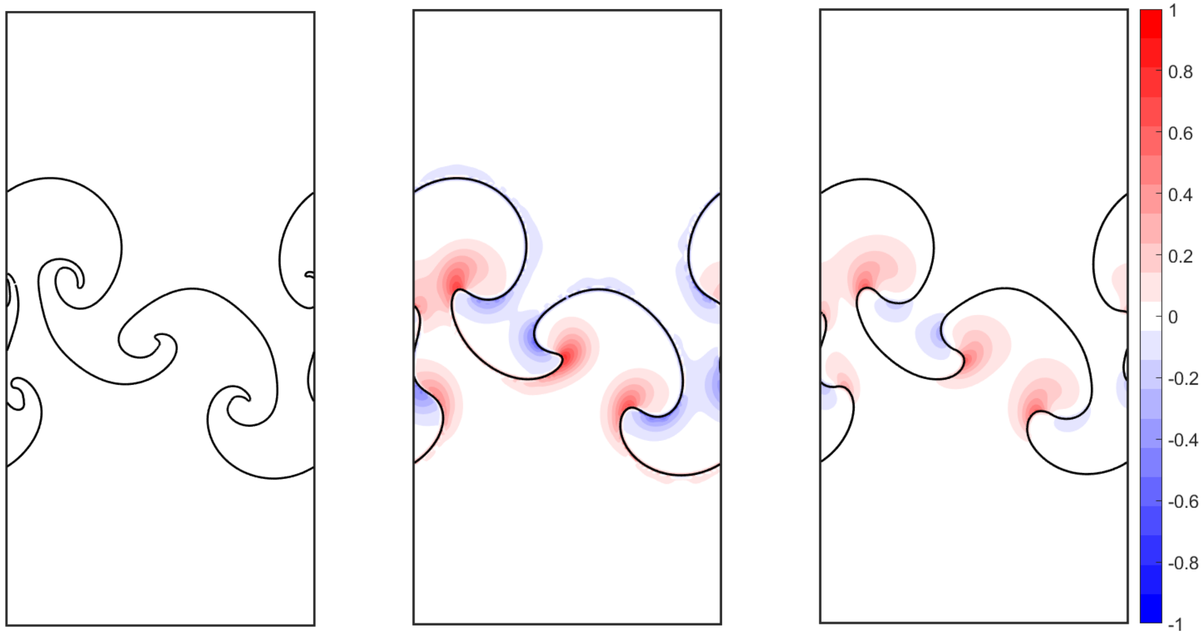
We model $u_f - u_I$ by diffusing the index function a little bit

$$\frac{\partial \alpha'}{\partial t} + \boldsymbol{u} \cdot \nabla \alpha' = \nabla \cdot D_e \nabla \alpha' + S_I$$ small scale index function

$$S_I = [\alpha](\boldsymbol{u}_f - \boldsymbol{u}_I) \cdot \boldsymbol{n}$$ interface source

$$\frac{\partial (\tilde{\rho} \tilde{\boldsymbol{u}})}{\partial t} + \nabla \cdot (\tilde{\rho} \tilde{\boldsymbol{u}} \tilde{\boldsymbol{u}}) = -\nabla \tilde{p} + \tilde{\rho} \boldsymbol{g} + \nabla \cdot \boldsymbol{\tau}_e$$ momentum

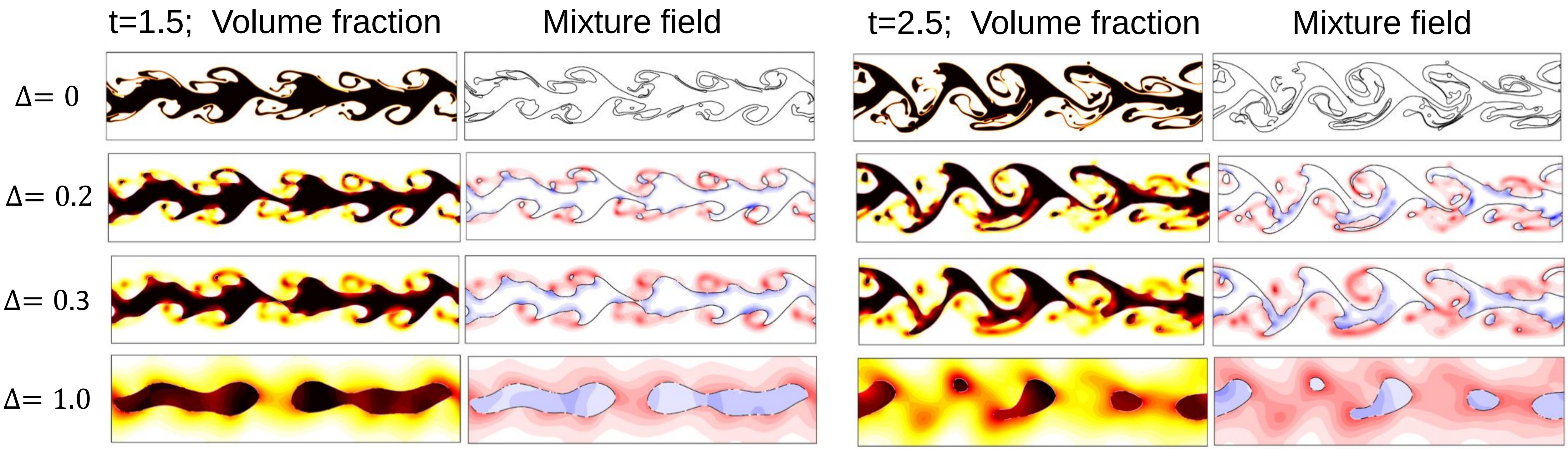
$$\boldsymbol{\tau}_e = \mu_e (\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T).$$ effective stress



DNS solution | Filtered DNS solution | Modeled solution

We start by picking the values of the subgrid terms manually and will do more rigorous modelling by machine learning in the future

Future work



- Using the coarsening method to analyze/characterize complicated, multi-scale flows, based on Interface Retaining Coarsening.
- Develop LES for multiphase flows based on our coarsening strategy and do rigorous modelling for subgrid terms. We have been successful in data-driven modelling for LES in 2D single phase turbulence.

Chen, Xianyang, Jiacai Lu, and Gretar Tryggvason. "Finding Closure Terms Directly from Coarse Data for 2D Turbulent Flow." *arXiv preprint arXiv:2104.09344* (2021).

Summary

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- We explore a strategy to coarsen multiphase flows in a consistent way while retaining a sharp, but simplified interface.
- The coarsening method preserves momentum, continuity and separates the large/small scales for the interface.
- The coarsening method preserves jump condition for the flow variables across the large-scale interface.
- Evolution equations for the coarsened field are derived assuming a simple homogeneous mixture model, and tested by evolving a Rayleigh-Taylor instability in time.

Xianyang Chen, Jiacai Lu *and* Gretar Tryggvason, "Interface retaining coarsening of multiphase flows", Physics of Fluids 33, 073316 (2021) https://doi.org/10.1063/5.0058776