INVESTIGATING ERRORS AND CONVERGENCE IN STOCHASTIC LAGRANGIAN-EULERIAN METHODS FOR DISPERSE MULTIPHASE FLOWS

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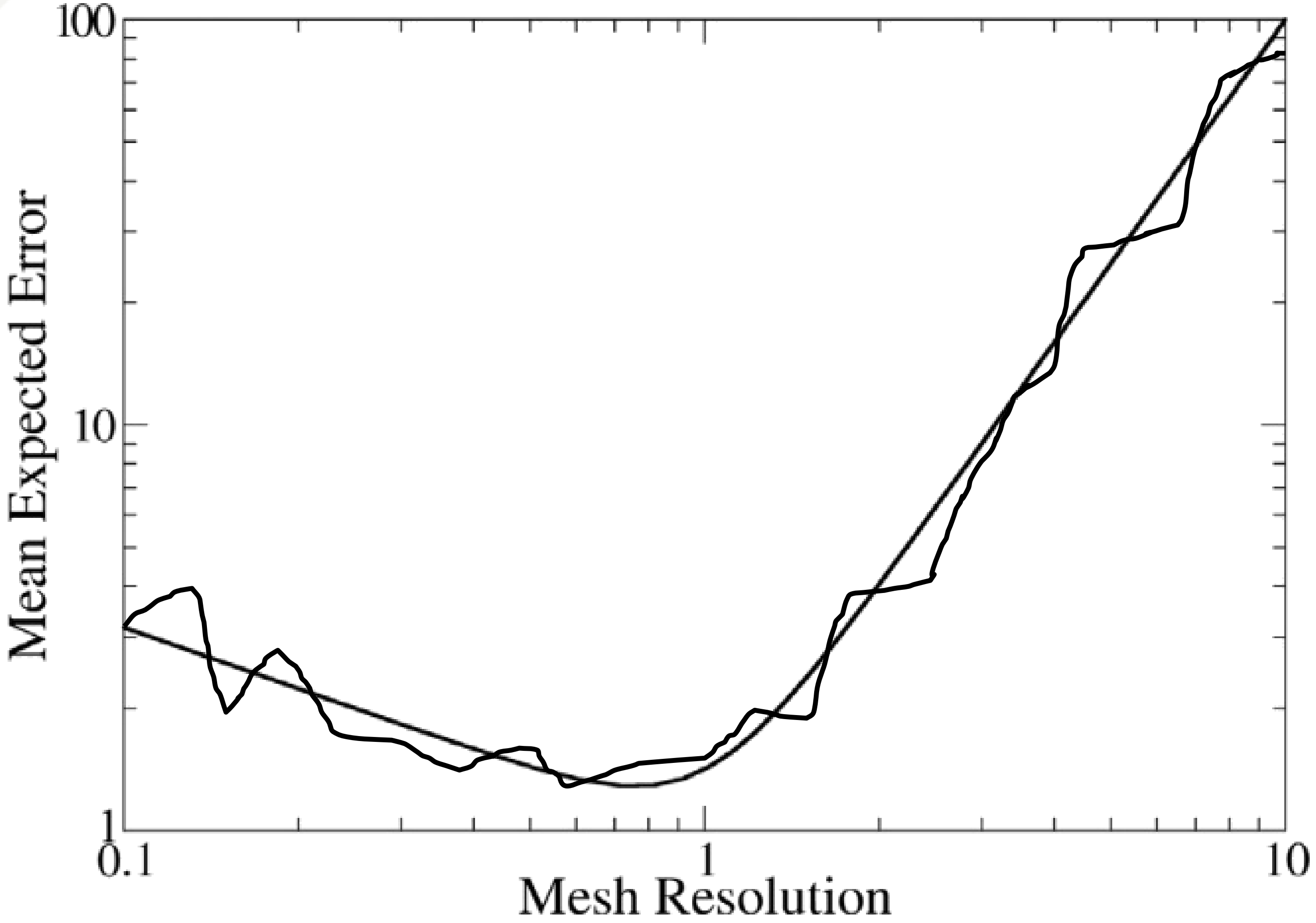
OUTLINE

- Introduction and Background
  - Motivation
  - Literature review
  - Derivation of Asymptotic Mean Square Error and L2 error norm estimates
- 2D static test case
  - Global error comparison
  - Local error comparison
- Future directions

MOTIVATION

PREDICT ERRORS & CONVERGENCE AS CELL SIZE AND PARCEL COUNT CHANGE



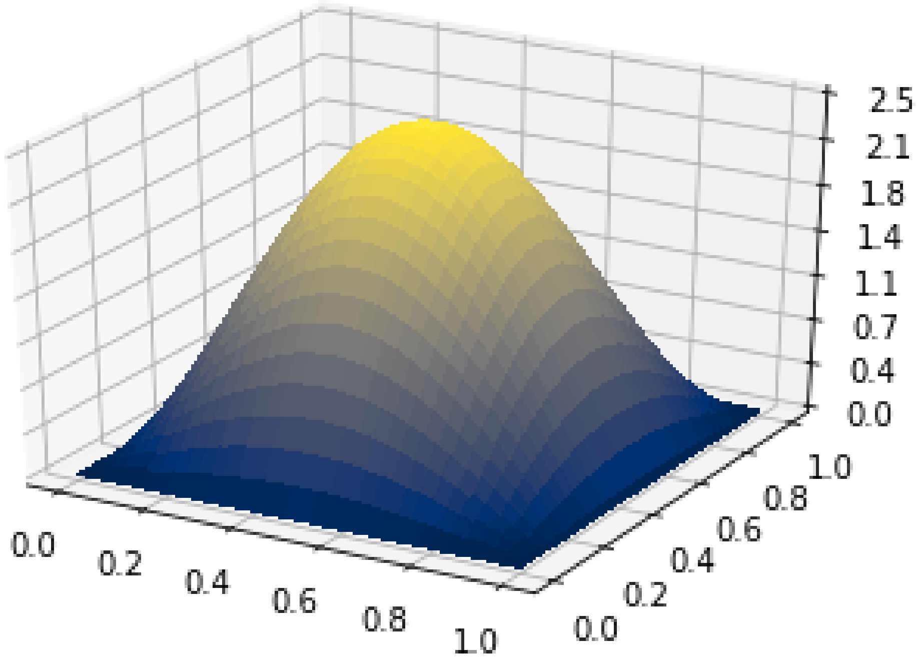
LITERATURE REVIEW

SINGLE TIME STEP – UNIFORM KERNEL

- Schmidt (2006) and Schmidt and Bedford (2018)*
- $E^2 = \frac{f}{n(\Delta x)^d} + \frac{1}{4}\frac{1}{12^2}\left[\sum_{m=1}^{d} \frac{\partial^2 f}{\partial x_m^2} \Delta x^2\right]$
- $n = \frac{b}{\Delta x^a}$
- $L_2^2 = \Delta x^d \sum_{cells} E^2$
- $L_2 \approx \sqrt{\frac{\Delta x^{a-d}}{b} \sum_{cells} f \Delta x^d + \frac{\Delta x^4}{576} \sum_{cells} \left[\sum_{m=1}^{d} \frac{\partial^2 f}{\partial x_m^2}\right]^2 \Delta x^{\mathrm{d}}}$
- $\Rightarrow c = \frac{a-d}{2}; \; a = 2c + d$

*Minor typo in Schmidt and Bedford 2018 fixed here

STATIC TEST CASE

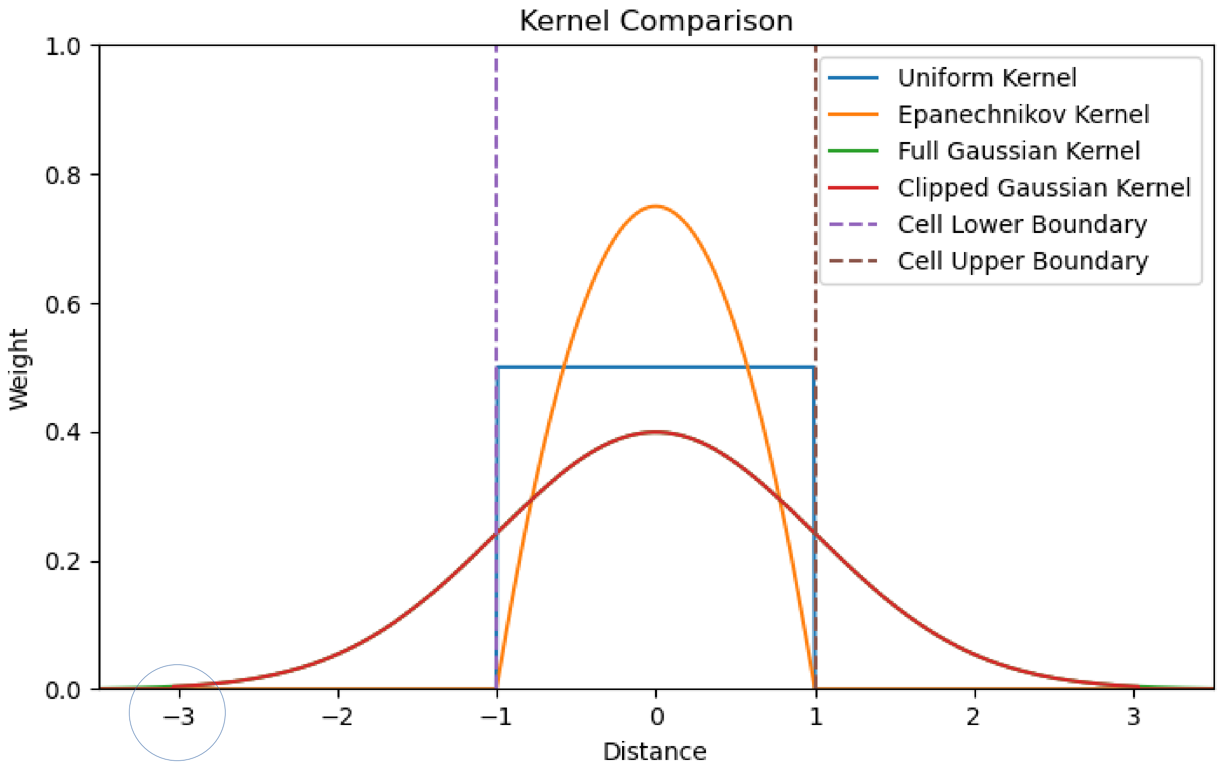


$$f_{exact}(x, y) = \frac{\pi^2}{4}\sin(\pi x)\sin(\pi y)$$

NUMBER OF CELLS VS NUMBER OF PARCELS REQUIRED

Nx	-1st order	0th order	1st order	2nd order
2	32	32	32	32
4	32	128	512	2,048
8	32	512	8,192	131,072
16	32	2,048	131,072	8,388,608
32	32	8,192	2,097,152	536,870,912
64	32	32,768	33,554,432	34,359,738,368

COMPARISON BETWEEN DIFFERENT KERNELS

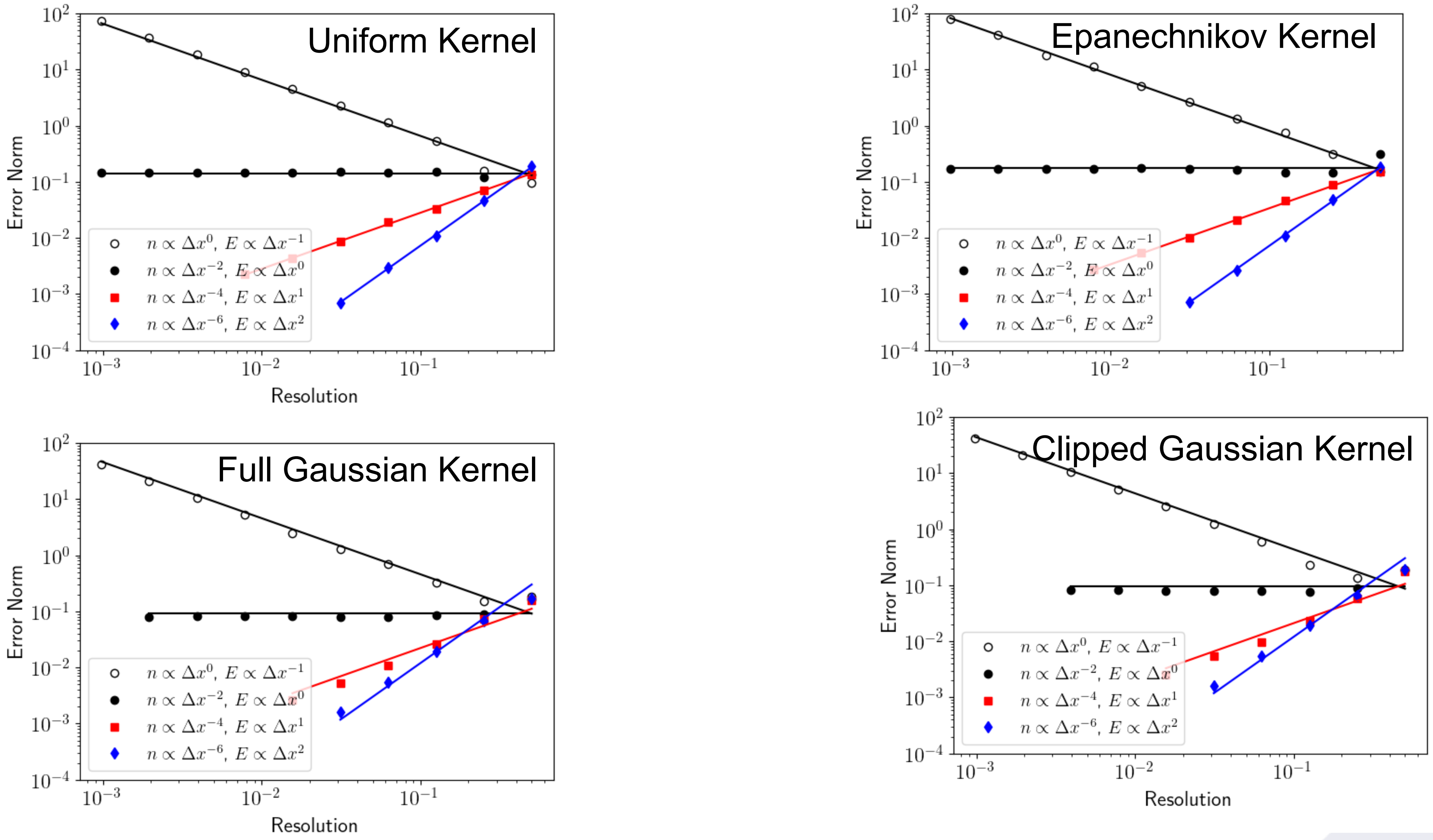


KERNEL PARAMETERS

$$ASME = M^2 \left\{ \frac{f(\vec{x})R\big(K(w)\big)^d}{n} \prod_{j=1}^{d} \frac{1}{h_j} + \frac{1}{4}\sigma_K^4 \left[\sum_{j=1}^{d} h_j^2 \frac{\partial^2 f}{\partial x^2} \right]^2 \right\}$$

Kernel	Variance	Roughness	Support	Integral
Boxcar	0.3333	0.5	[-1,1]	1
Epanechnikov	0.2	0.6	[-1,1]	1
Gaussian	1	0.2821	[-∞,∞]	1
Gaussian Clipped	0.9734	0.2821	[-3.03,3.03]*	0.9976

GLOBAL ERROR COMPARISON



ASYMPTOTIC MEAN SQUARE ERROR

$$ASME = M^2 \left\{ \frac{f(\vec{x})R\big(K(w)\big)^d}{n} \prod_{j=1}^{d} \frac{1}{h_j} + \frac{1}{4}\sigma_K^4 \left[\sum_{j=1}^{d} h_j^2 \frac{\partial^2 f}{\partial x^2} \right]^2 \right\}$$

- Statistical Error
- "Noise"

- Spatial Error
- "Bias"

KERNEL ERROR COMPARISONS, NX=32, NP=536,870,912

Kernel	Expected L2	Empirical L2
Boxcar	0.000769	0.000693
Epanechnikov	0.000649	0.000719
Gaussian	0.001909	0.001589
Gaussian Compact	0.001859	0.001583

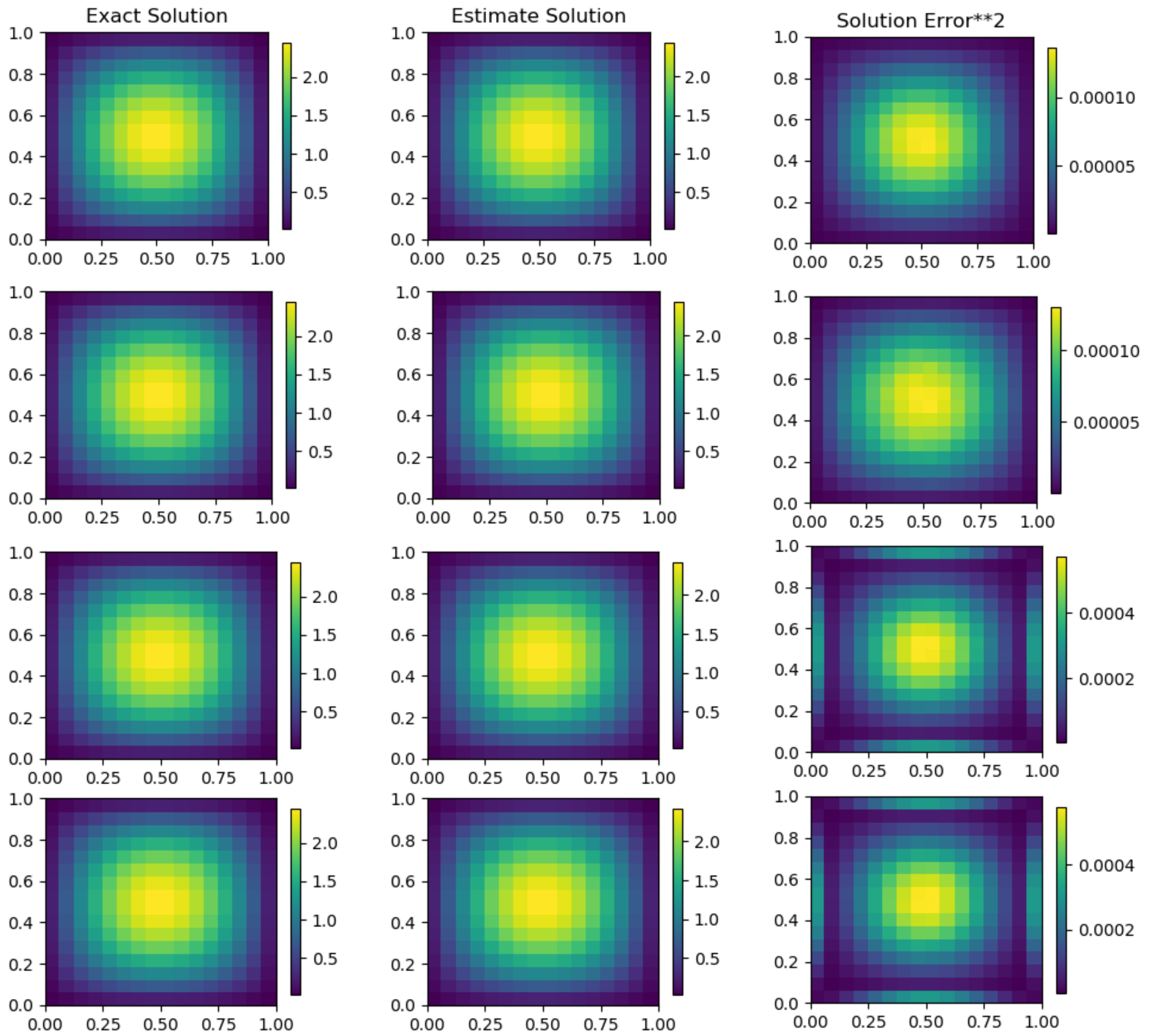
LOCAL ERROR COMPARISON

Uniform Kernel

Epanechnikov Kernel

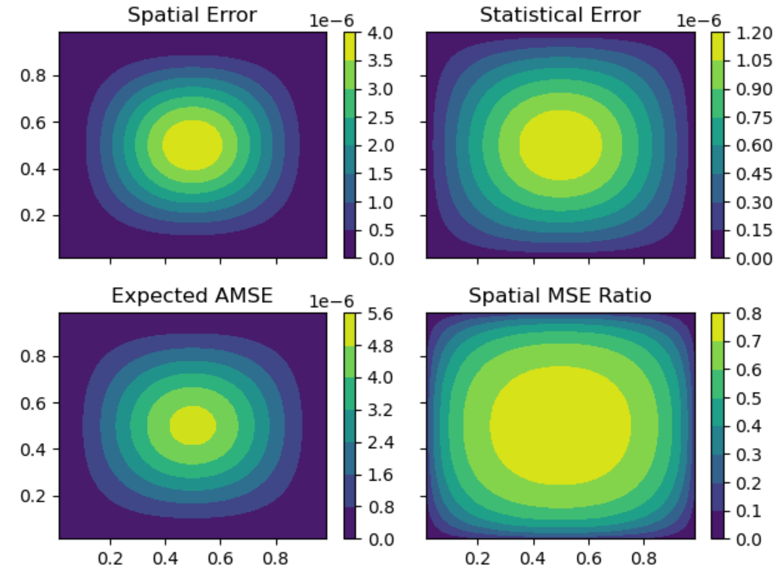
Full Gaussian Kernel

Clipped Gaussian Kernel

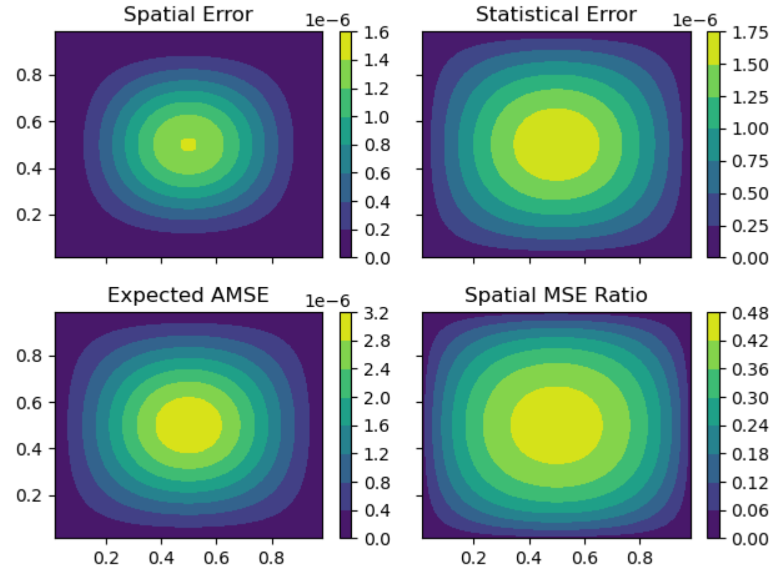


SPATIAL AND STATISTICAL ERROR

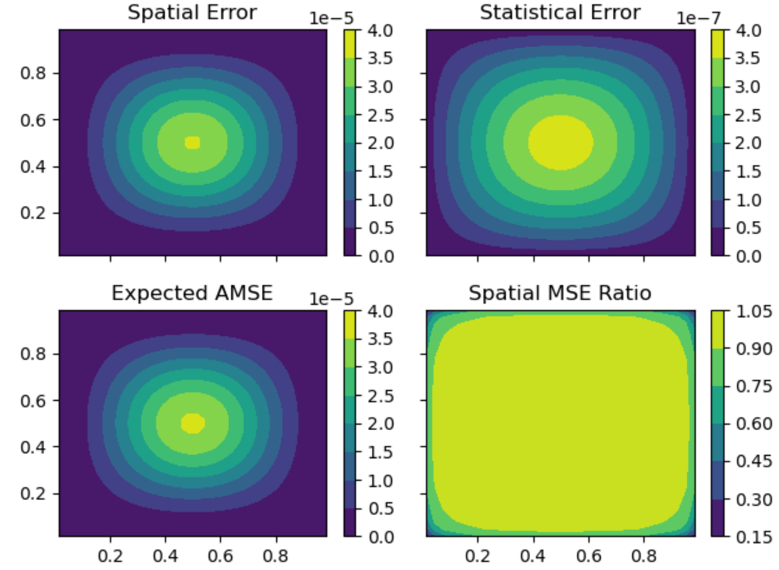
Uniform Kernel



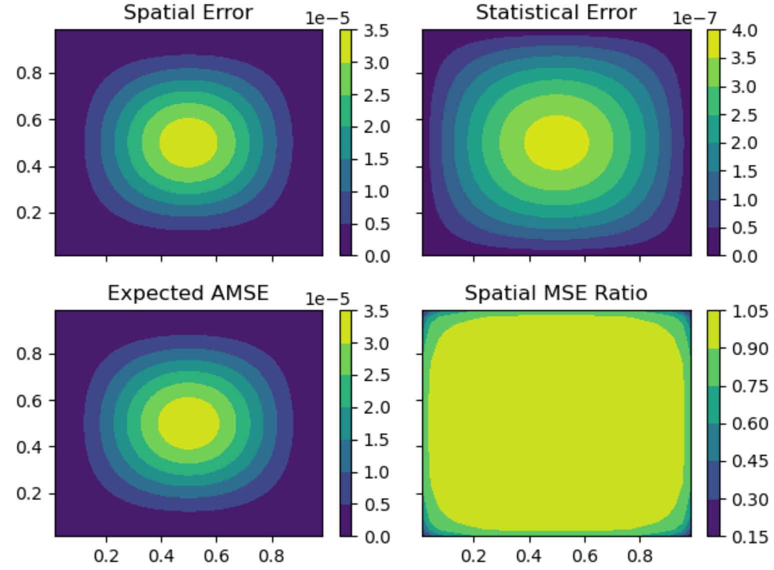
Epanechnikov Kernel



Full Gaussian Kernel



Clipped Gaussian Kernel



COMPARISON OF SPATIAL AND STATISTICAL ERROR

Kernel	Expected Spatial Error	Expected Statistical Error
Uniform	0.001005	0.000489
Epanechnikov	0.000362	0.000704
Gaussian	0.009049	0.000156
Compact Gaussian	0.008574	0.000156

FUTURE DIRECTIONS

- Transient Flow
- 3D simulations
- Higher order kernels

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QUESTIONS?