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Enriched finite element formulation for discontinuous electric field in electrohydrodynamic problems

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CIMNE International Centre for Numerical Methods in Engineering

KRATOS MULTI-PHYSICS

DROPLET-DYNAMICS

OUTLINE

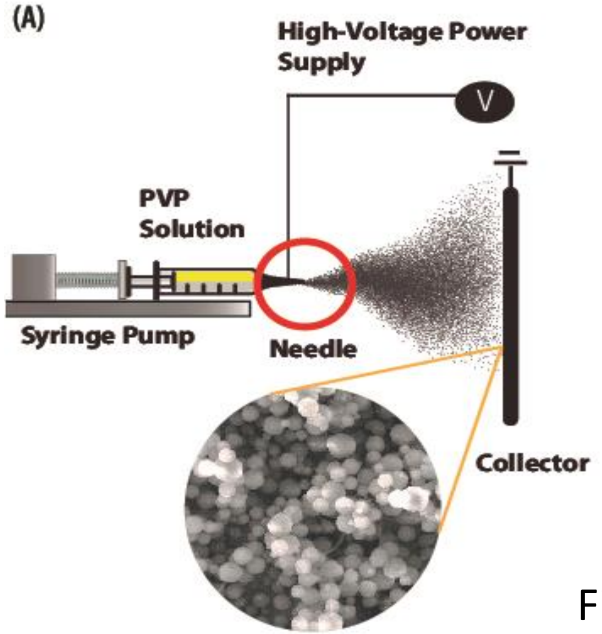
- Introduction
  - Electrohydrodynamics
- Motivation
- Numerical method
- Results
- Conclusions

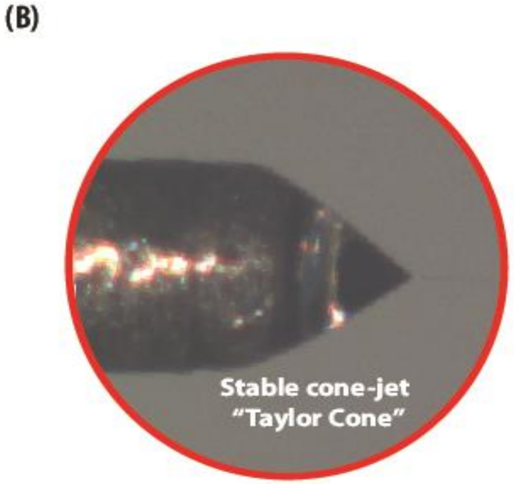
INTRODUCTION

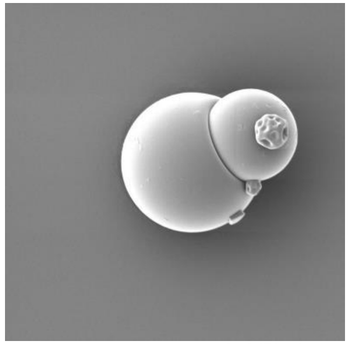
WHAT IS ELECTROHYDRODYNAMIC (EHD)

Electrohydrodynamic [EHD] of liquids is a transport phenomenon, which describes the motion of the liquids subjected to electric field.

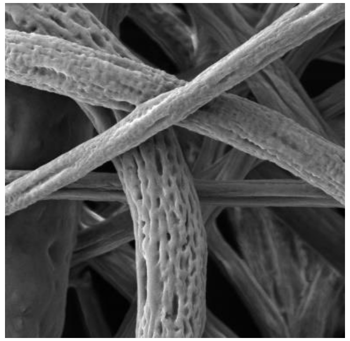
- electrospinning
- electrospray







micro/nano particles



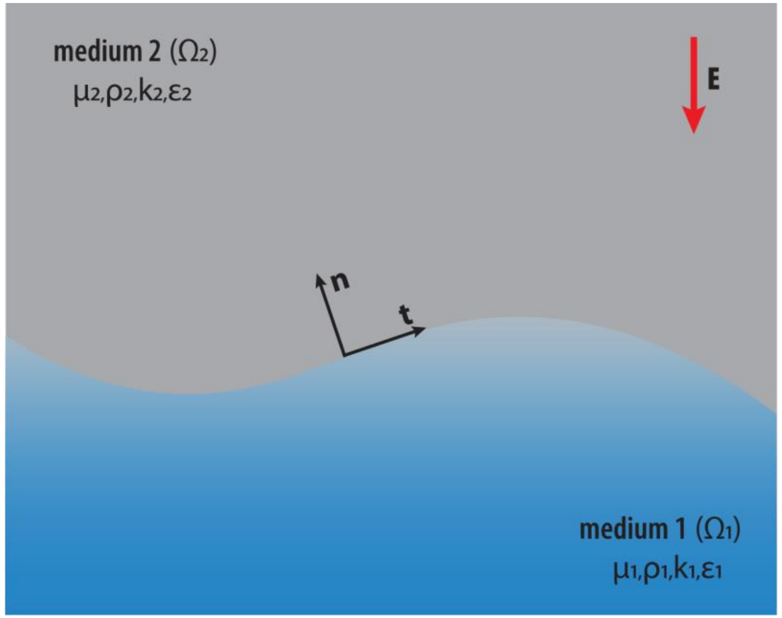
porous fibers

CHALLENGE..!!!

Find the OPERATIONAL PARAMETERS (**onset voltage**)

MOTIVATION

Modeling



FLUIDS

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\frac{\partial(\rho \boldsymbol{u})}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}\boldsymbol{u}) = \nabla \cdot \sigma + \mathbf{f}$$

ELECTRIC FIELD

<u>leaky-dielectric model (Melcher and Taylor)</u>

$$\nabla \cdot (k\mathbf{E}) = 0$$

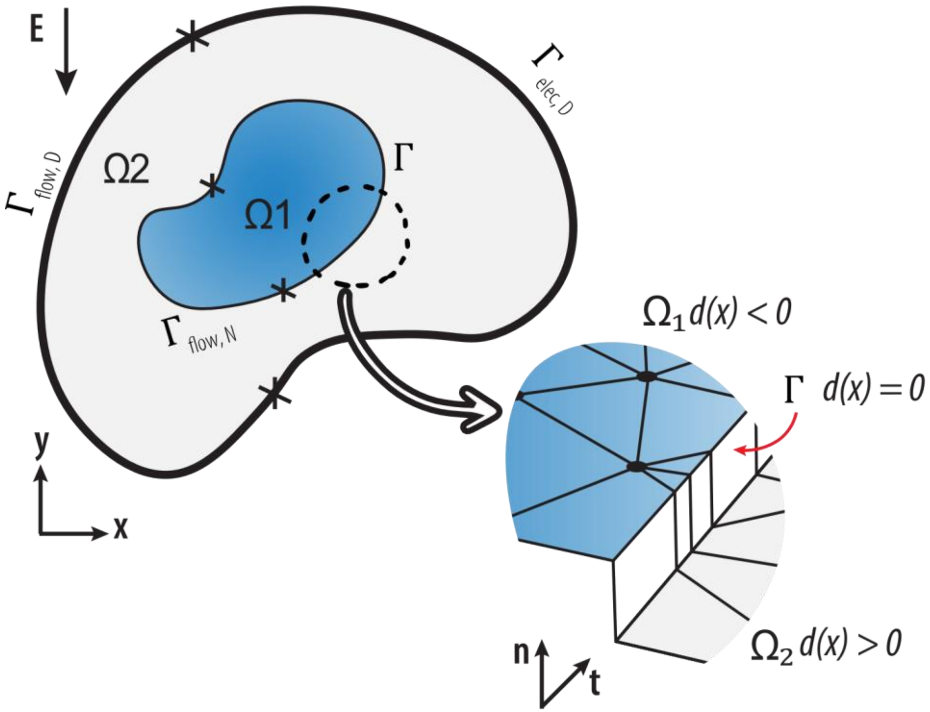
Electric force

$$\boldsymbol{\tau}^e = \varepsilon \left(\mathbf{E}\mathbf{E} - \frac{1}{2}(\mathbf{E}\cdot\mathbf{E})\,\mathbf{I} \right)$$

CHALLENGE..!!!

Sharp representation of the **discontinuous gradient** of the electric potential (electric field **E)**

Modeling the electro-mechanical coupling



electro-mechanical coupling

Interfacial force

$$[\![\boldsymbol{n} \cdot \boldsymbol{\tau}^e]\!] = \left[\!\!\left[\varepsilon(n \cdot \mathrm{E})\mathrm{E} - \frac{1}{2}\varepsilon(\mathrm{E} \cdot \mathrm{E})\, n \right]\!\!\right]$$

Body force

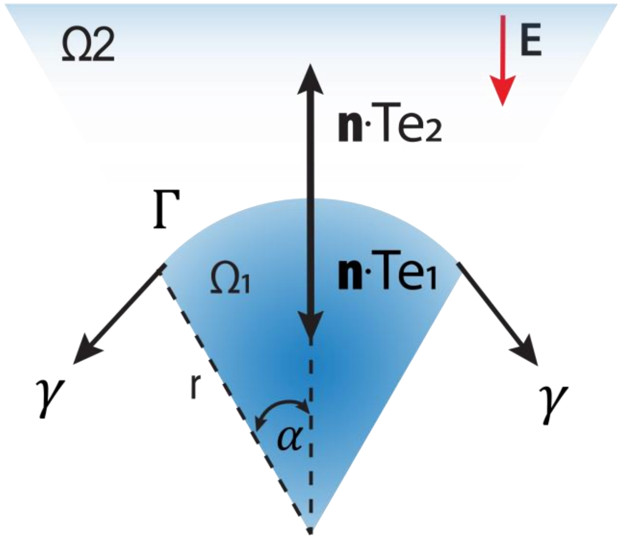
$$\boldsymbol{\nabla} \cdot \boldsymbol{\tau}^e = \nabla \cdot (\varepsilon \mathbf{E})\mathbf{E} - \frac{1}{2}(\mathbf{E} \cdot \mathbf{E})\, \nabla \varepsilon$$

$$\nabla \cdot \mathbf{E} = 0$$

Since both the permittivity and conductivity are piecewise constant, one immediately sees that the **electric force is non-zero only in the vicinity of the interface.**

NUMERICAL METHODS

Modeling the electro-mechanical coupling



The **strong form** of the governing equations can be summarized as:

$$\nabla \cdot (k \nabla \phi) = 0 \quad \text{in} \quad \Omega$$
$$\phi = \tilde{\phi} \quad \text{on} \quad \partial\Omega_{elec,D}$$
$$\mathbf{n} \cdot (k \nabla \phi) = 0 \quad \text{on} \quad \partial\Omega_{elec,N}$$
$$\frac{\partial \rho \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = \nabla \cdot \sigma + \mathbf{f} \quad \text{in} \quad \Omega$$
$$\nabla \cdot \mathbf{u} = 0 \quad \text{in} \quad \Omega$$
$$\mathbf{u}(\mathbf{x}, t) = \mathbf{u}_D \quad \text{on} \quad \partial\Omega_{flow,D}$$
$$\mathbf{t}(\mathbf{x}, t) = \tilde{\mathbf{t}} \quad \text{on} \quad \partial\Omega_{flow,N}$$
$$[\![\mathbf{t}(\mathbf{x}, t)]\!] = -[\![\mathbf{n} \cdot \mathbb{T}_e]\!] + \gamma\kappa\mathbf{n} \quad \text{on} \quad \Gamma$$

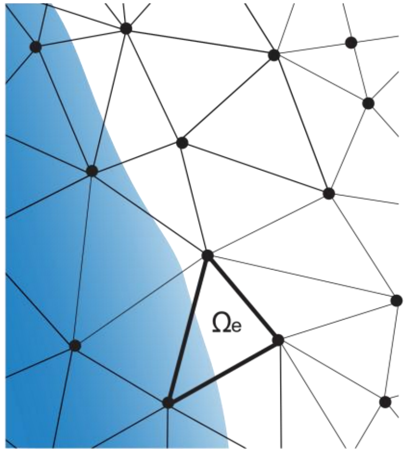
$$\int_\Omega \nabla s \cdot (\kappa \nabla \phi)\, d\Omega = 0$$

$$\int_\Omega \rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) \cdot \mathbf{w} d\Omega = -\int_\Omega p \nabla \cdot \mathbf{w} d\Omega + \int_\Omega \mu (\nabla \mathbf{u} + \nabla \mathbf{u}^T) : \nabla \mathbf{w} d\Omega$$
$$- \int_\Gamma \left[\!\left[\epsilon (\mathbf{n} \cdot \mathbf{E}) \mathbf{E} - \frac{\epsilon}{2} (\mathbf{E} \cdot \mathbf{E}) \mathbf{n} \right]\!\right] \cdot \mathbf{w} d\Gamma + \int_\Gamma \gamma \kappa \mathbf{n} \cdot \mathbf{w} d\Gamma$$

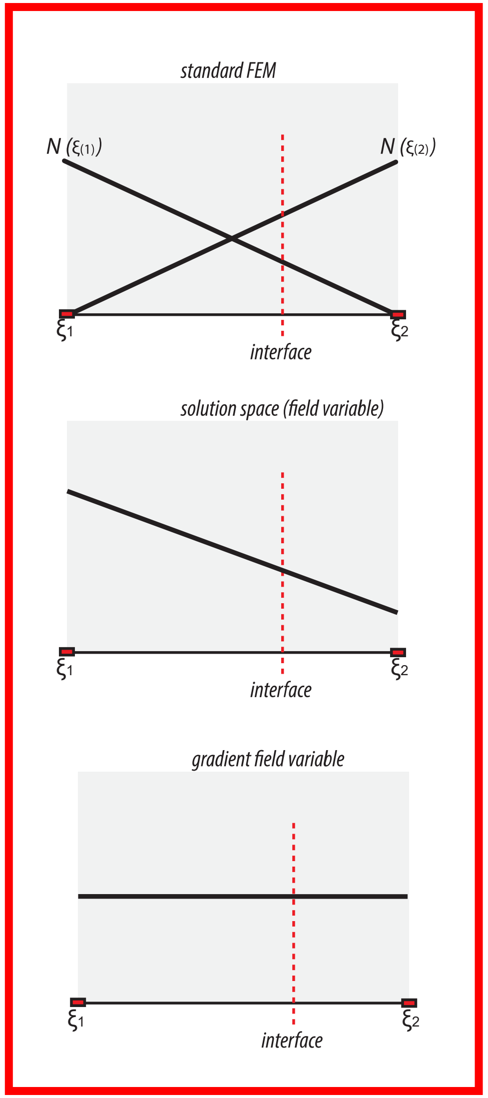
For **those elements that are not cut**, the source term that include the traction vector **(t)** is canceled out by compatibility and equilibrium condition between the neighboring elements

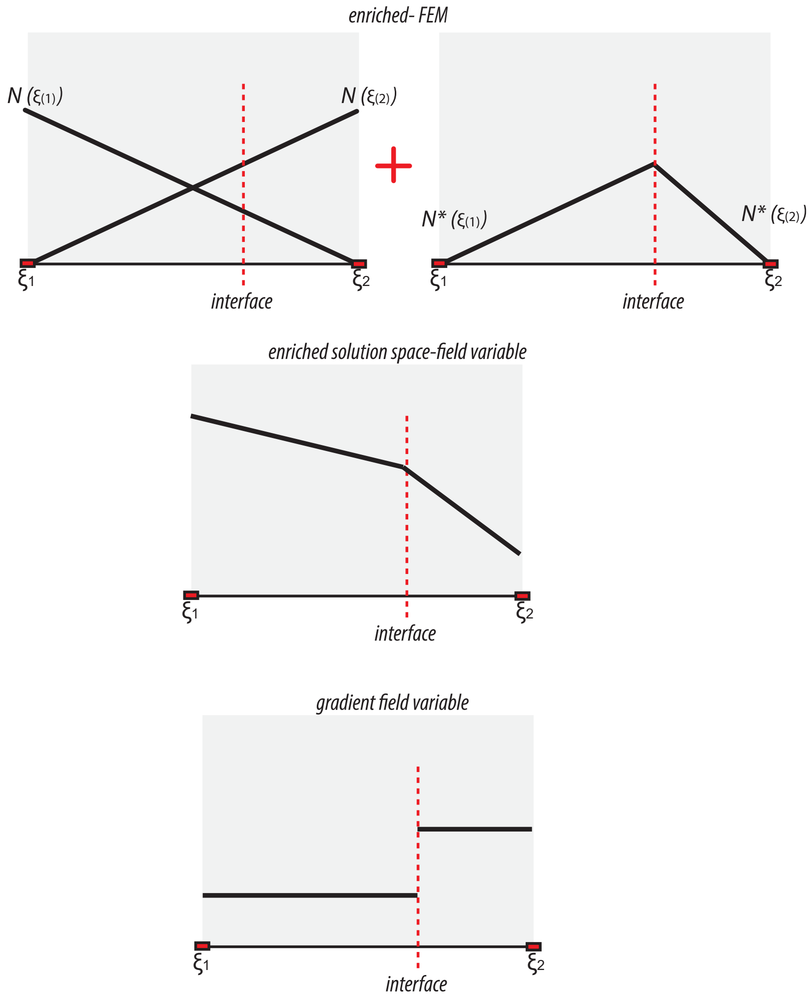
NUMERICAL METHODS

Finite element space

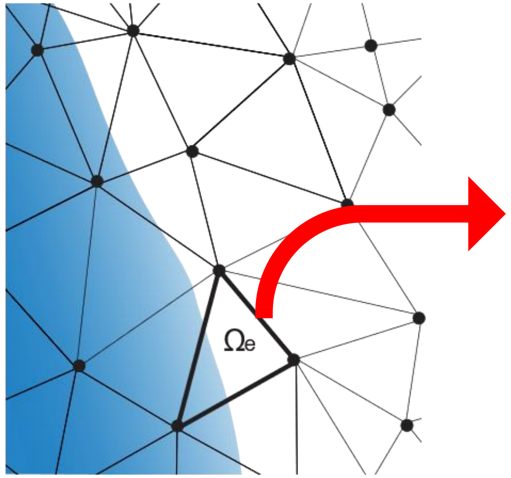


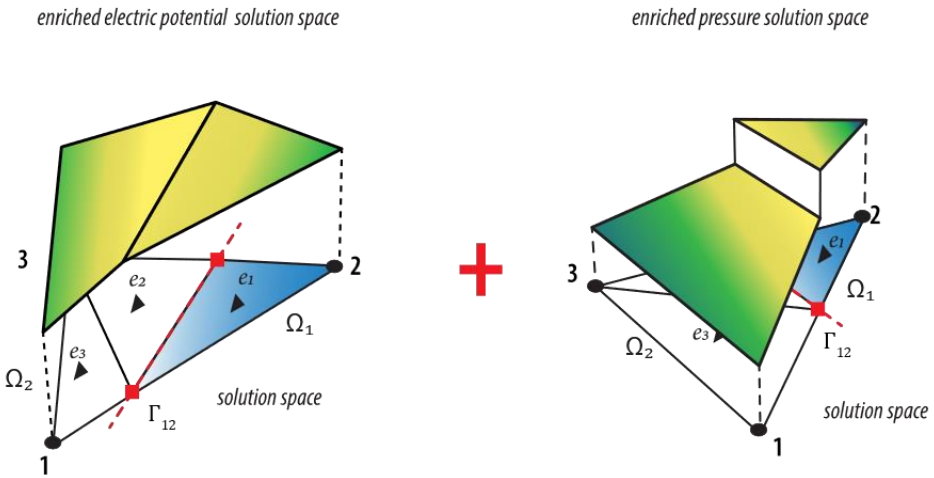
The main problem originates from the fact that **FEM** can represent continuous gradients only, while in the problem at hand gradient field passing across the material interface is no longer continuous





Enriched finite element space





discretization procedure

$$\phi^h(x) = \underbrace{\sum_{i \in e.n} N_i(x)\phi_i}_{standard} + \underbrace{\bar{N}(x)\phi^*}_{enriched}$$

→ **ONE DoF**

$$p^h(\mathbf{x},t) = \underbrace{\sum_{i \in e.n} N_i(\mathbf{x})p_i(t)}_{standard} + \underbrace{\sum_{i \in e.n} \bar{N}_{p,i}(\mathbf{x})p_i^*(t)}_{enriched}$$

→ **THREE DoFs**

In the implementation of the present work, the enriched DoFs associated to the discontinuities of the **EHD** problem are condensed at the elemental level

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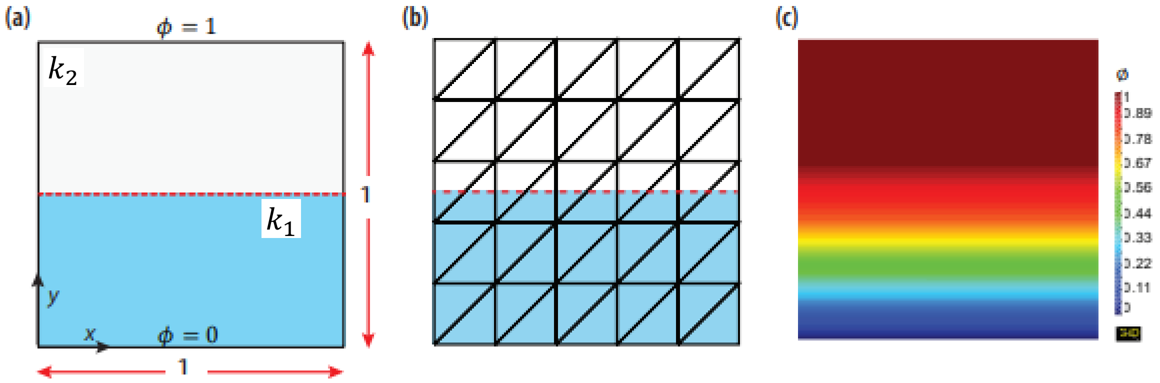
Two materials between horizontal electrodes in a square domain

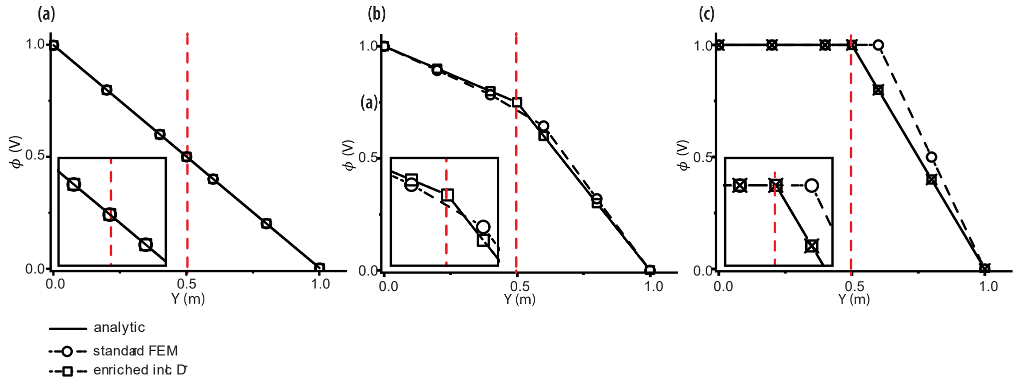


$\mathbf{Q} = \boldsymbol{k_1/k_2}$

Figure: planar test for bi-material problem. **(a)** Sketch of the problem. **(b)** Nonconforming mesh with element size h = 0.2, employed for standard FEM and E-FEM. **(c)** Approximation of the electric potential field for 1/Q ≈ 0 using E-FEM with the inclusion of inter-elemental D+ term in the tangent matrix

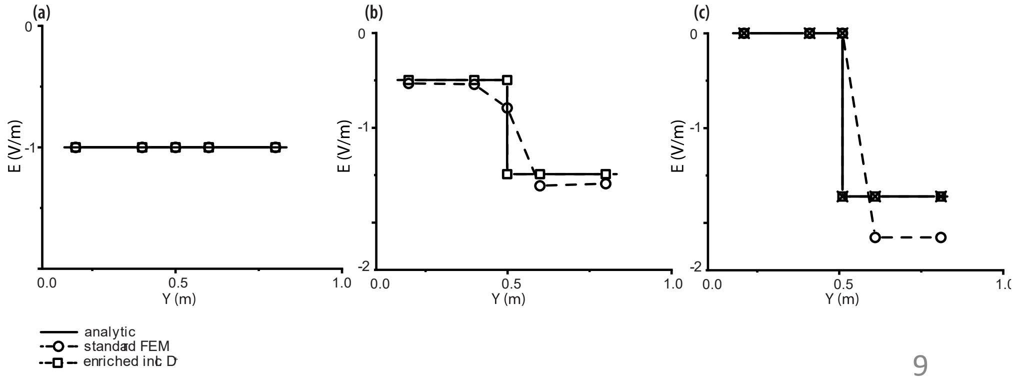
electric potential (∅) distribution

Q=1 **Q=3** **1/Q ≈ 0**



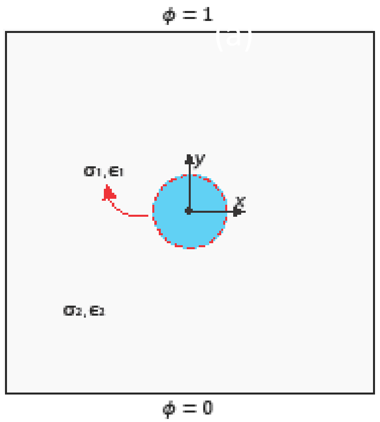
electric field (∇∅) distribution

Q=1 **Q=3** **1/Q ≈ 0**



ACCURACY-TEST

Two materials circular interface



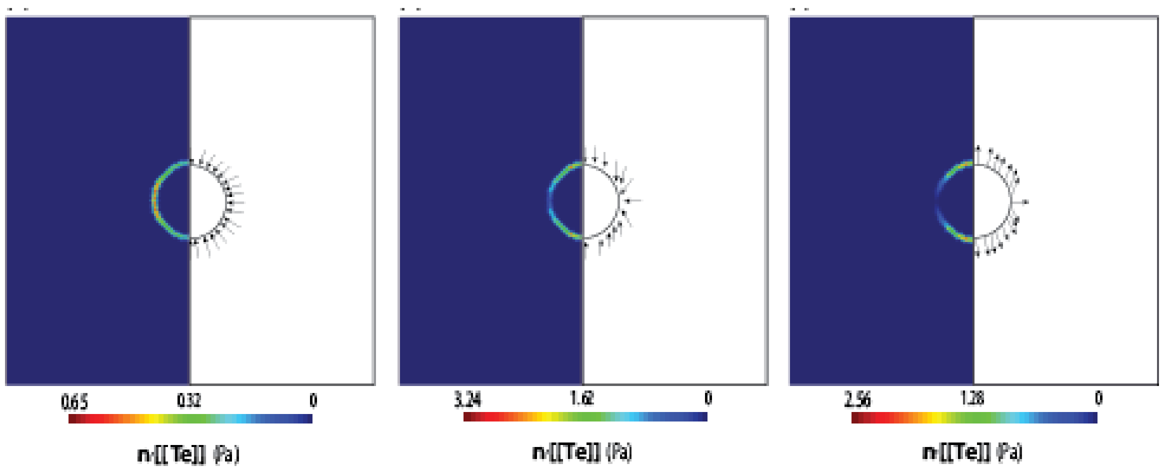


Figure: Numerical contours of electric potential and electric force . (a) Q = R, (b) Q > R, and (c) Q < R

System	ϵ_1 (F/m)	σ_1 (S/m)	ϵ_2 (F/m)	σ_2 (S/m)
S1 $(Q = R)$	1	1	2	2
S2 $(Q > R)$	$2.66\epsilon_0$	2.67×10^{-12}	$3.24\epsilon_0$	1.06×10^{-11}
S3 $(Q < R)$	$3.24\epsilon_0$	1.06×10^{-11}	$2.66\epsilon_0$	2.67×10^{-12}

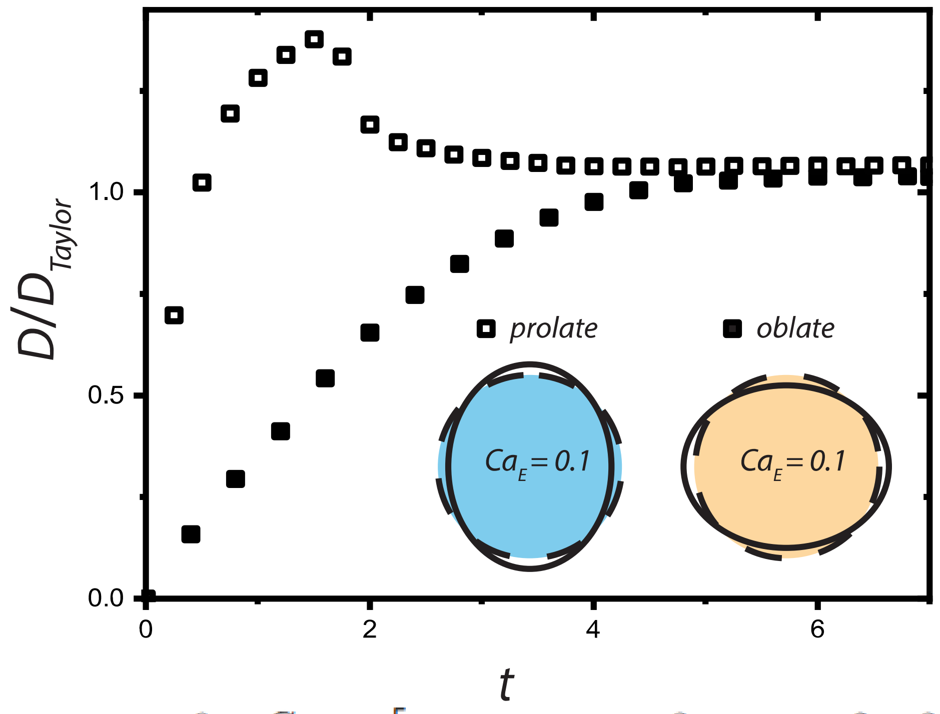
$$\mathcal{O} = \frac{\ln([\![\mathbf{n}\cdot\mathbb{T}_{e,2}]\!]/\ln([\![\mathbf{n}\cdot\mathbb{T}_{e,1}]\!])}{\ln(N_2/N_1)}$$

Ro/h ≈ 1.5
Ro/h ≈ 5.5
Ro/h ≈ 11

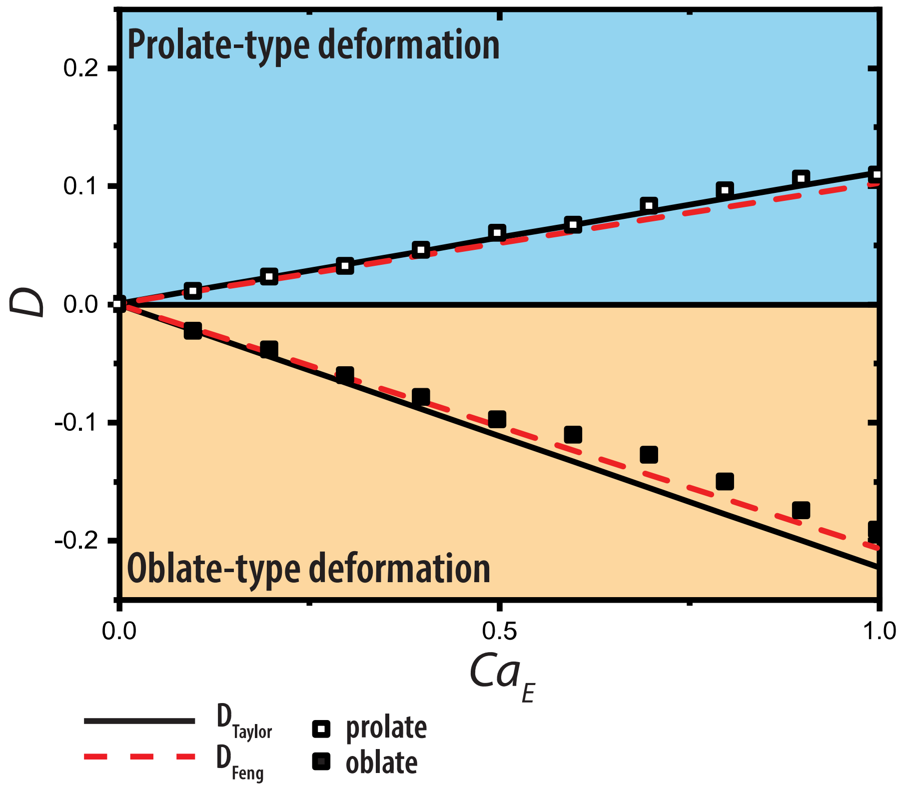
$\ln(N_2/N_1)$	error $\lvert 1 - [\![\mathbb{T}_e]\!]/[\![\mathbb{T}_e^{ex}]\!]\rvert$	$\mathcal{O}$
$\boldsymbol{Q = R}$		
-	0.221	-
2	0.107	1.05
4	0.02	1.21
$\boldsymbol{Q > R}$		
-	0.361	-
2	0.131	1.462
4	0.006	2.224
$\boldsymbol{Q < R}$		
-	0.193	-
2	0.109	0.824
4	0.003	2.59

ACCURACY-TEST

Benchmark case: Droplet deformation



$$D_{Taylor} = \frac{9}{16}\frac{Ca_E}{(2+R)^2}\left[1 + R^2 - 2Q + \frac{3}{5}(R-Q)\frac{2+3\beta}{1+\beta}\right]$$



Smeared interface approach has reasonably accurate solutions for Ro/h > 20, while E-FEM (**sharp interface**) solution on coarse mesh (Ro/h ≈ 8) gives an approximate error of 2%.

ACCURACY-TEST

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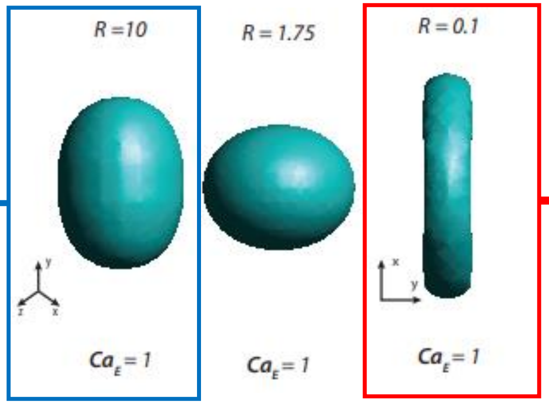
Conductivity effect on droplet deformation



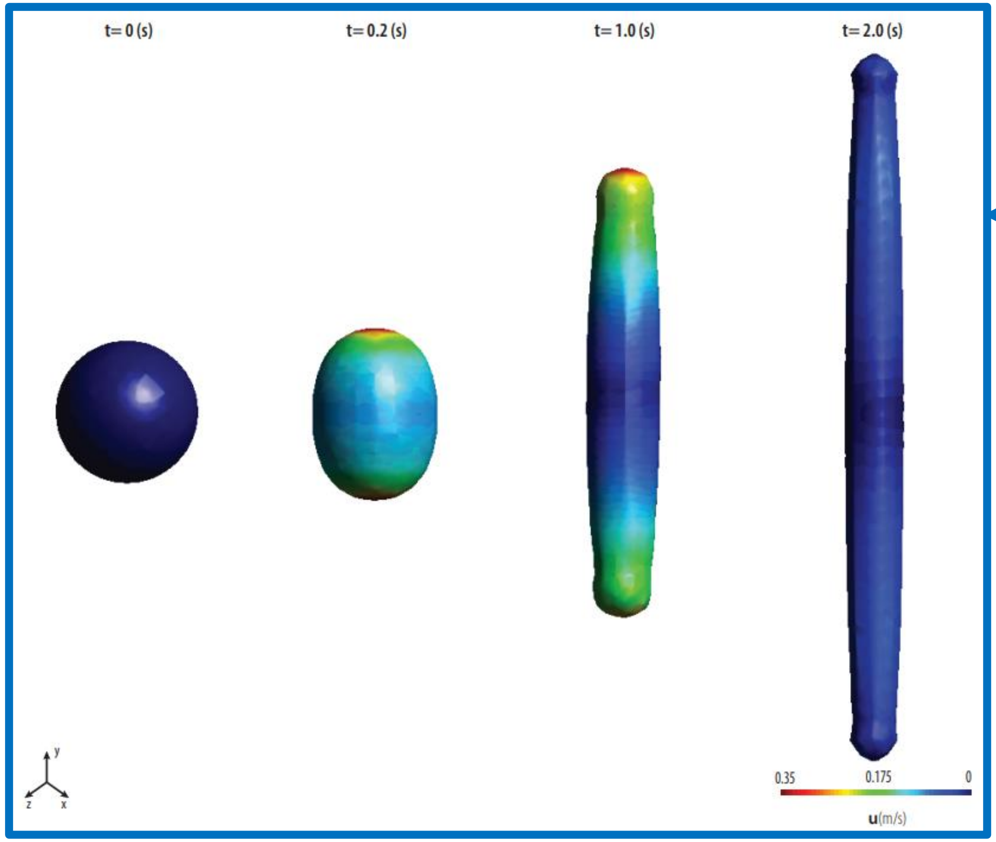


Figure: Snapshots at different time steps of the deformed droplet, for R =10.

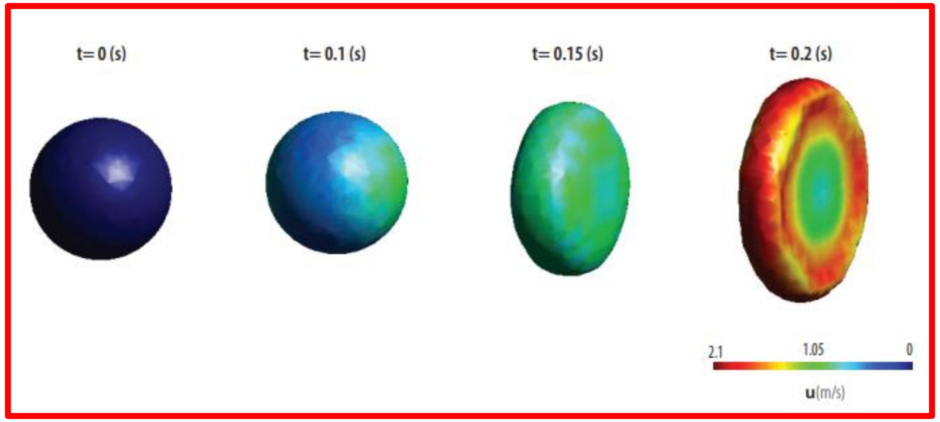


Figure: Snapshots at different time steps of disk-type deformation (R = 0.1).

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CONCLUSIONS

Overall, one can conclude that:

- The proposed E-FEM method accurately represents the **discontinuous fields in EHD.**

- The method is capable of dealing with a **wide range of property ratios (liquid-liquid & gas-liquid).**

- The proposed scheme is capable of providing accurate results for arbitrary interface orientation, exhibiting **second-order of accuracy.**

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THANKS