Steady and Pulsating Flow of Cement Slurries

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Challenges in Petroleum Industry

**Industrial Disaster:** Deepwater Horizon explosion  
**Location:** Gulf of Mexico, Louisiana, United States  
**Date:** April 20, 2010  
**Death:** 11  
**Injuries:** 17

“the Chief Council’s team is certain that the Macondo cement failed” (Chief Counsel’s Report, 2011, pgs 95 – 96)

**Problem:**  
- The offshore oil rig Deepwater Horizon experienced **Gas Migration** after cementing  
- Lead to loss of well control and/or blowouts

**Background - Well Cementing**

**Well Cementing**
- Process of placing a cement slurry in the annulus space between the well casing and the surrounding formations
- Primary function is to provide zonal isolation.
- Designed to have a hydrostatic pressure higher than formation pressure and lower than formation fracture pressure

**Operation Environment**
- High temperature & high pressure (200°C and 150 MPa in deep wells)
- Weak or porous formations, corrosive fluids, formation gas

**Problems**
- Gas migration into wellbore cement
- Permanent pathways can form

**Objectives**
- Understanding the **rheological properties** of cement in oil well applications
  - **Comprehensive model** for cement slurry
  - **Important parameters** that affect the rheology of cement
- Understanding the process of **gas migration** in the hydrating cement

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### Well Cement Properties

#### Cement Physical Properties

<table>
<thead>
<tr>
<th>Cement properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement powder density</td>
<td>3.15 g/cm³</td>
</tr>
<tr>
<td>Cement slurry density</td>
<td>1.442 g/cm³</td>
</tr>
<tr>
<td>Cement particle size</td>
<td>0.1 to 100 µm</td>
</tr>
<tr>
<td>Compressive strength</td>
<td>20 - 40 Mpa</td>
</tr>
<tr>
<td>Maximum solid concentration</td>
<td>0.65</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>2716-3971</td>
</tr>
</tbody>
</table>

#### Cement Chemical Properties

<table>
<thead>
<tr>
<th>Mineral phase</th>
<th>Chemical formula</th>
<th>Abbreviation</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tricalcium silicate (Alite)</td>
<td>Ca₃SiO₅</td>
<td>C₃S</td>
<td>40-70%</td>
</tr>
<tr>
<td>Dicalcium silicate (Belite)</td>
<td>Ca₂SiO₄</td>
<td>C₂S</td>
<td>15-45%</td>
</tr>
<tr>
<td>Tricalcium aluminate (Aluminate)</td>
<td>Ca₃Al₂O₆</td>
<td>C₃A</td>
<td>1-15%</td>
</tr>
<tr>
<td>Tetracalcium aluminoferrite (Ferrite)</td>
<td>Ca₂AlO₅, Ca₂FeO₅</td>
<td>C₄AF</td>
<td>0-18%</td>
</tr>
<tr>
<td>Magnesium oxide (Periclase)</td>
<td>MgO</td>
<td>MgO</td>
<td>2%</td>
</tr>
<tr>
<td>Calcium Oxide (Free lime)</td>
<td>CaO</td>
<td>CaO</td>
<td>2%</td>
</tr>
</tbody>
</table>

#### Cross-section of a Cement Particle

![Cross-section of a Cement Particle](image)

Barron, 2012

#### Cement Hydration Process

1. **Cement** + **H₂-rich water** → 2. **Rapid dissolution of cement component in water** → 3. **Early nucleation and flocculation of hydrated product** → 4. **Early development of microstructure**

Chakraborty et al., 2016

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Rheological Behavior of Non-Newtonian Fluids

Shear Thinning (Pseudoplastic)
- Ketchup
- Viscosity decreases with increasing shear rate

Shear Thickening (Dilatant)
- Cornstarch and water mixture
- Viscosity increases with increasing shear rate

Thixotropic
- Yogurt
- Viscosity decreases with stress over time

Rheoplectic
- Printer ink
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\[ \eta = \frac{\tau}{\dot{\gamma}} = K\dot{\gamma}^{n-1} \]

Hershel-Bulkley

Rheology of Cement Slurry

Goals:
- Develop a **comprehensive rheological model** for cement slurry
- Determine **important parameters** that affect rheological behavior

Rheology of cement slurry:
- Viscosity depends on the **shear rate**, **particle concentration**, ...
- Cement has a **yield stress**
- Cement shows **thixotropic** behavior

**In house constitutive model for cement slurry**

\[ T = T_v + T_y \]

**Total stress:**
\[ T_v = -p I + \mu_0 \left( 1 - \frac{\phi}{\phi_m} \right)^{-\beta} (1 + \lambda^n) \left[ 1 + \alpha tr A_1^2 \right]^m A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \]

**Viscous stress:**
\[ T_y = \left[ m_1 \phi^2 \left( \phi - \phi_{perc} \right) \phi m (\phi_m - \phi) \right] \times \left( -175 w/c + 137 \right) + K \left[ \Pi A_1 \right]^{n-1} \]

**Yield stress:**

\[ \Pi = \frac{1}{A_1} \]
Mathematical Model - Governing Equations

- **Conservation of mass**
  \[
  \frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{v}) = 0
  \]
  \(\rho\): density of cement slurry
  \(\mathbf{v}\): velocity vector, \(\text{div}(\mathbf{v}) = 0\) for an isochoric motion

- **Conservation of linear momentum**
  \[
  \rho \frac{d}{dt} \mathbf{v} = \text{div}\mathbf{T} + \rho \mathbf{b}
  \]
  \(d/dt\): total time derivative, given by \(\frac{d(\cdot)}{dt} = \frac{\partial (\cdot)}{\partial t} + \text{grad}(\cdot)\mathbf{v}\)
  \(\mathbf{b}\): body force vector
  \(\mathbf{T}\): Cauchy stress tensor given by the constitutive equation

- **Conservation of angular momentum**
  \(\mathbf{T} = \mathbf{T}^T\)

- **Convection - diffusion equation**
  \[
  \frac{\partial \phi}{\partial t} + \text{div}(\phi \mathbf{v}) = f
  \]
  \(\phi\): volume fraction
  \(f\): diffusive particle flux

Constitutive Relations

\[
\mathbf{T}_v = -p\mathbf{I} + \mu_{eff}(\phi, A_1)A_1 + \alpha_1 A_2 + \alpha_2 A_1^2
\]

I. For the **viscous stress tensor** $T$

$$T = T_y + T_v$$

$T_y$: yield stress – future work

$T_v$: viscous stress, which is dependent on shear rate, particle volume fraction, temperature, pressure, cement hydration, etc.

A modified second grade (Rivlin-Ericksen) fluid model is applied for viscous stress of cement slurry (Massoudi & Tran, 2016)

$$T_v = -pI + \mu_{eff}(\phi, A_1)A_1 + \alpha_1 A_2 + \alpha_2 A_1^2$$  \hspace{1cm} (5)

$p$: pressure

$\phi$: volume fraction

$A_n$: n-th order Rivlin-Ericksen tensors

where $A_1 = \nabla v + \nabla v^T$  \hspace{1cm} $A_2 = \frac{dA_1}{dt} + A_1 \nabla v + \nabla v^T A_1$

$\alpha_1, \alpha_2$: normal stress coefficients

$\mu_{eff}$: effective viscosity, which is dependent on volume fraction (Krieger 1959) and shear rate

$$\mu_{eff}(\phi, A_1) = \mu_0 \left(1 - \frac{\phi}{\phi_m}\right)^{-\beta} \left[1 + \alpha \text{tr} A_1^2\right]^m$$

$\mu_0$: viscosity of the cement slurry without particles; $\phi_m$: maximum volume concentration of solids; $\beta, m$: material parameters

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II. For the **diffusive particle flux** \( f \)

\[
f = -\text{div} N
\]  
(6)

\( N \): flux vector, related to the movement of the particles (Philips et al, 1992)

\[
N = N_c + N_\mu + N_b = -a^2 \phi K_c \nabla (\dot{\gamma} \phi) - a^2 \phi^2 \dot{\gamma} K_\mu \nabla (\ln \mu_{\text{eff}}) - D \nabla \phi
\]

particles collision spatially varying viscosity Brownian diffusive flux

\( D \) is the diffusion coefficient (diffusivity), which is the function of \( \dot{\gamma} \) and \( \phi \)

\[
D(\dot{\gamma}, \phi) = \eta \| A_1 \|^2 \cdot D_0 [K_1 + K_2 (1 - \phi)^2 + K_3 (\phi_m - \phi)^2 H(\phi_m - \phi)]
\]

(Bridges and Rajagopal 2006; Garboczi and Bentz 1992)

\( a \): particle radius; \( K_c \) and \( K_\mu \): empirically coefficients; \( D_0 \): the diffusivity parameter

\( K_1, K_2 \) and \( K_3 \): fitting coefficients, \( H \): Heaviside function, \( H(x) = 1 \) for \( x > 0 \), \( H(x) = 0 \) for \( x \leq 0 \)

Substitute two constitutive relations (5) (6) into convection-diffusion equation (4)
Cement Slurry Model

Example 1: Steady Flow of a Cement Slurry

MODEL DESIGN

- Constitutive cement model – cement flow at offshore wellbore conditions
- Cement slurry modeled as non-Newtonian fluid
- Viscosity depends on the shear rate and particle concentration
- Study the impact of parameters on behavior of cement slurry

OUTCOMES

- Parametric study results indicate that the following significantly affect the velocity and volume fraction:
  - Angle of inclination $\theta$
  - Maximum packing fraction of cement particles
  - Pressure and gravity terms

Parametric Study

Effect of Inclination Angle, $\theta$

Parametric Study

Effect of maximum packing fraction $\phi_m$

Maximum packing fraction $\phi_m$:
Volume fraction of aggregates in closest-packing at which the relative viscosity approaches infinity.

Parametric Study

Effect of $K_c/K_\mu$

Effect of $m$

Effect of $R_0$

Effect of $R_1$

The motion is unsteady and in transient state
- The flow is assumed to be one-dimensional
- The velocity and the volume fraction forms:
  \[
  \begin{align*}
  \phi &= \phi(r, t) \\
  \mathbf{v} &= v(r, t) \mathbf{e}_z
  \end{align*}
  \]
Parametric Study

Effect of time cycles

Distribution of velocity & volume fraction at different time cycles in the pipe

Parametric Study

Effect of $m$

Effect of Pressure

Simulations & Experiments

Channel Flow Air in Fluid Simulations

Wellbore Simulation Chamber

- Max 1500 psi (~depth 1850 ft)
- 20°C to 60°C

@ University of Pittsburgh

Conclusions

Viscous stress:
\[ T_v = -pI + \mu_0 \left( 1 - \frac{\phi}{\phi_m} \right) (1 + \lambda^n) [1 + \alpha t A_1^2]^m A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \]

Yield stress:
\[ T_y = \frac{m_1 \phi^2 (\phi - \phi_{perc})}{\phi_m (\phi_m - \phi)} \times (-175 w/c + 137) \times \frac{n}{\Pi A_1^{1/2}} + K \left[ \Pi A_1^{n-1} \right] A_1 \]

Comprehensive Constitutive Model for Cement Slurry

Gas Migration in Well Cementing

Cement Rheology

Viscous stress:

Yield stress:

Parametric Study with CFD

Mitigate the Geotechnical Infrastructure Hazard

Gas flow potential factor

Flow condition 1
Minor

Flow condition 2
Moderate

Flow condition 3
Severe
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