UNIVERSITY OF LIVERPOOL

ENHANCING INSIGHTS INTO GRANULAR FLOWS USING PROPER ORTHOGONAL DECOMPOSITION

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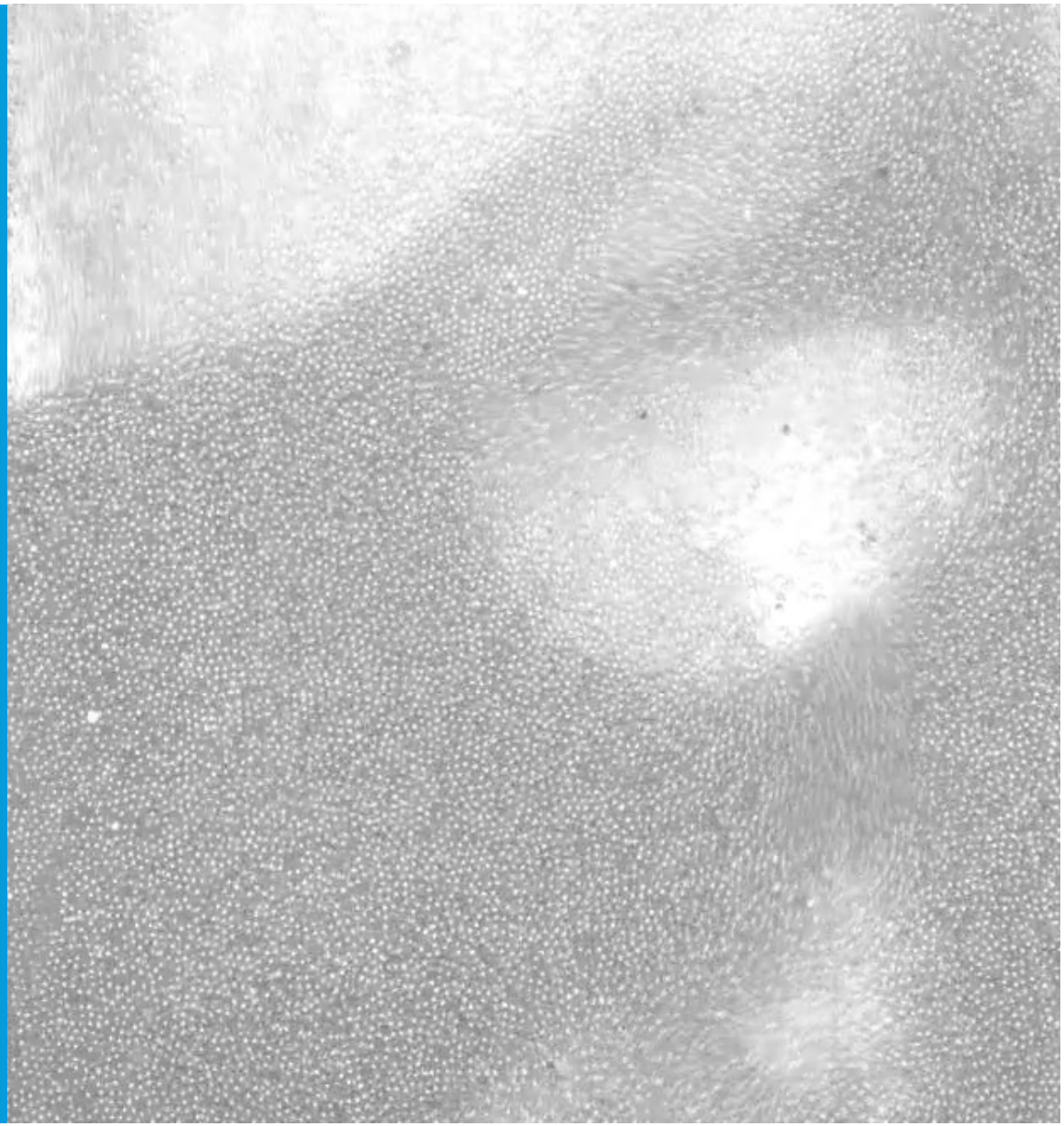
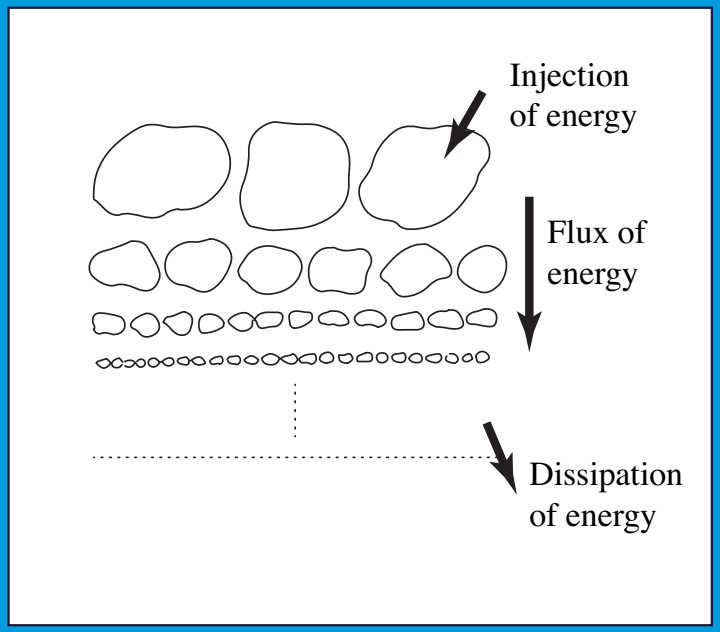
MFIX Online Event
01/08/2023

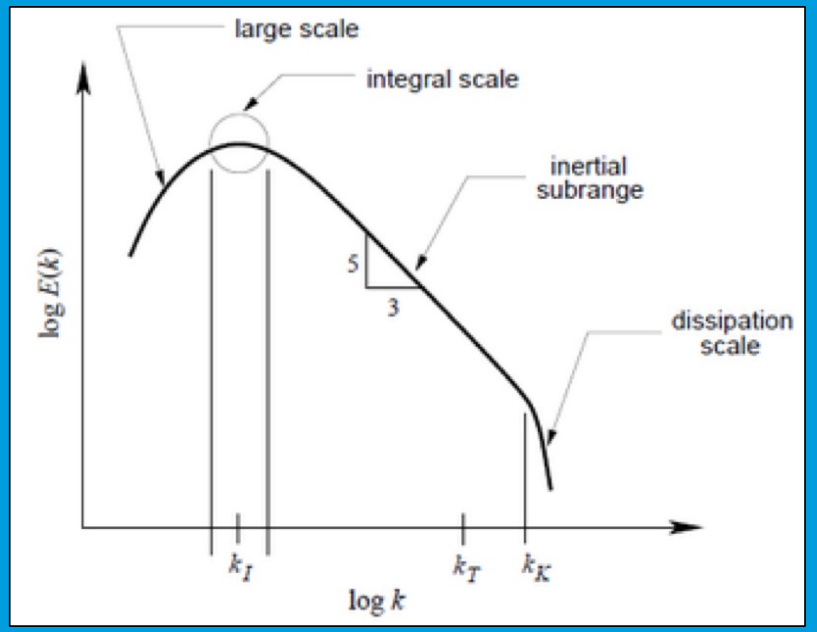
MOTIVATION



FLUID MECHANICS

$$\rho \frac{\partial \mathbf{u}}{\partial t} = -\nabla p + \rho g + \mu \nabla^2 \mathbf{u}$$

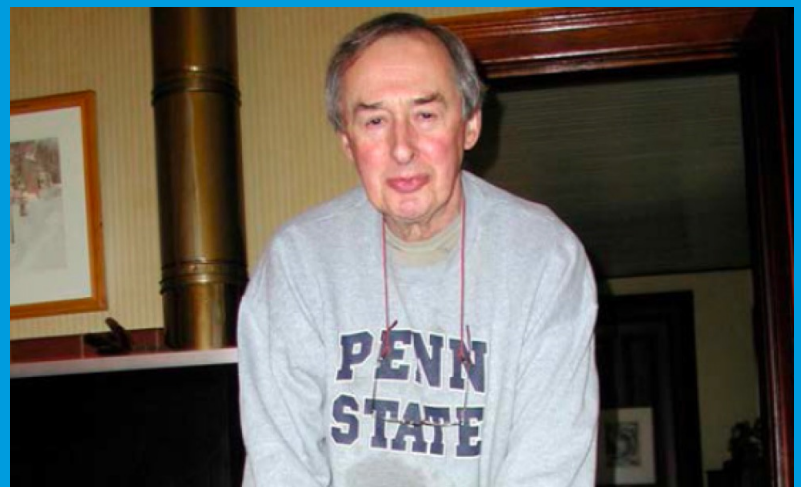


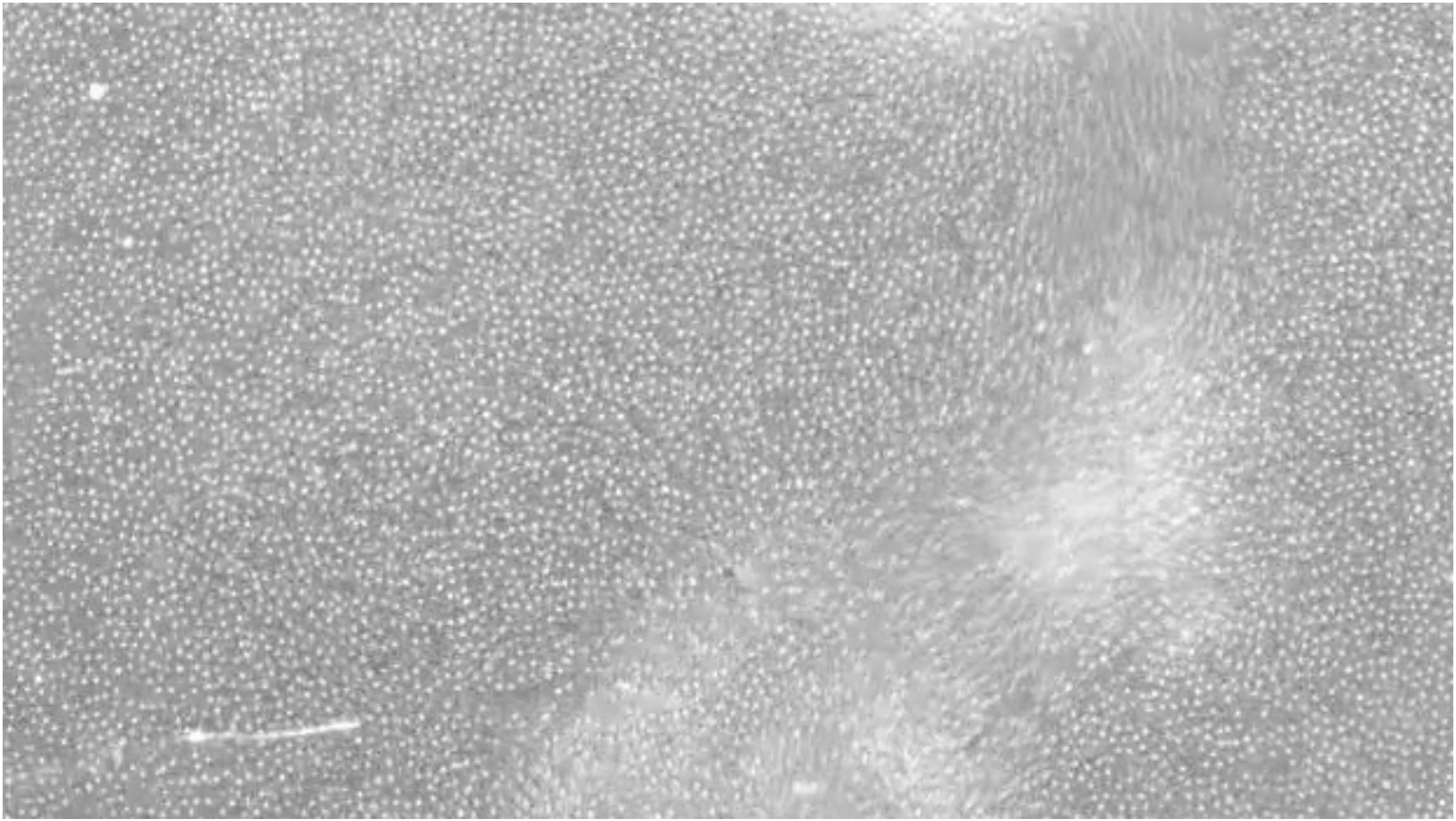


COHERENT STRUCTURES?

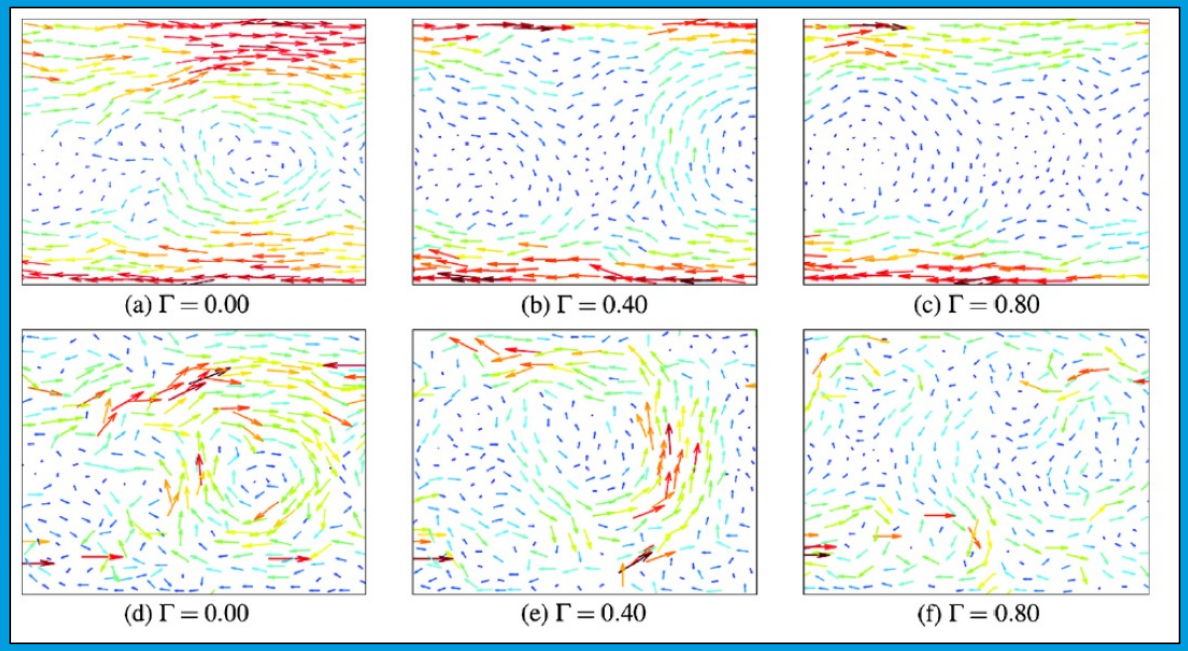
Richardson (1926)
“Big whirls have little whirls,
That feed on their velocity;
And little whirls have lesser whirls,
And so on to viscosity.”

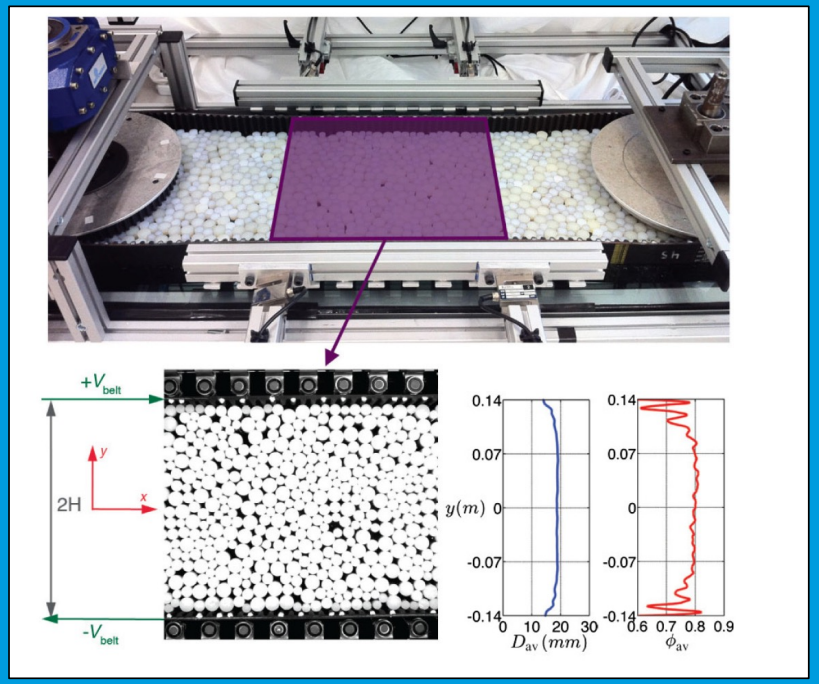
Betchov (1976):
“Big whirls lack smaller whirls,
To feed on their velocity.
They crash and form the finest curls,
Permitted by viscosity.”





COHERENT STRUCTURES IN SOLIDS?

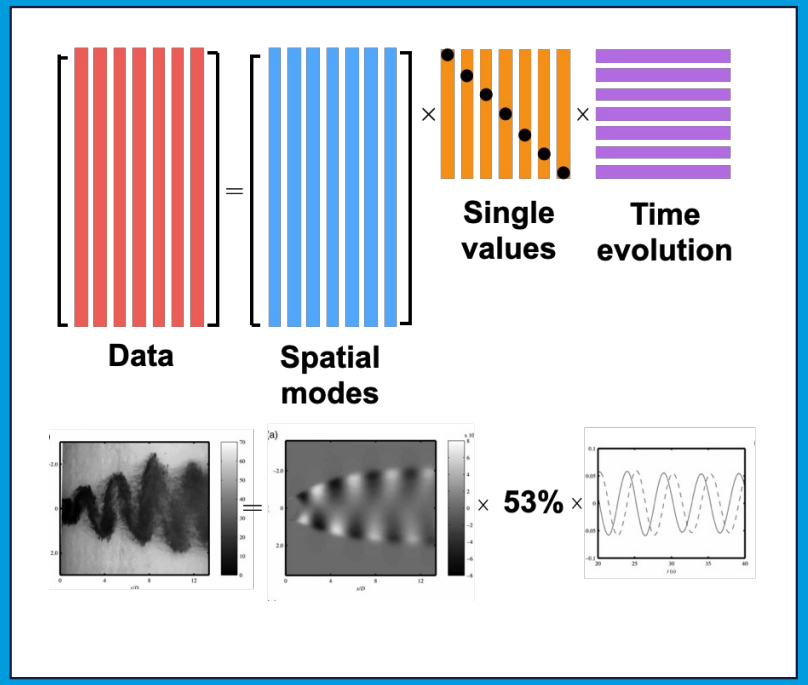


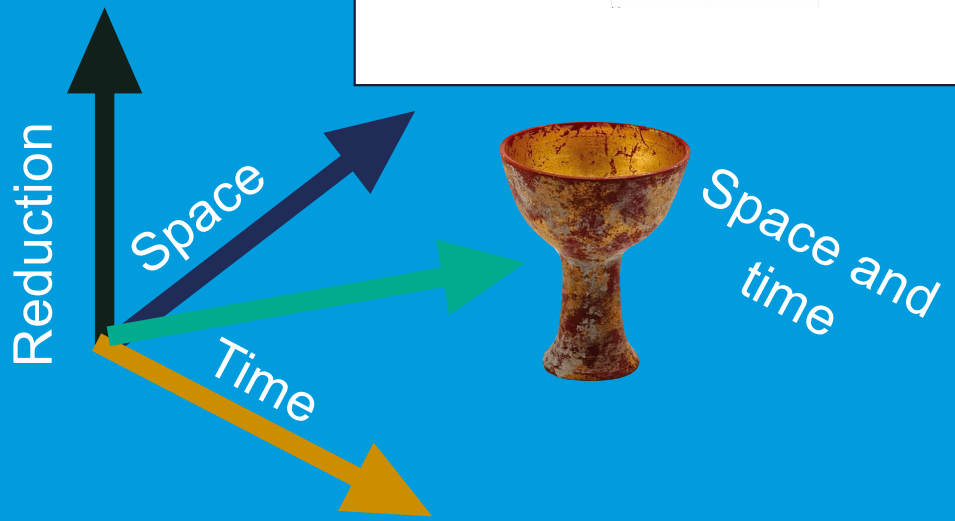


Miller et al. 2013 - Eddy Viscosity model?

MODAL DECOMPOSITIONS

- In fluids there are two main techniques.
- POD (Proper Orthogonal Decomposition) - space?
- DMD (Dynamic Mode Decomposition) - time?
- The main aim in a modal decomposition technique to extract regions of coherence.





PROPER ORTHOGONAL DECOMPOSITION

$$\mathbf{X} = \mathbf{USV}^* \quad \text{where} \quad \mathbf{X} \in \mathbb{R}^{N \times T}$$

$$\mathbf{X} = \mathbf{USV}^{\mathrm{T}} \in \mathbb{R}$$

We want our modes to represent space i.e.

$$\mathbf{U} \in \mathbb{R}^{N \times N} \quad \text{therefore temporally} \quad \mathbf{V} \in \mathbb{R}^{T \times T}$$

Provides that both U and V are orthogonal such that:

$$\mathbf{V}\mathbf{V}^T = \mathbf{V}^T\mathbf{V} = \mathbf{I}$$

$$\mathbf{U}\mathbf{U}^T = \mathbf{U}^T\mathbf{U} = \mathbf{I}$$

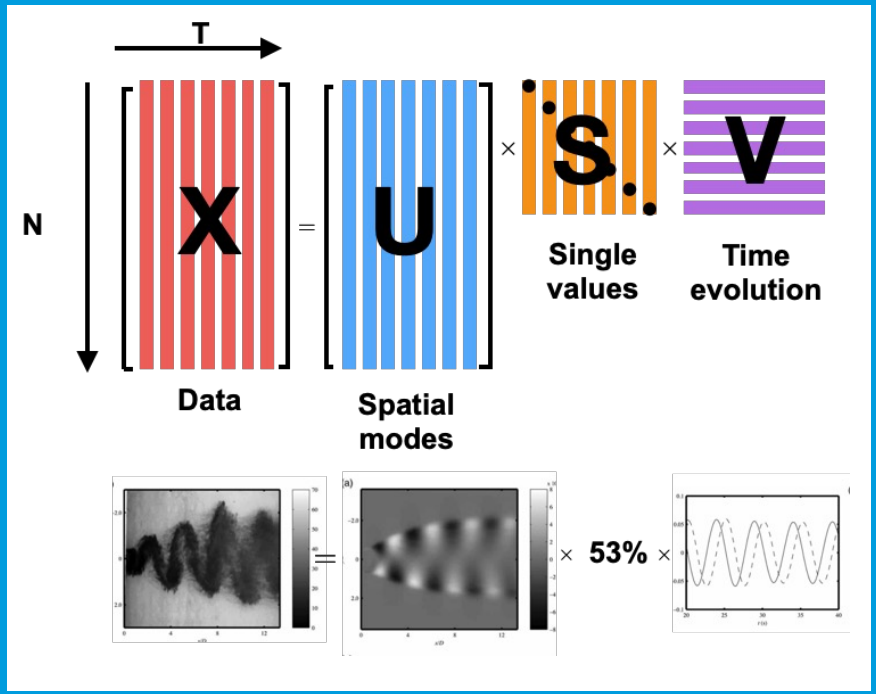
So to create this we could say:

$$\mathbf{X}^T\mathbf{X} = \mathbf{V}\mathbf{S}^T\mathbf{U}^T\mathbf{U}\mathbf{S}\mathbf{V}^T = \mathbf{V}(\mathbf{S}\mathbf{S}^T)\mathbf{V}^T$$

$$\mathbf{X}\mathbf{X}^T = \mathbf{U}\mathbf{S}\mathbf{V}^T\mathbf{V}\mathbf{S}^T\mathbf{U}^T = \mathbf{U}(\mathbf{S}^T\mathbf{S})\mathbf{U}^T$$

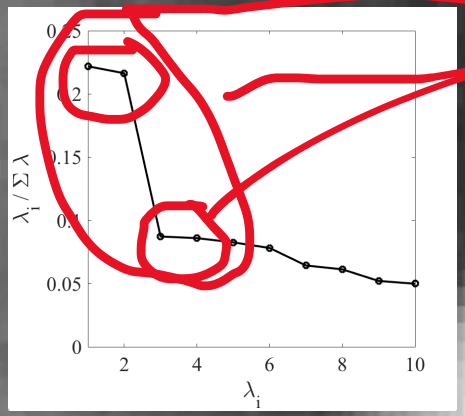
This then proves that U and V determined from the Eigen decomposition of XX^T or X^TX are orthogonal descriptions of X:

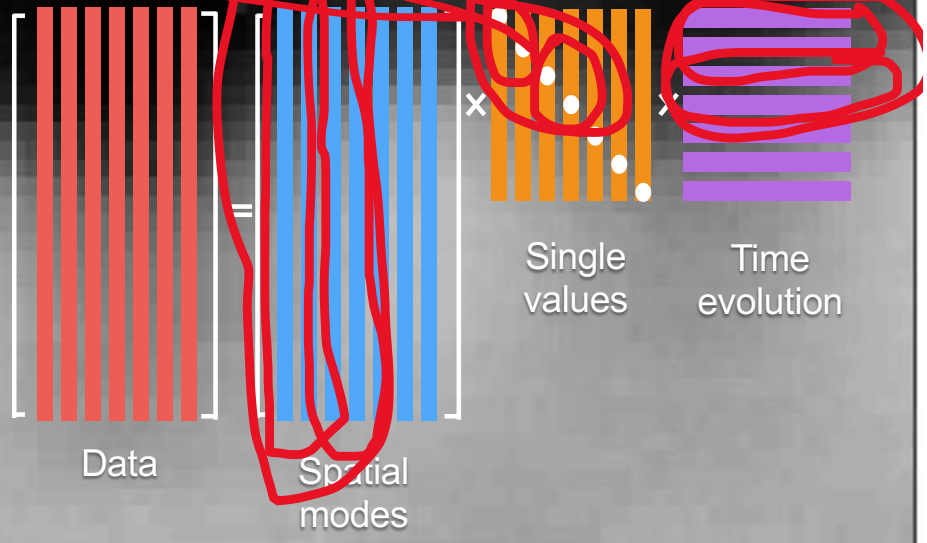
$$\mathbf{X} = \mathbf{U}\mathbf{S}\mathbf{V}^T$$

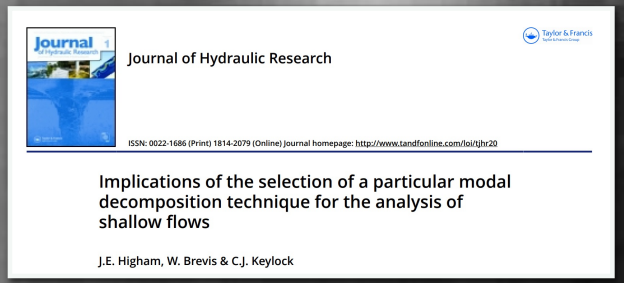


Therefore U relates to the orthogonal spatial picture

Therefore V relates to the orthogonal spatial pictures evolution






Journal of Hydraulic Research

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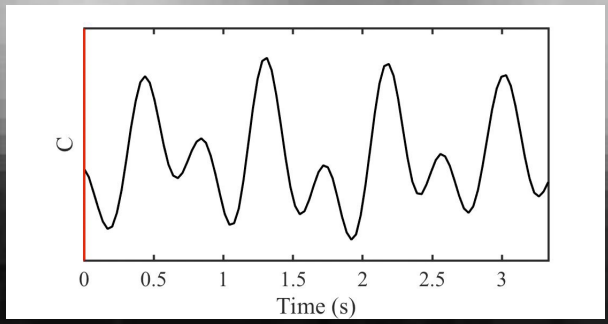
Implications of the selection of a particular modal decomposition technique for the analysis of shallow flows

J.E. Higham, W. Brevis & C.J. Keylock



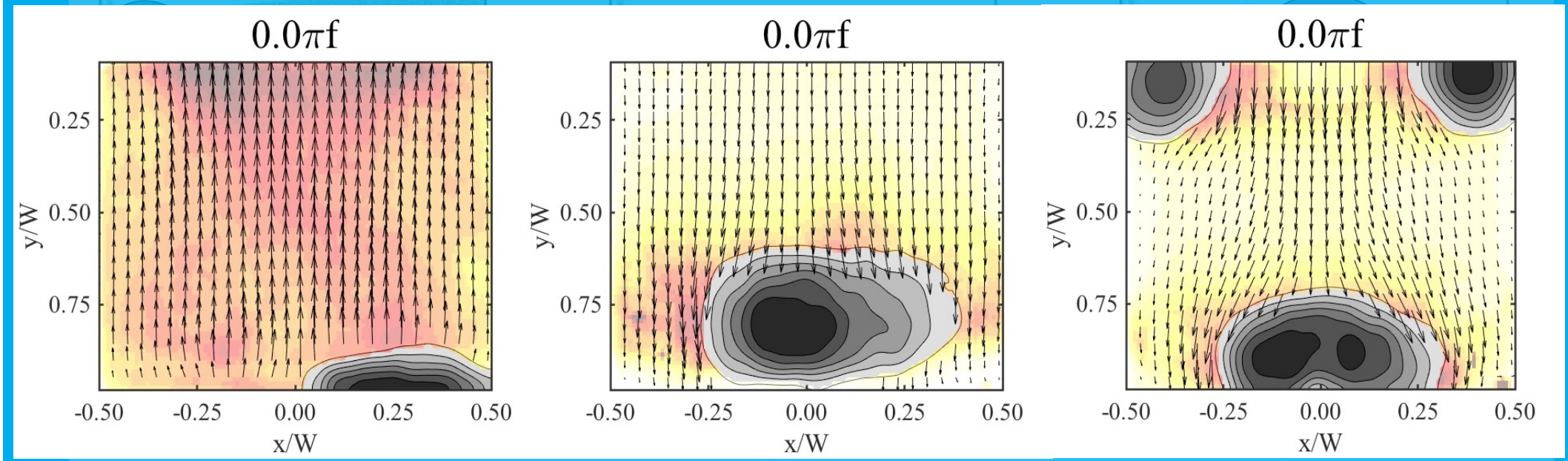
*all Mehrdad's idea..

EXPERIMENTAL WORK

liverpool.ac.uk | We are the original redbrick

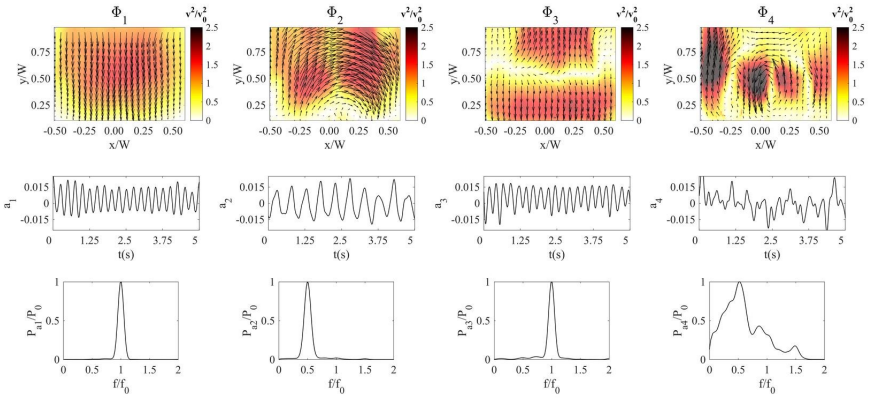
https://link.springer.com/article/10.1007/s10035-020-01037-7

PATTERNS

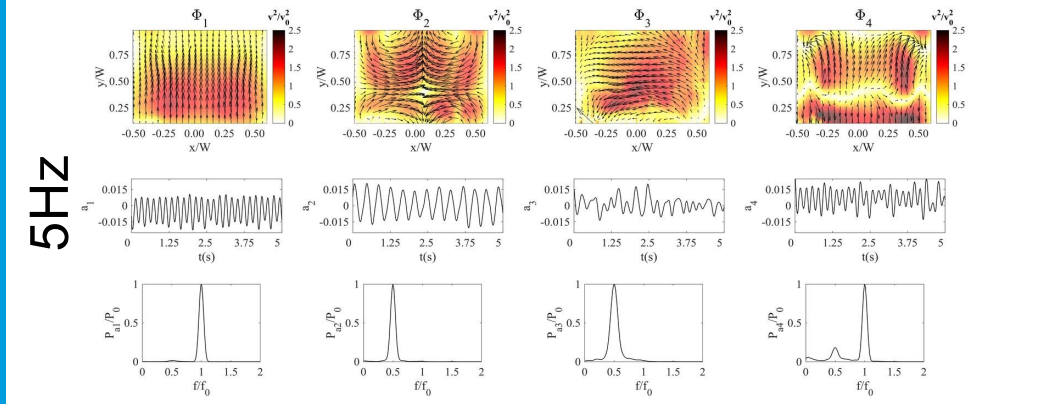

0.0πf
0.25
0.50
0.75
y/W
-0.50
-0.25
0.00
0.25
0.50
x/W
0.0πf
0.25
0.50
0.75
y/W
-0.50
-0.25
0.00
0.25
0.50
x/W
0.0πf
0.25
0.50
0.75
y/W
-0.50
-0.25
0.00
0.25
0.50
x/W

POD RESULTS

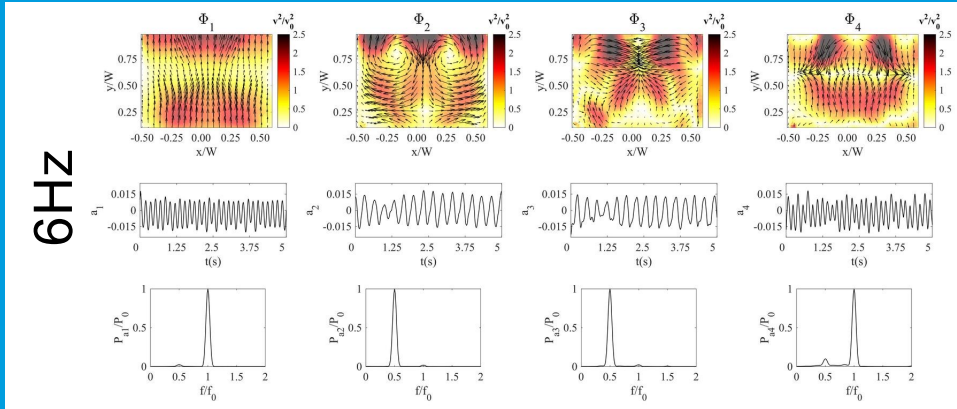
4Hz



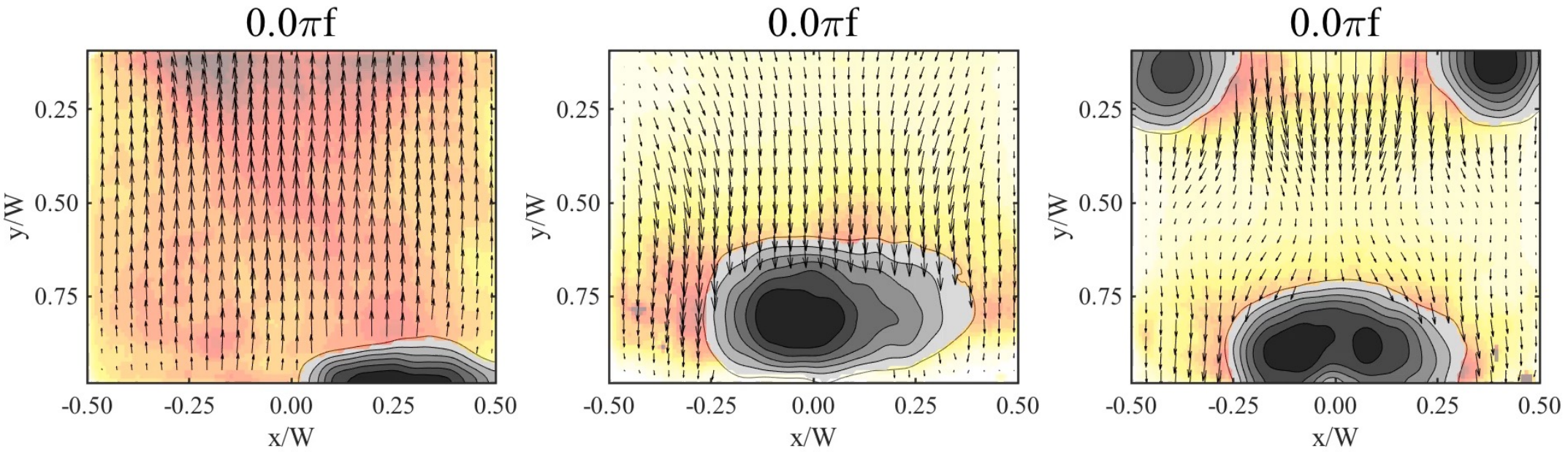
5Hz



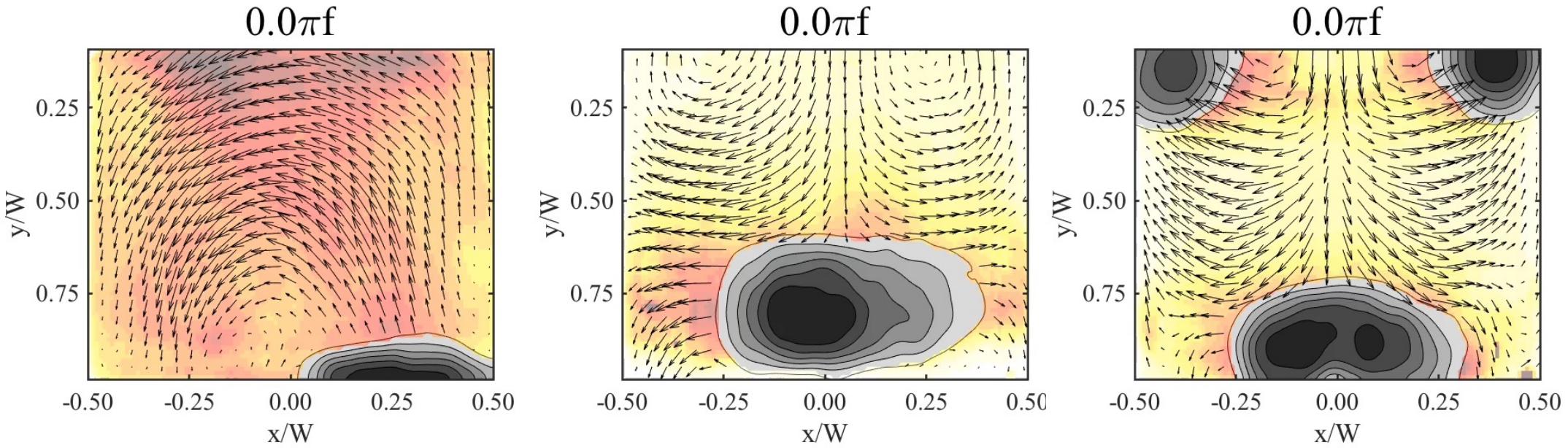
6Hz



POD: HARMONICS

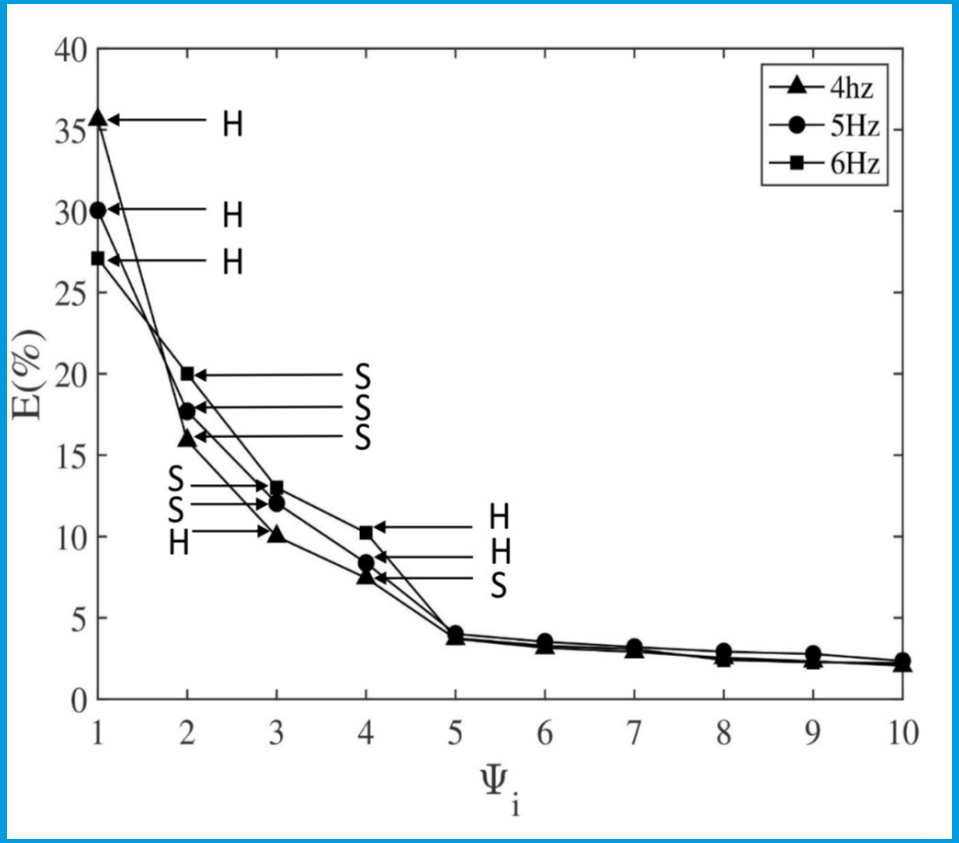


POD: HALF HARMONICS



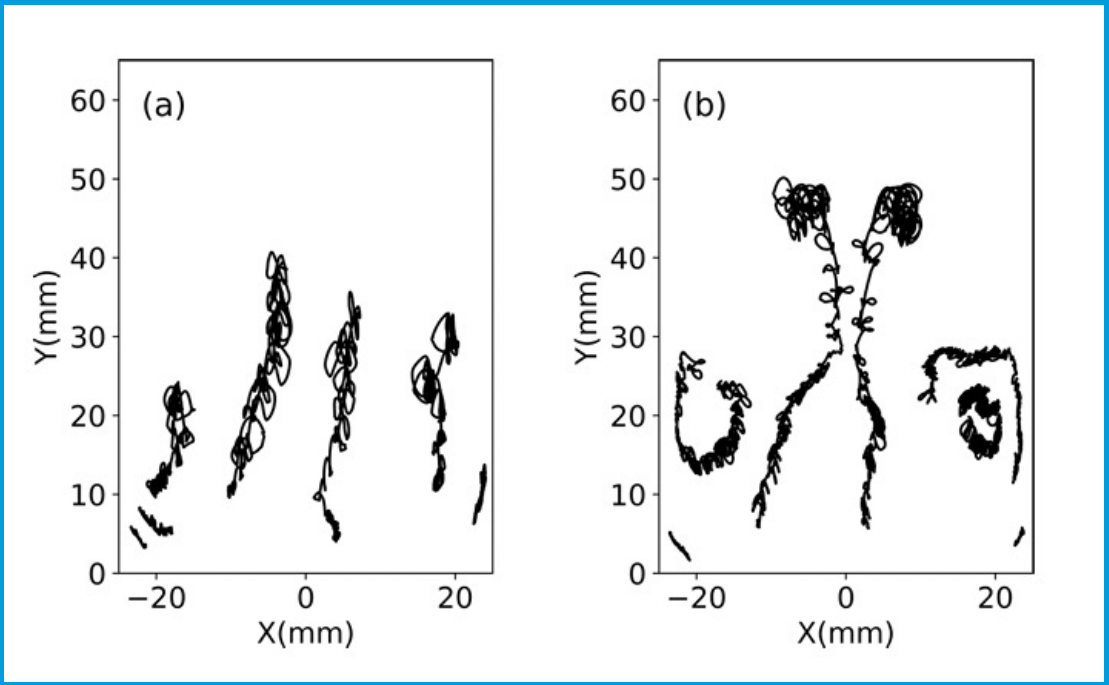
POD RESULTS

- Change in energy in harmonic (H) and sub-harmonic (S) modes with frequency.
- H: Transverse motion, S: Lateral motion
- Change in the bubble pattern and an improved mixing of solids as the frequency is increased (in the observed range)
- Possible suppression of chaos…?



CHAOTIC SYSTEM

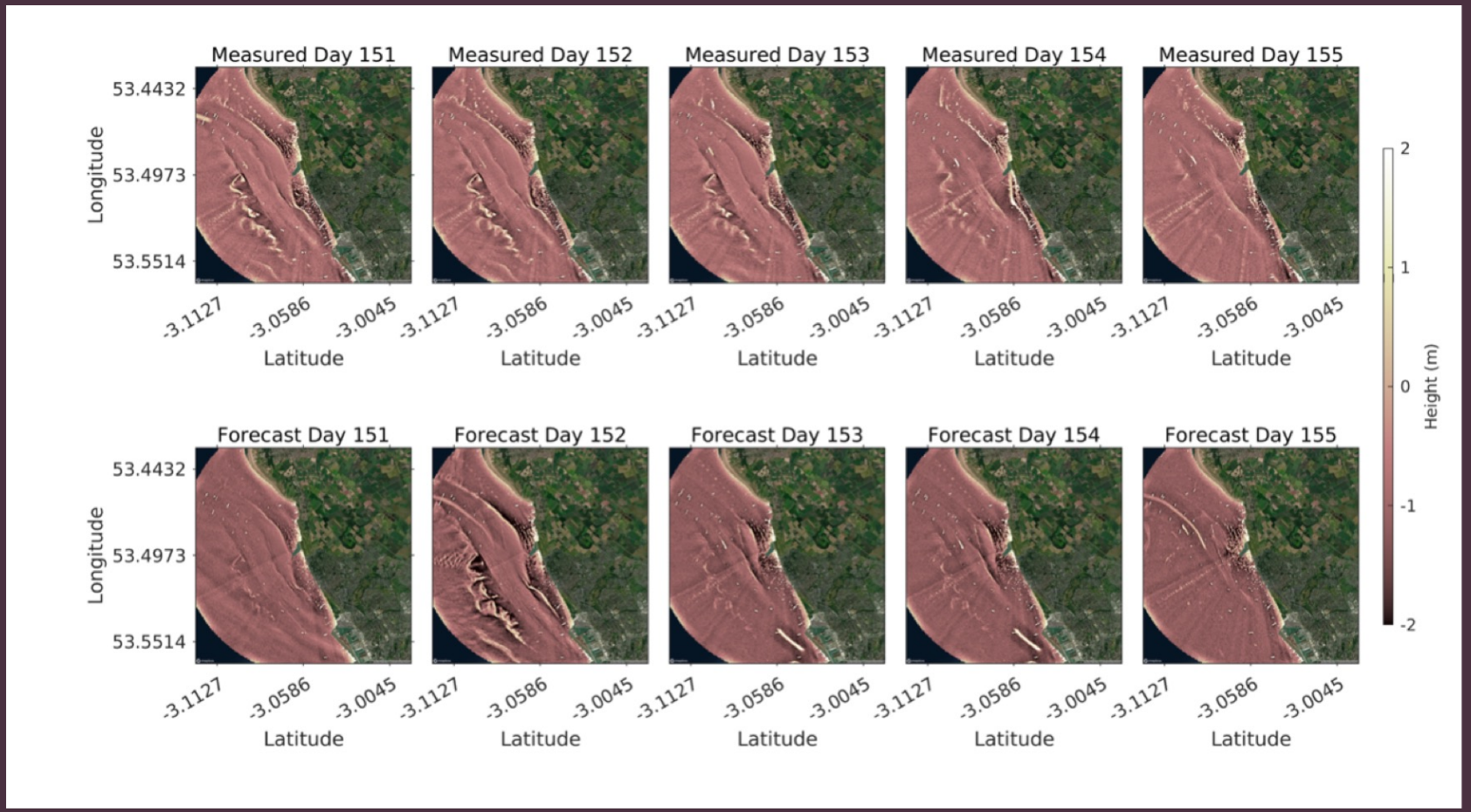
- We found in the system the way particles move in this system is unusual and not like regular diffusion.
- The different in frequency enhance the diffusion and mixing.
- The system showed some interesting behaviours like not following normal rules for particle motion and changing its behaviour over time.



https://journals.aps.org/pre/abstract/10.1103/PhysRevE.103.043103

CURRENT WORK

EIGENSHORES



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THANK YOU

We are the projects
that make headlines
And people that
make history

We are the original redbrick