Numerical study on fluidization of binary mixture of particles using SOM- KTGF-MP

(second-order moment of kinetic theory of granular flow for multi-type particles)

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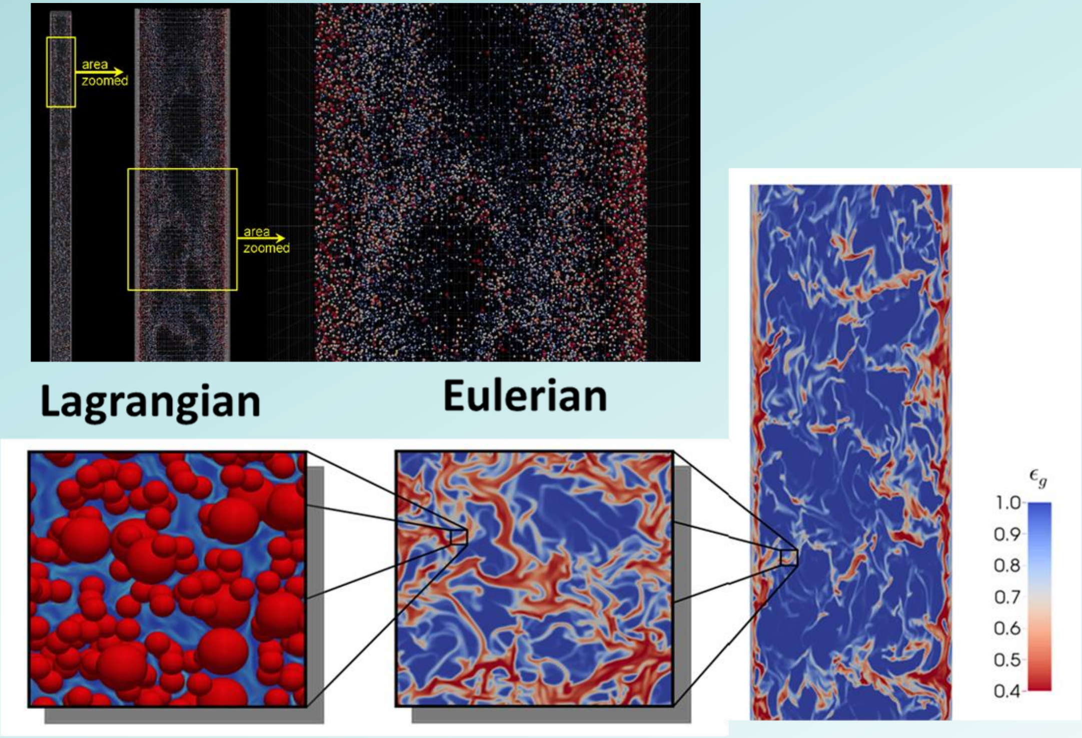
Kinetic theory of granular flow (KTGF)

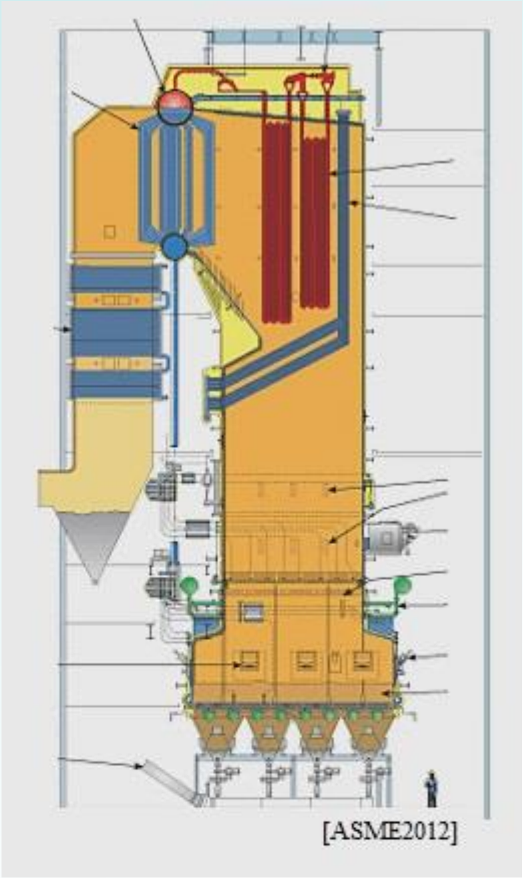
Numerical method for fluidization

- **Lagrangian (CFD-DEM …)**
- **Eulerian (TFM with KTGF)**



Particle distribution   Particle concentration

Large-scale simulation for fluidized bed using Eulerian method with KTGF



Challenge of Eulerian method for fluidization

Multi-scale structure of gas-particle flow in fluidized risers

Core–annulus pattern

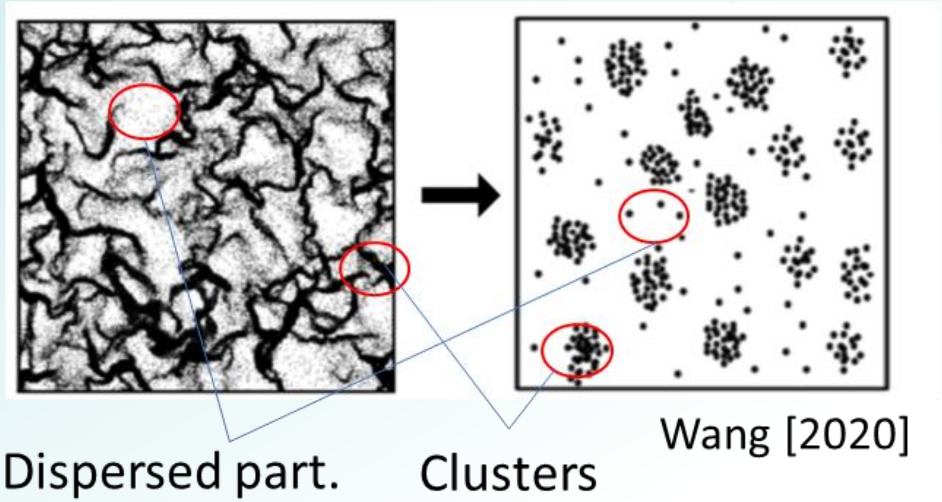
- dilute particles flow upward in the core of the reactor.
- dense particles flow downward close to the wall.

EMMS model for drag correction [Li 1987]

- multi-scale structure: dilute & dense
- drag force modelling for dilute & dense phases separately.

Multi-scale structure in risers [EMMS 1987]

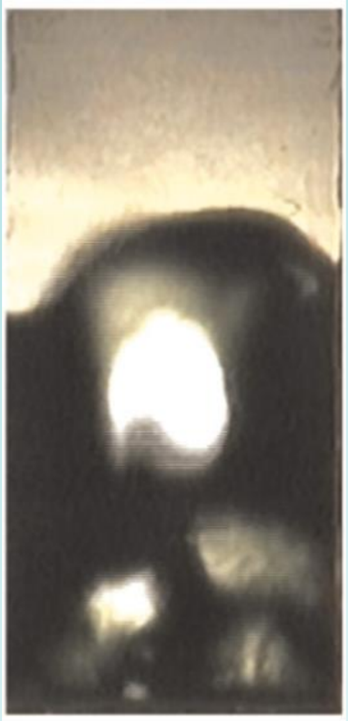
- Dense phase: particle clusters
- Dilute phase: dispersed particles



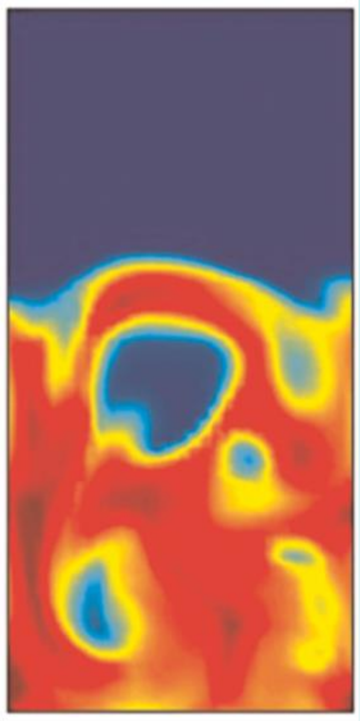
KTGF simulation for dense & dilute phase

Particles in dense phase concentration

well predicted by KTGF



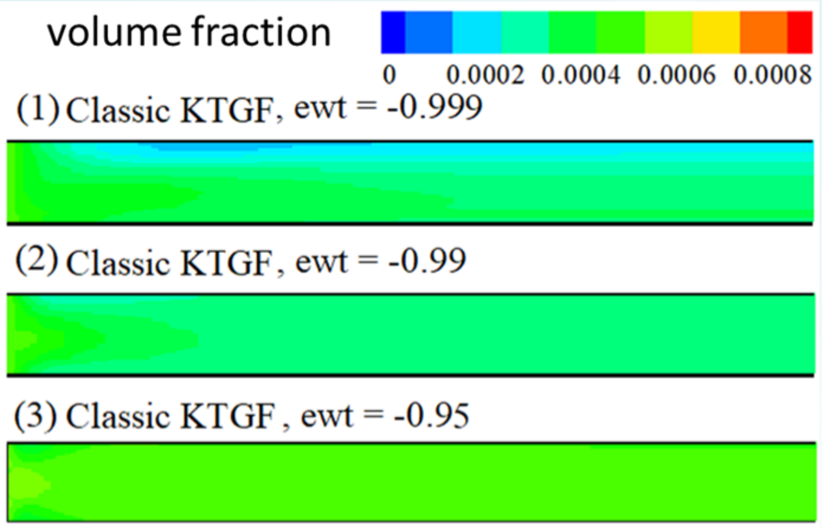
Exp.

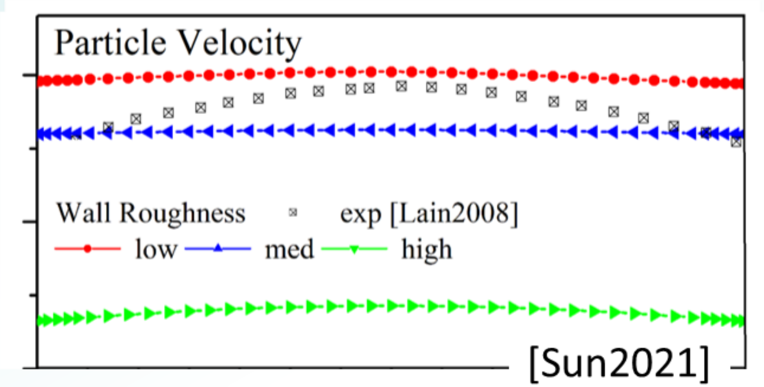


KTGF (Jung 2006)

Particles in dilute phase concentration

failed in predicting velocity gradient using classic KTGF

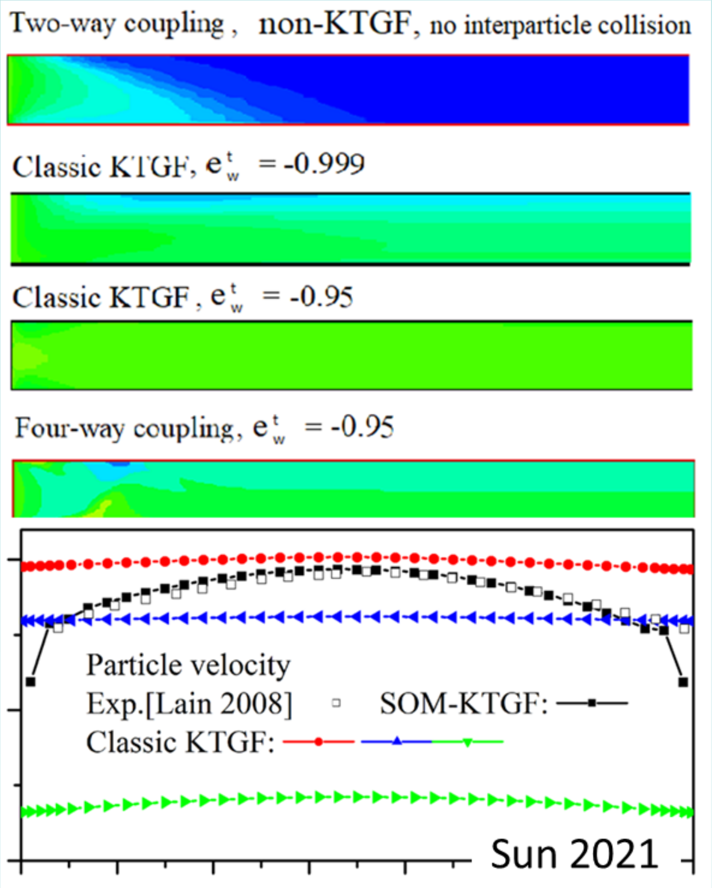




Second-order moment KTGF (SOM-KTGF)

Particles in dilute phase concentration

Velocity gradient were well predicted using SOM-KTGF



[Powder Tech. 390: 354-368]

SOM-KTGF model

Solve M_{ij} by PDE

$$\frac{\partial}{\partial t}(\varepsilon_s\rho_s M_{ij}) + \frac{\partial}{\partial x_k}(\varepsilon_s\rho_s u_{sk} M_{ij}) =$$

$$-\frac{\partial}{\partial x_k}(\varepsilon_s\rho_s A_s M_{kij}) - \frac{\partial}{\partial x_i}(\varepsilon_s\rho_s B_s M_{jkk}) - \frac{\partial}{\partial x_j}(\varepsilon_s\rho_s B_s M_{ikk})$$

$$-\frac{\partial}{\partial x_k}\left(\kappa^c \frac{\partial\Theta}{\partial x_k}\delta_{ij}\right) - \frac{\partial}{\partial x_i}\left(\kappa^c \frac{\partial\Theta}{\partial x_j}\right) - \frac{\partial}{\partial x_j}\left(\kappa^c \frac{\partial\Theta}{\partial x_i}\right)$$

$$-2\beta\left(M_{ij} - \overline{C_{gi}C_{sj}}\right) + \chi_{ij}$$

SOM-KTGF extends the capability of KTGF to predict particles from the dense to median concentration.

Conservation equation for particles, KTGF

Conservation of momentum in KTGF

$$\frac{\partial}{\partial t}(\varepsilon_s\rho_s u_{si}) + \frac{\partial}{\partial x_j}(\varepsilon_s\rho_s u_{si}u_{sj}) = -\frac{\partial}{\partial x_j}(\rho_s\varepsilon_s A_s M_{ij})$$

$$-\varepsilon_s\frac{\partial p_g}{\partial x_i} - \frac{\partial p_s^c}{\partial x_i} + \frac{\partial \tau_{sij}^c}{\partial x_j} + \beta u_{gs,i} + \varepsilon_s\rho_s g_i$$

Second-order moment of fluctuating velocity of particles,

$$M_{ij} = \begin{bmatrix} C_iC_i & C_iC_j & C_iC_k \\ C_iC_j & C_jC_j & C_jC_k \\ C_iC_k & C_jC_k & C_zC_z \end{bmatrix}$$

Fluctuating velocity of particles

$$C_i = u_i - \overline{u_i}$$

$$A_s = 1 + \frac{4}{5}(1+e)\varepsilon_s g_0$$

Classic-KTGF, solve M_{ij} by

Normal stress, ***i*** **=** ***j***

by Granular temperature, *θ*, *PDE*.

$$\langle M_{ii}\rangle = M_{ii} + M_{jj} + M_{kk} = 3\theta$$

Shear stress **,** ***i*** **≠** ***j***

by pseudo kinetic(eddy) viscosity

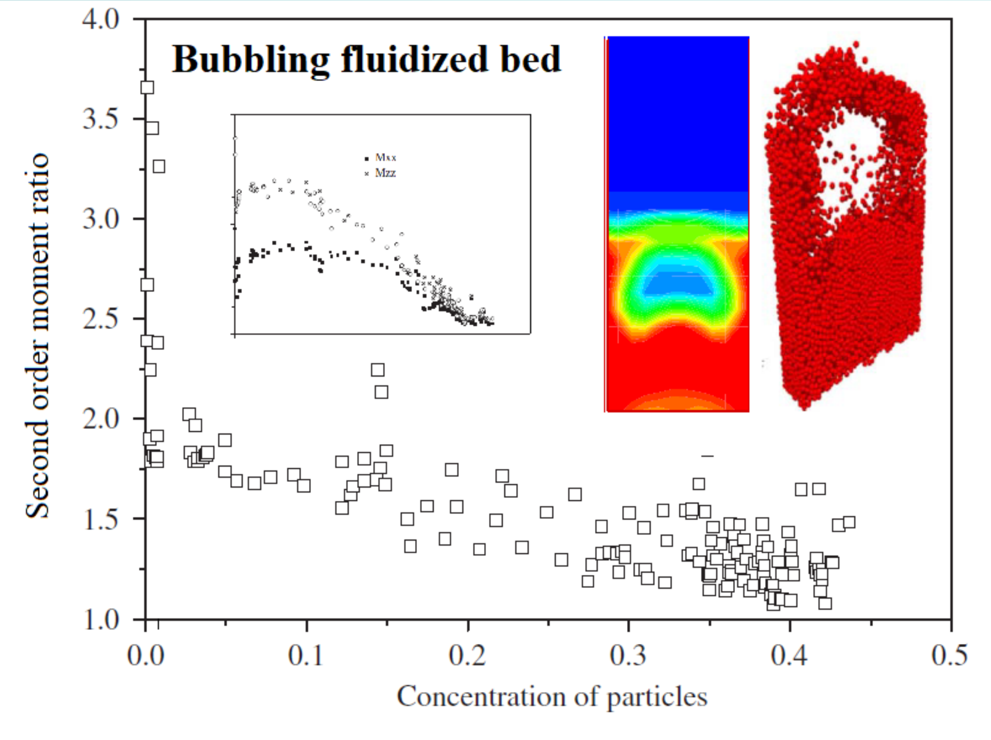
$$M_{ij} = \left[\frac{10\rho_s d_p\sqrt{\theta\pi}}{96(1+e)g_0}\right] \times \left\{\frac{1}{2}[\nabla u_s + (\nabla u_s)^T]\right\}$$

Boussinesq hypothesis

Equilibrium status can be achieved by frequent interparticle collisions in dense particle flow.

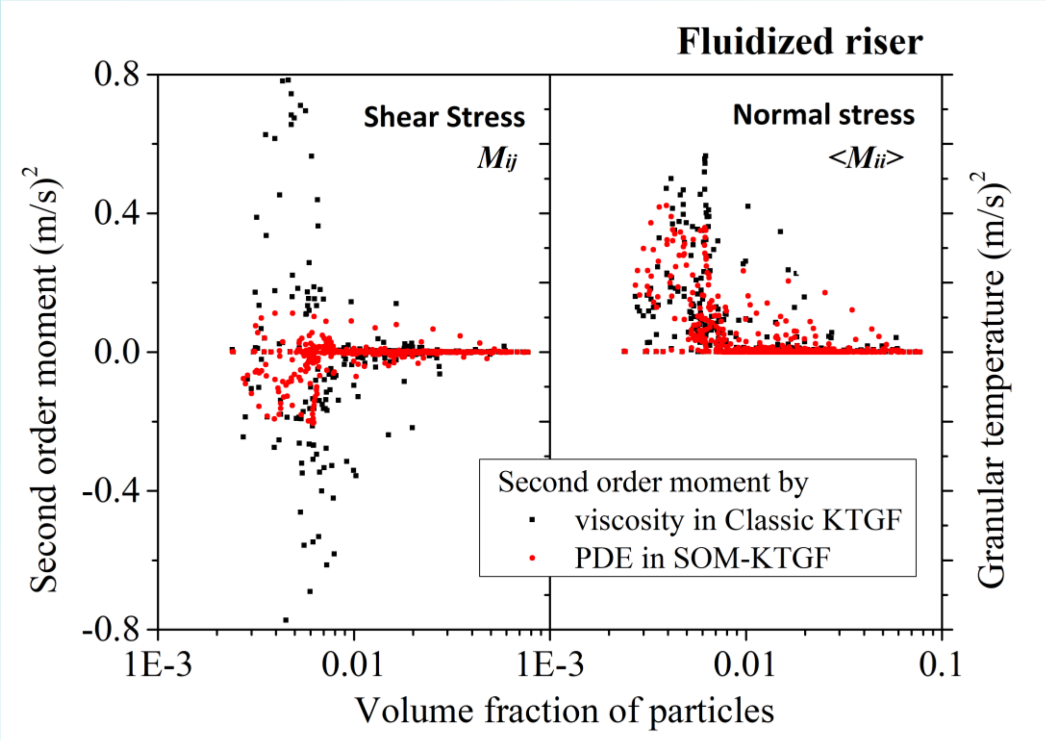
Prediction using classic KTGF & SOM-KTGF

Normal stress



Anisotropic ratio of normal stress > 1, high when volume fraction α_s< 0.01.

Shear stress

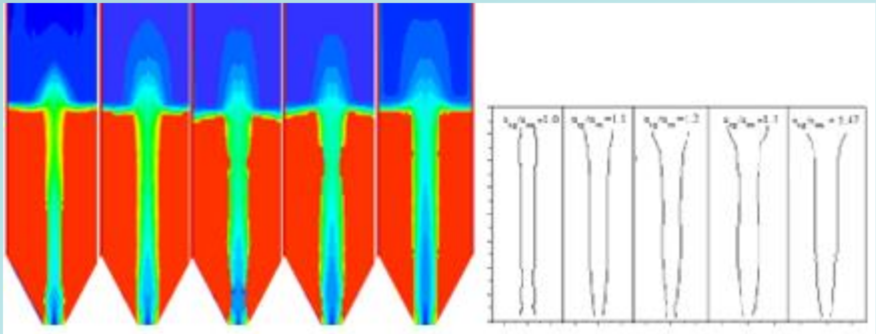


Shear stress by Classic KTGF higher than stress by SOM-KTGF, esp. when α_s < 0.01.

SOM-KTGF is essential for dilute-phase particles, when α_s < 0.01.

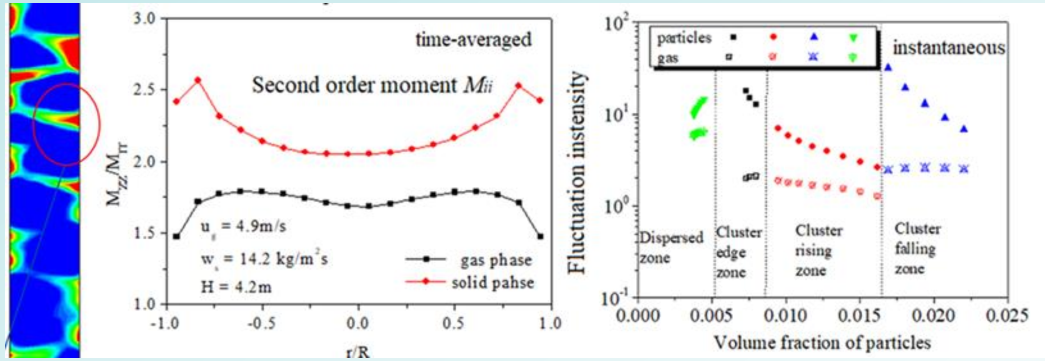
Simulation with SOM-KTGF(FW-SOM)

Spouted fluidized bed



- High fidelity prediction.
- Able to describe the interface between the spouted and annulus zone.
- Volume fraction of particles across dense to dilute regions.

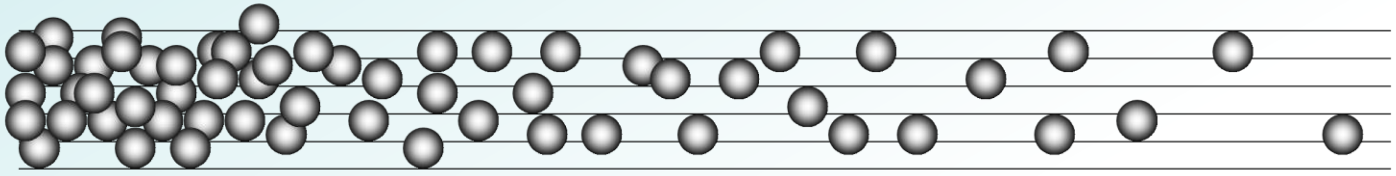
Fluidized riser



- Four-way coupled SOM (FW-SOM)
- Increasing with the particle volume fraction, anisotropic ratios of turbulent intensity,
  - increase in dilute phase
  - reduce in dense phase.

<u>SOM-KTGF well predicts both dense & dilute phases, from dense to median concentrations.</u>

Dense concentration inter-particle collision

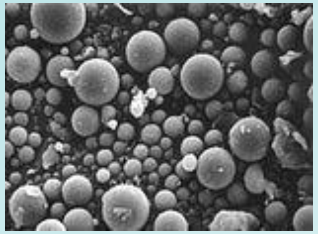


Dilute concentration Non-equilibrium

SOM-KTGF for Multi-type Particles

Conservation PDE

Continuity equation



$$\frac{\partial}{\partial t}(\varepsilon_m \rho_m) + \frac{\partial}{\partial x_i}(\varepsilon_m \rho_m u_{m,i}) = 0$$

Momentum equation

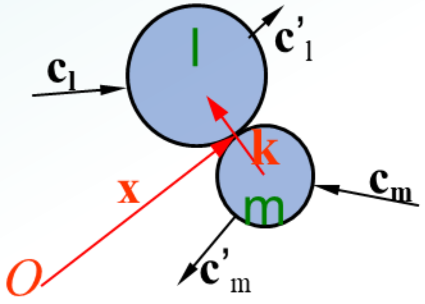
$$\frac{\partial}{\partial t}(\alpha_m \rho_m u_{mi}) + \frac{\partial}{\partial x_j}(\alpha_m \rho_m u_{m,i} u_{m,j}) = \sum_{l=1,m} \left[\chi(m_m C_{m,i})_l\right] - \frac{\partial}{\partial x_j}\left(\alpha_m \rho_m M_{m,ij} + \sum_{l=1,m} \left[\psi(m_m C_{m,i})_l\right]\right) + F_{m,i} - \alpha_m \frac{\partial p_g}{\partial x_i}$$

Second-order moment conservation

$$\frac{\partial}{\partial t}(\alpha_m \rho_m M_{m,ij}) + \frac{\partial}{\partial x_k}(\alpha_m \rho_m u_{m,k} M_{m,ij}) = +\Pi_{d,m,ij} + \sum_{l=1}^{M} \left[\chi(m_m C_{m,i} C_{m,j})_l\right] - \frac{\partial}{\partial x_k}\left(\alpha_m \rho_m M_{m,kij} + \sum_{l=1}^{M} \left[\psi_k(m_m C_{m,i} C_{m,j})_l\right]\right) - \left\{\left[\alpha_m \rho_m M_{m,ik} + \sum_{l=1}^{M} \left[\psi_k(m_m C_{m,i})_l\right]\right] \frac{\partial u_{m,j}}{\partial x_k} + \left[\alpha_m \rho_m M_{m,jk} + \sum_{l=1}^{M} \left[\psi_k(m_m C_{m,j})_l\right]\right] \frac{\partial u_{m,i}}{\partial x_k}\right\}$$

SOM-KTGF-MP

- Derived based on kinetic theory with interparticle collisions.
- Momentum exchange between particles and multi-type particles, $\chi(m_m C_{m,i})_l$.
- Energy/Stress exchange between particles and multi-type particles, $\chi(m_m C_{m,i} C_{m,j})_l$.



SOM-KTGF-MP and classic KTGF-MP

Flux due to collisions in momentum equations (pressure tensor)

SOM-KTGF-MP

$$\psi_j\left(mC_{m,i}\right)_l = \frac{\pi d_{ml}^3}{6}\frac{m_l m_m}{m_0}(1+e_{ml})g_{ml}n_l n_m\left(\frac{m_l m_m}{4\Theta_l\Theta_m}\right)^{\frac{3}{2}} \times \left\{ \begin{array}{l} R_0\delta_{ij} \\ +\frac{1}{5}\frac{m_l^2 m_m^2}{m_0^2}\left(\frac{a_{l,ij}^{(2)}}{\Theta_m^2}+\frac{a_{m,ij}^{(2)}}{\Theta_l^2}\right)R_3 \\ -\frac{m_l m_m}{m_0}\frac{d_{ml}}{5}\frac{1}{\sqrt{\pi}}R_1\left[\begin{array}{l}\frac{1}{\Theta_l}\left(\frac{\partial u_{l,k}}{\partial x_k}+\frac{\partial u_{l,i}}{\partial x_j}+\frac{\partial u_{l,j}}{\partial x_i}\right)+ \\ \frac{1}{\Theta_m}\left(\frac{\partial u_{m,k}}{\partial x_k}+\frac{\partial u_{m,i}}{\partial x_j}+\frac{\partial u_{m,j}}{\partial x_i}\right)\end{array}\right] \end{array} \right\}$$

Classic KTGF-MP (Iddir & Arastoopour [2005])

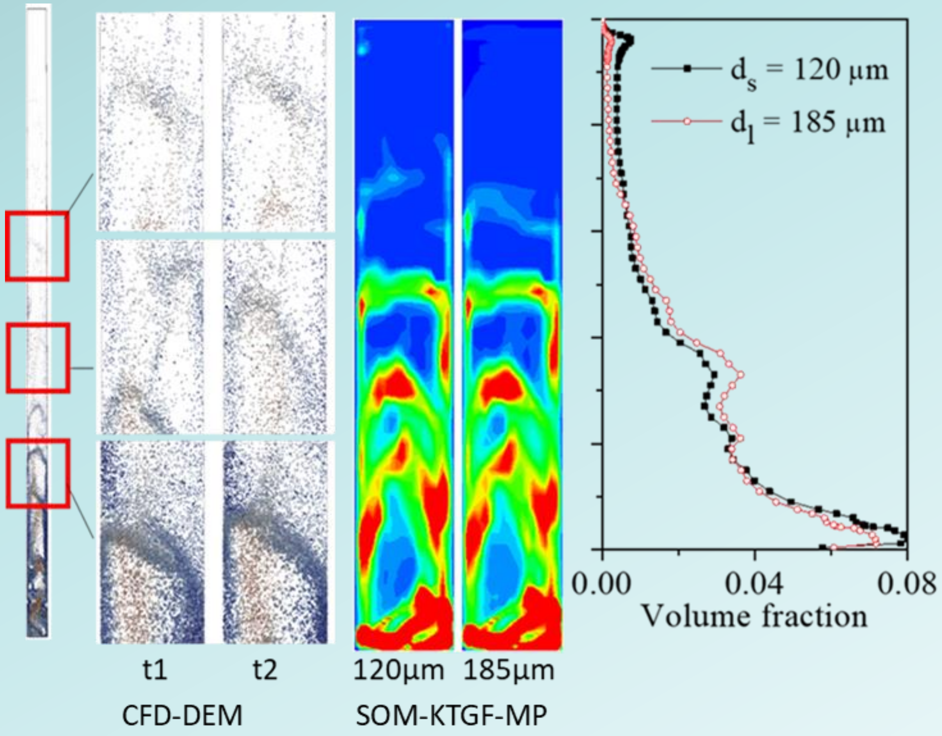
$$\psi\left(mC_{m,i}\right)_l = \frac{d_{ml}^3}{48}\frac{m_l m_m}{m_0}(1+e_{ml})g_{ml}n_l n_m\left(\frac{m_l m_m}{\Theta_l\Theta_m}\right)^{\frac{3}{2}} \times \left\{\pi R_0\bar{I} - \frac{\sqrt{\pi}}{3}d_{ml}\frac{m_l m_m}{m_0}R_1\left[\begin{array}{l}\frac{6}{5}\left(\frac{\nabla^s\overrightarrow{v_m}}{\Theta_m}+\frac{\nabla^s\overrightarrow{v_l}}{\Theta_l}\right) \\ +\left(\frac{\nabla\cdot\overrightarrow{v_m}}{\Theta_m}+\frac{\nabla\cdot\overrightarrow{v_l}}{\Theta_l}\right)\bar{I}\end{array}\right]\right\}$$

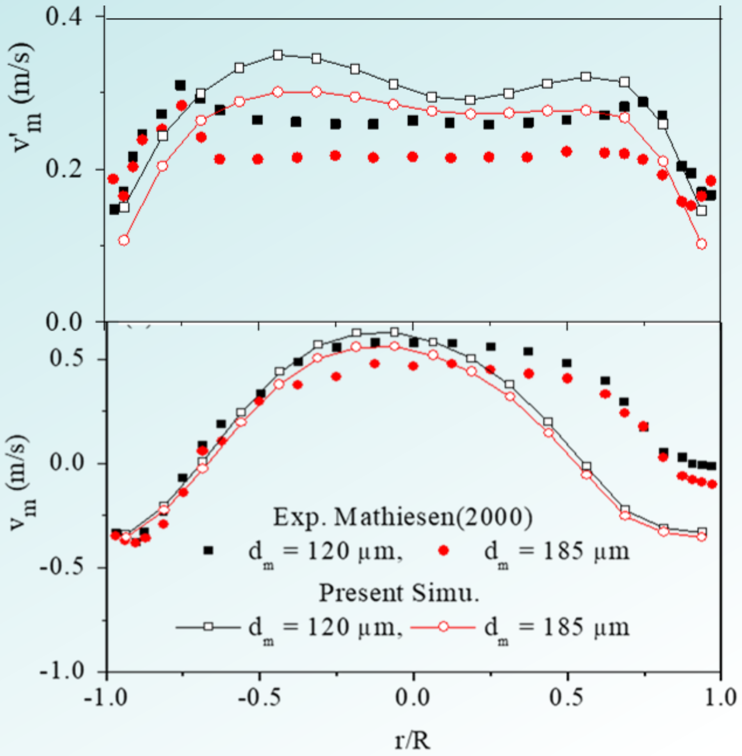
SOM-KTGF ($l = m$)

$$\psi_j\left(m_m C_{m,i}\right)_m = 2\rho_m\alpha_m^2(1+e_m)g_0\Theta_m\delta_{ij} - \frac{4}{5}\rho_m\alpha_m^2\sigma_m(1+e_m)g_0\sqrt{\frac{\Theta_m}{\pi}}\left[\begin{array}{l}\left(\frac{\partial u_{mi}}{\partial x_j}+\frac{\partial u_{mj}}{\partial x_i}\right) \\ +\frac{\partial u_{mk}}{\partial x_k}\delta_{ij}\end{array}\right] + \frac{4}{5}\rho_m\alpha_m^2(1+e_m)g_0 a_{m,ij}^{(2)}$$

SOM-KTGF-MP for fluidization

Fluidized riser with two sizes of particles.



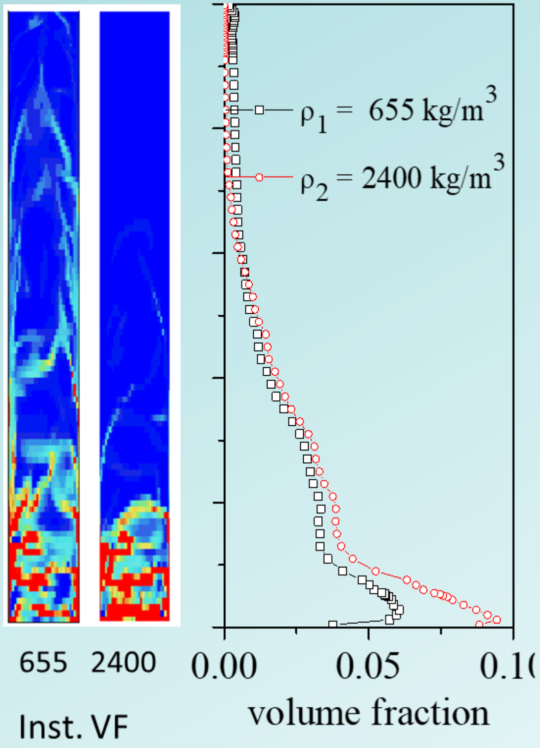


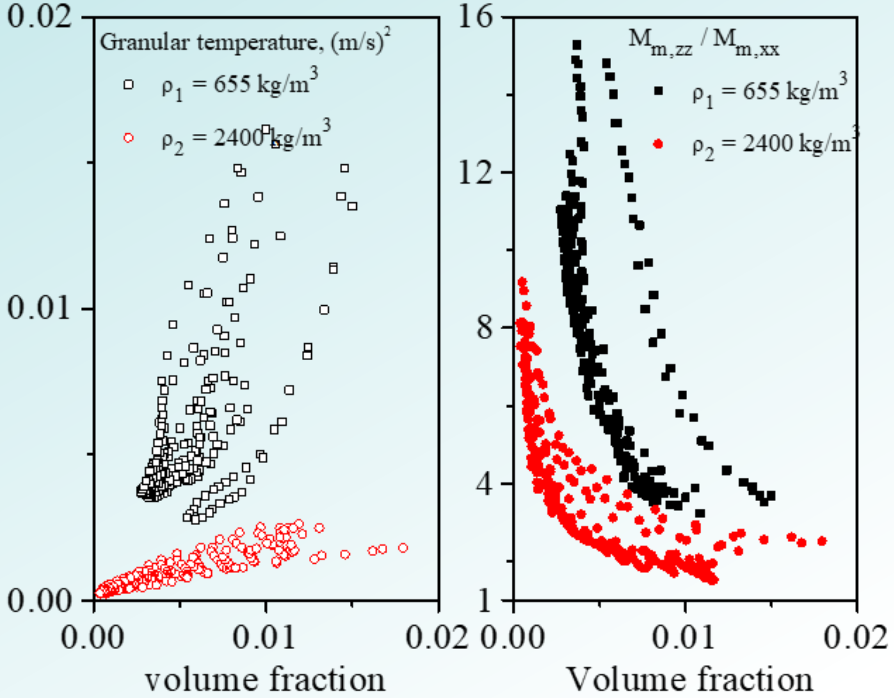
- Binary particles moving together.
- d_p = 120 µm, 185 µm.

- Simulation comparison with experimental data.
- Velocities of binary particles were close.

SOM-KTGF-MP for fluidization

Fluidized riser with two types (density) of particles.





- More high-density particles at bottom.
- ρ_p = 655 kg/m^3, 2400 kg/m^3.

- Granular temp. higher for low-density particles.
- Anisotropic turbulence high when $\alpha_s < 0.01$.

Conclusion

- SOM-KTGF, as an Eulerian method, extends the capability of KTGF to predict particles from the dense to median concentrations.
- SOM-KTGF (FW-SOM) can well predict both the dense and dilute phases in fluidization with multi-scale structures.
- SOM-KTGF is essential in the dilute phase of particles in fluidization,
    - when the volume fraction of particles is less than 0.01,
    - where exist particle-particle collisions, but less frequent to be equilibrium and unable to satisfy the Boussinesq hypothesis.
- SOM-KTGF-MP was proposed to predict particle flows with various types of particles with different diameters and densities, in the Eulerian framework.
- The simulations of two-size and two-type particles in fluidized risers demonstrated the capability of SOM-KTGF-MP.

Acknowledge

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