A kinetic-based model for incompressible, polydisperse, fluid-particle flows

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Outline

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Background

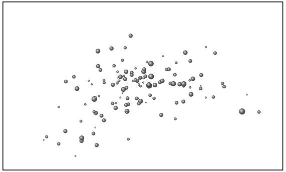
- Modelling of particle-laden flows in industrial systems
- Computational expense of simulations is prohibitive due to number of droplets required
- **Euler-Euler models** provide a continuum description for a suspension of droplets
- **Quadrature method of moments** (QMOM) models the particle phase at a **mesoscopic level** by using a kinetic-based approach for closure of the flux and source terms
- Able to include a variety of important physical effects in dense suspensions such as **collisions**, **heat transfer**, and **added mass**
- Offers a **detailed and accurate** means of modelling industrial particle-laden flow systems

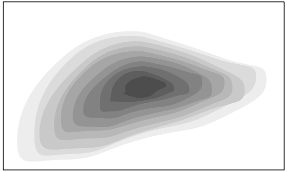

New Vista Research

[1] Marchisio D. *et al.*; AICHE J. (2003)

Overview of QMOM development

Marchisio *et al.* 2003	Procedure first applied to population balance equations
Fox 2008	Direct QMOM applied to dilute gas-particle flows
Yuan & Fox 2011	Conditional QMOM (CQMOM) developed using 1-D adaptive quadrature of conditional velocity moments
Kong & Fox 2020	Application of QMOM to polydisperse gas-particle flows by reconstructing the particle size distribution
Fox & Laurent 2022	Hyperbolic QMOM (HyQMOM) developed to ensure globally hyperbolic moment closure
Boniou *et al.* 2023	Application to high-speed fluid particle flows
Fox *et al.* 2023	Generalised QMOM (GQMOM) developed allowing for an arbitrary number of quadrature nodes
Fox *et al.* 2024	Extension to polydisperse high-speed fluid-particle flows





Eulerian description of the particle phase

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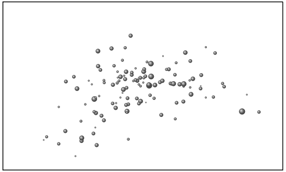

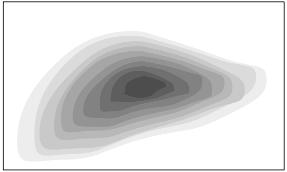

Eulerian description of the particle phase

... application to polydisperse incompressible fluid-particle flows

Kinetic model for polydisperse particles

Generalised population balance equation for mass-velocity-internal-energy NDF[2]

$$
\begin{aligned}
&\partial_t f + \partial_{\boldsymbol{x}} \cdot \left(\boldsymbol{u} f - P_p \frac{\partial f}{\partial \boldsymbol{u}} \right) \\
&\quad + \frac{\partial}{\partial \boldsymbol{u}} \cdot \left[\frac{1}{\tau_p(\xi)} (\boldsymbol{u}_f - \boldsymbol{u}) f - \frac{1}{\rho_e} (\partial_{\boldsymbol{x}} \hat{p}_f + \boldsymbol{F}_{pf}) f - \frac{1}{\rho_e \alpha_p^\star} (\partial_{\boldsymbol{x}} \cdot \boldsymbol{P}_{pfp}) f \right] \\
&\quad + \frac{\partial}{\partial e} [A_e(\xi) f] = \frac{\partial^2}{\partial \boldsymbol{u} \partial \boldsymbol{u}} : [\boldsymbol{B}_u(\xi) f] + C + F + S
\end{aligned}
$$

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Generalised population balance equation for mass-velocity-internal-energy NDF[2]

$$\partial_t f + \partial_{\boldsymbol{x}} \cdot \left(\boldsymbol{u} f - P_p \frac{\partial f}{\partial \boldsymbol{u}} \right) + \frac{\partial}{\partial \boldsymbol{u}} \cdot \left[\frac{1}{\tau_p(\xi)} (\boldsymbol{u}_f - \boldsymbol{u}) f - \frac{1}{\rho_e} (\partial_{\boldsymbol{x}} \hat{p}_f + \boldsymbol{F}_{pf}) f - \frac{1}{\rho_e \alpha_p^\star} (\partial_{\boldsymbol{x}} \cdot \boldsymbol{P}_{pfp}) f \right] + \frac{\partial}{\partial e} [A_e(\xi) f] = \frac{\partial^2}{\partial \boldsymbol{u} \partial \boldsymbol{u}} : [\boldsymbol{B}_u(\xi) f] + C + F + S$$

- Spatial free transport and particle pressure
- Surface forces: fluid drag, buoyancy, pfp-pressure
- Source terms correspond to collisions, friction, and added mass

[2] Marchisio D.L. & Fox R.O.; CUP (2013)

Outline

1D model equations

Derived for 4 mass-conditioned velocities and 6 half-order mass moments[3]

24 conserved variables

- Fluid: $\rho_f \alpha_f^\star$, $\rho_f \alpha_f^\star u_f$, $\rho_f \alpha_f^\star k_f$, $\rho_f \alpha_f^\star E_f$
- Particle volume fraction: α_p
- Half-order mass moments: $\mathcal{M}_0$, $\mathcal{M}_{1/2}$, $\mathcal{M}_1$, $\mathcal{M}_{3/2}$, $\mathcal{M}_2$, $\mathcal{M}_{5/2}$, $\mathcal{M}_3$
- Mass-weighted velocities: $\mathcal{U}_0^1$, $\mathcal{U}_1^1$, $\mathcal{U}_2^1$, $\mathcal{U}_3^1$
- Kinetic energies: $\mathcal{K}_0$, $\mathcal{K}_1$, $\mathcal{K}_2$, $\mathcal{K}_3$
- Mass-internal-energy moments: $\mathcal{E}_0^e$, $\mathcal{E}_1^e$, $\mathcal{E}_2^e$, $\mathcal{E}_3^e$

[3] Fox R.O. *et al.*; IJMF (2024)

Incompressible fluid phase

Barotropic assumption

- Treat fluid as **weakly compressible** to retain density-based solver formulation
- Fluid pressure defined by barotropic equation of state:

$$p_f = \rho_f \Theta_f$$

- $\Theta_f \gg u_f^2$ is constant
- Density nearly constant when Mach number $Ma = u_f / \sqrt{\Theta_f} \approx 0.01$
- Estimate u_f using slip velocity due to gravity
- Fluid phase eigenvalues $u_f \pm \sqrt{\Theta_f}$ control the time step through CFL condition

Particle pressure terms

Frictional pressure

- Assumed the same as in the monodisperse case[4]

$$P_{fr} = C_{fr}\alpha_p^n \left(1 + n\alpha_f\right) h_{fr}(\alpha_p)$$

[4] Boniou V. *et al.* (2023)

[5] Fox R.O. *et al.*; IJMF (2024)

[6] Santos A. *et al.* *et al.*; Mol. Phys. (1999)

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Collisional pressure

- Employed as polydisperse model[5]

$$P_c = \frac{\beta}{\rho_p}\int \xi p_c(\xi) n(\xi)\, d\xi$$

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Radial distribution function

- Polydisperse expression used[6]

$$g(\xi,\zeta) = \frac{1}{\alpha_f} + \left(g_0(\alpha_f) - \frac{1}{\alpha_f}\right)\frac{\langle d_p^2\rangle}{\langle d_p^3\rangle}\frac{2d_p(\xi)d_p(\zeta)}{d_p(\xi)+d_p(\zeta)}$$

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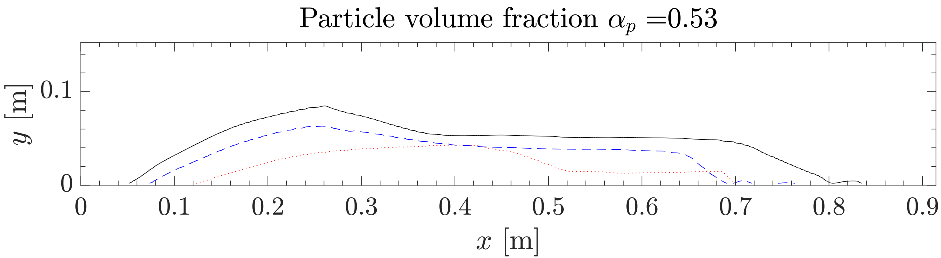
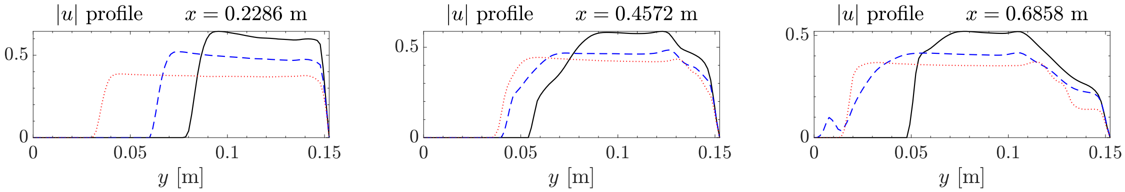
Dune contours



- Behaviour of different particle sizes within sedimentary flow of sand immersed in water
- Contours of the critical particle volume fraction for —— $d_p = 297$ micron, - - - - $d_p = 250$ micron, · · · · · · $d_p = 210$ micron.

Dune velocity profiles



- Behaviour of different particle sizes within sedimentary flow of sand immersed in water
- Cross-sectional profiles of particle velocity magnitude at locations: $x = 0.2286$ m; $x = 0.4572$ m; $x = 0.6858$ m; for —— $d_p = 297$ micron, - - - - $d_p = 250$ micron, · · · · · · $d_p = 210$ micron.

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Summary

Conclusions

- **Density-based solver** for **polydisperse liquid-particle flows**
- **Barotropic equation of state** used to specify fluid pressure for a **weakly compressible flow**
- **Frictional and collisional pressures** are important in capturing the transition from a dense particle suspension to granular flow

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- **Density-based solver** for **polydisperse liquid-particle flows**
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Outlook

- Improve numerical flux splitting schemes to limit diffusion
- Replace correlations for frictional and collisional pressures with models from granular rheology

Questions

Thank you for your attention

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CoMFRE Center for Multiphase Flow Research and Education

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